Computer Algebra Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.5-Secant/117-4.5.11-e-x-^m-a+b-sec-c+d-x^n-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [83]. This is test number [117].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (83)	0.00 (0)
Mathematica	95.18 (79)	4.82 (4)
Fricas	75.90 (63)	24.10 (20)
Maple	61.45 (51)	38.55 (32)
Maxima	57.83 (48)	42.17 (35)
Mupad	56.63 (47)	43.37 (36)
Giac	51.81 (43)	48.19 (40)
Sympy	44.58 (37)	55.42 (46)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

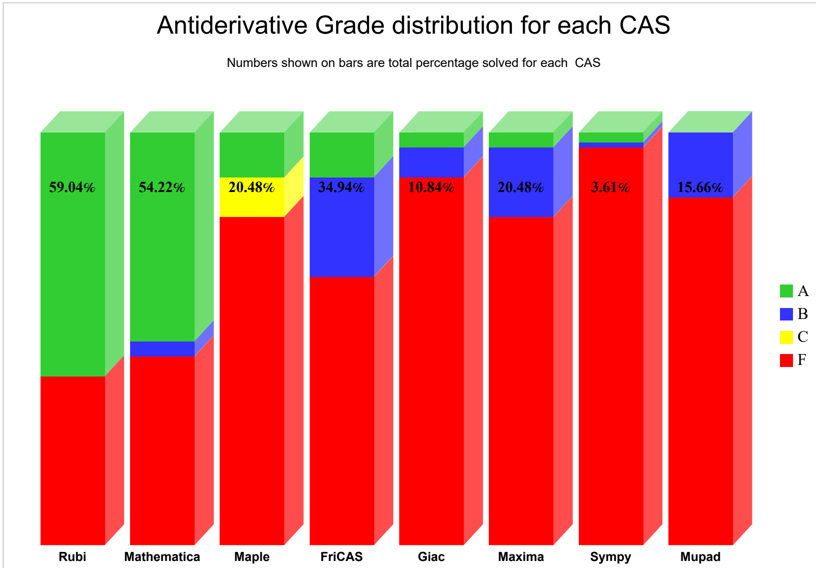
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

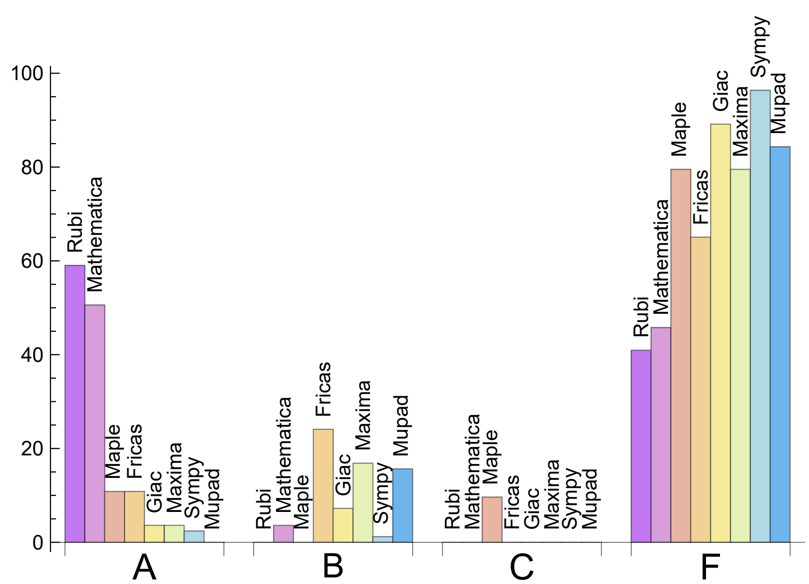
System	% A grade	% B grade	% C grade	% F grade
Rubi	59.036	0.000	0.000	40.964
Mathematica	50.602	3.614	0.000	45.783
Maple	10.843	0.000	9.639	79.518
Fricas	10.843	24.096	0.000	65.060
Giac	3.614	7.229	0.000	89.157
Maxima	3.614	16.867	0.000	79.518
Sympy	2.410	1.205	0.000	96.386
Mupad	0.000	15.663	0.000	84.337

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Fricas	20	100.00	0.00	0.00
Maple	32	100.00	0.00	0.00
Maxima	35	54.29	8.57	37.14
Mupad	36	0.00	100.00	0.00
Giac	40	100.00	0.00	0.00
Sympy	46	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.31
Giac	0.52
Rubi	0.58
Maple	0.62
Maxima	2.74
Sympy	4.98
Mathematica	10.88
Mupad	13.47

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	21.19	1.01	17.00	0.94
Giac	45.81	1.29	20.00	1.11
Mupad	74.91	1.58	22.00	1.22
Maple	164.18	1.44	18.00	1.00
Rubi	310.49	1.00	79.00	1.00
Mathematica	348.47	1.14	54.00	1.10
Fricas	411.59	2.29	44.00	2.11
Maxima	1447.38	38.13	287.50	7.92

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

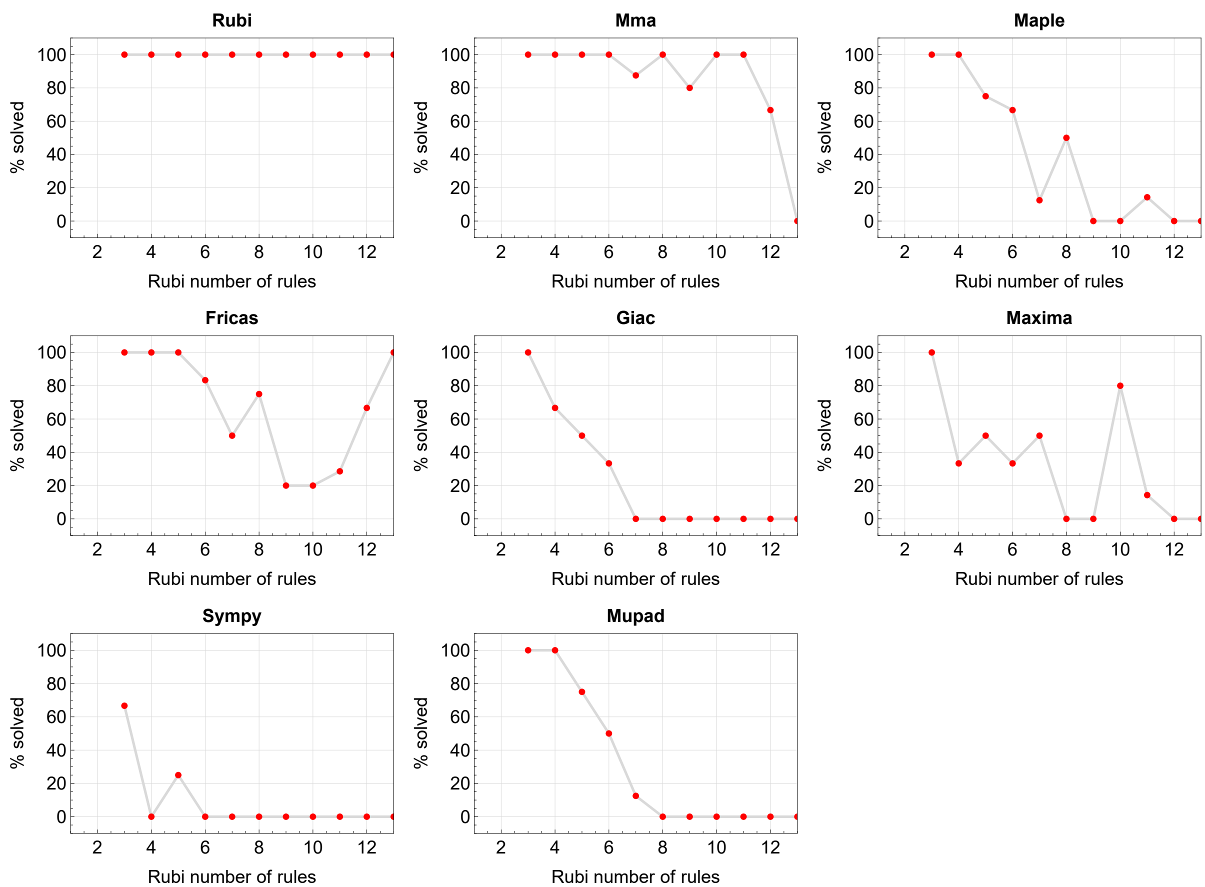


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

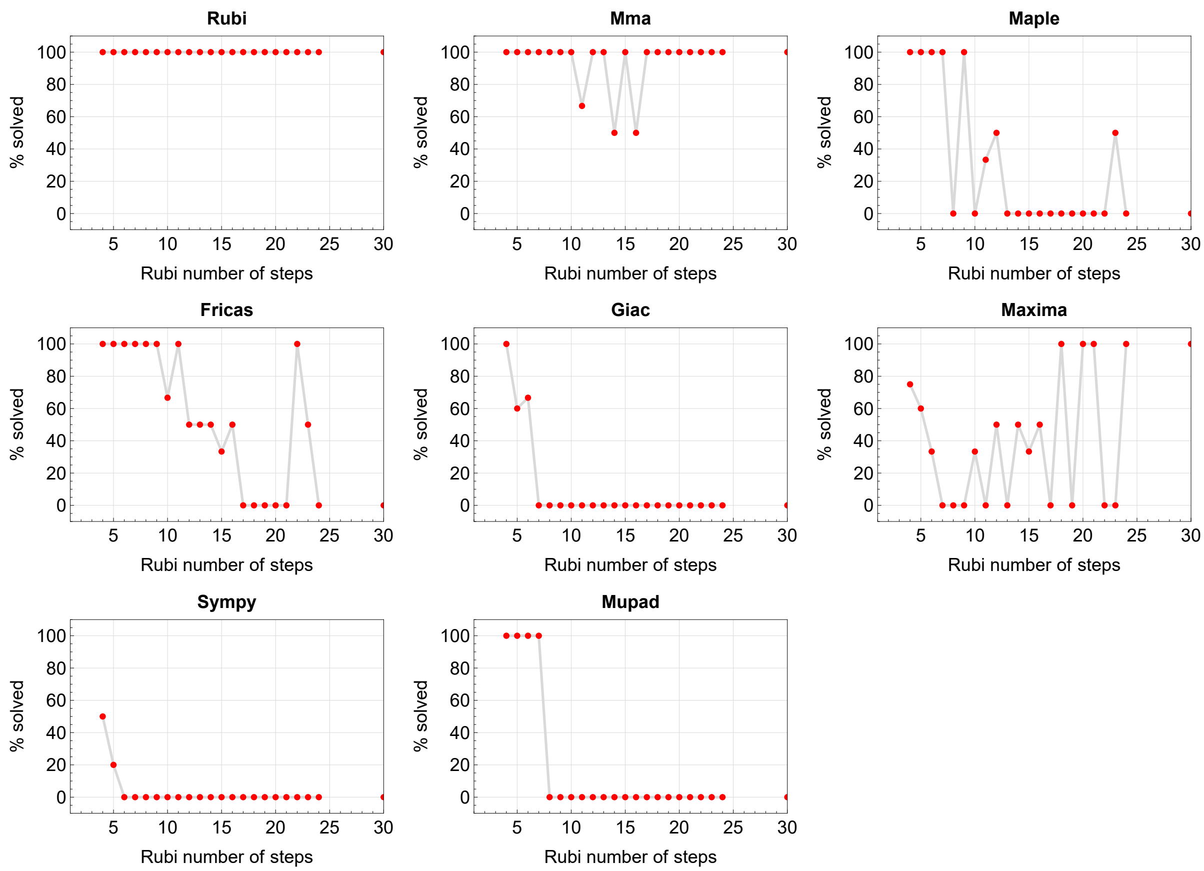


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

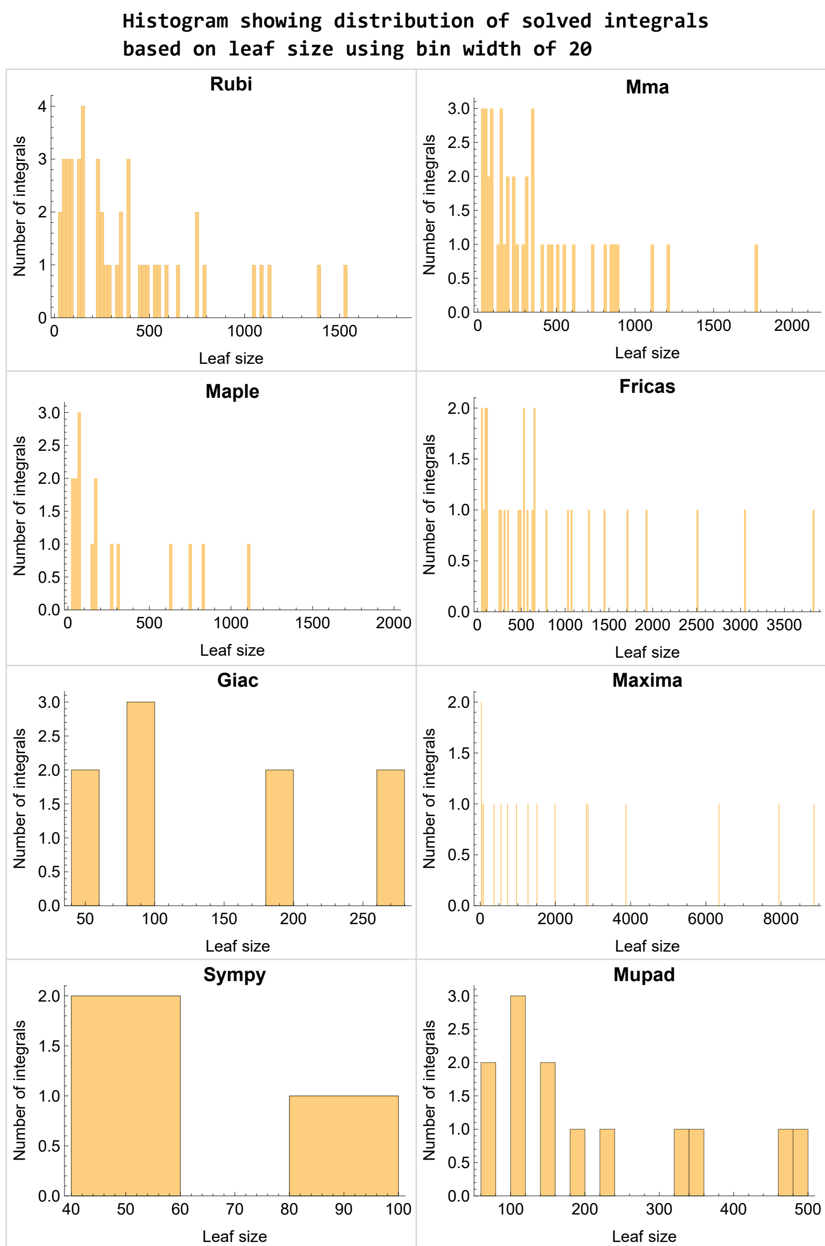


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

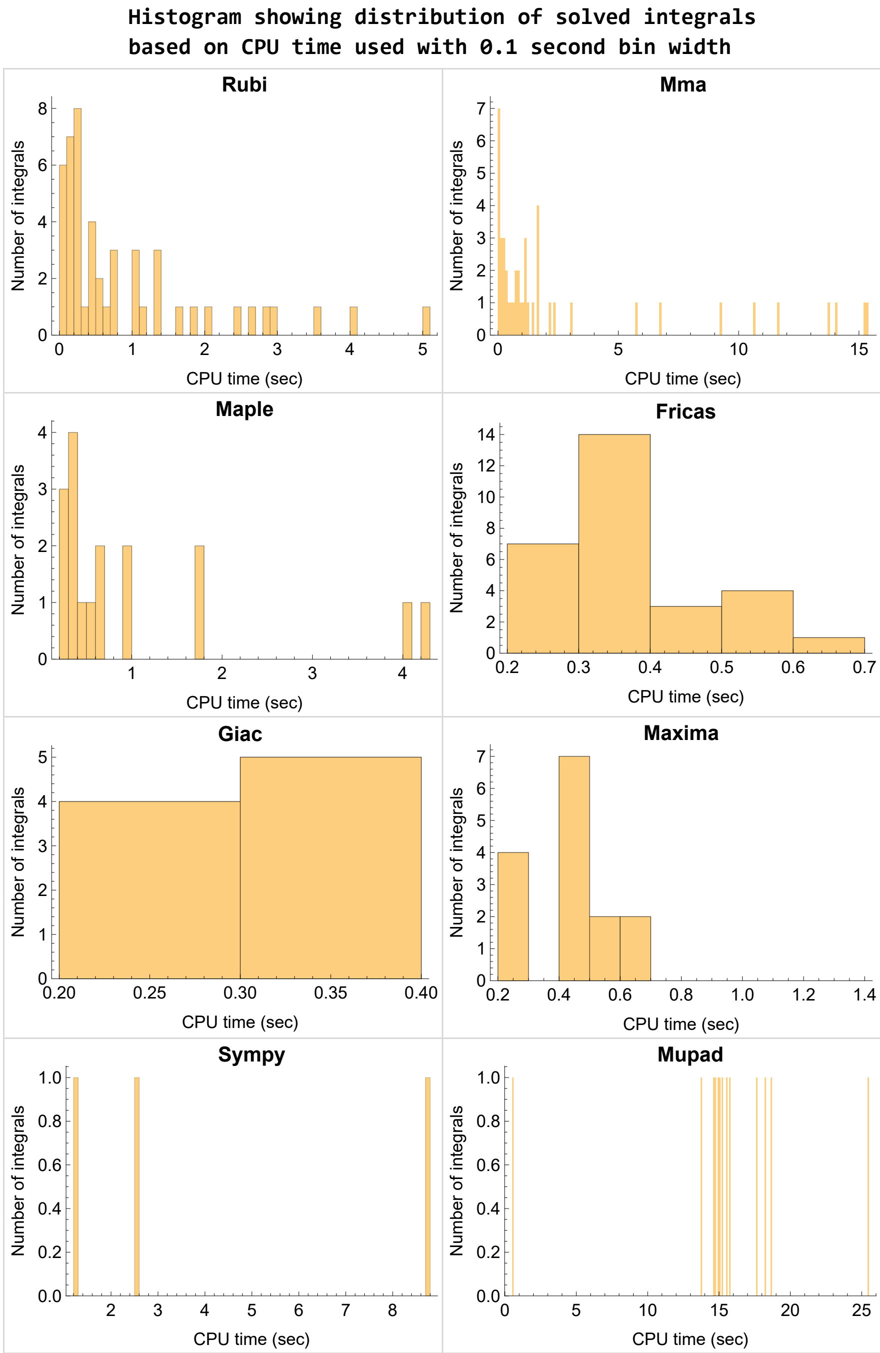


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

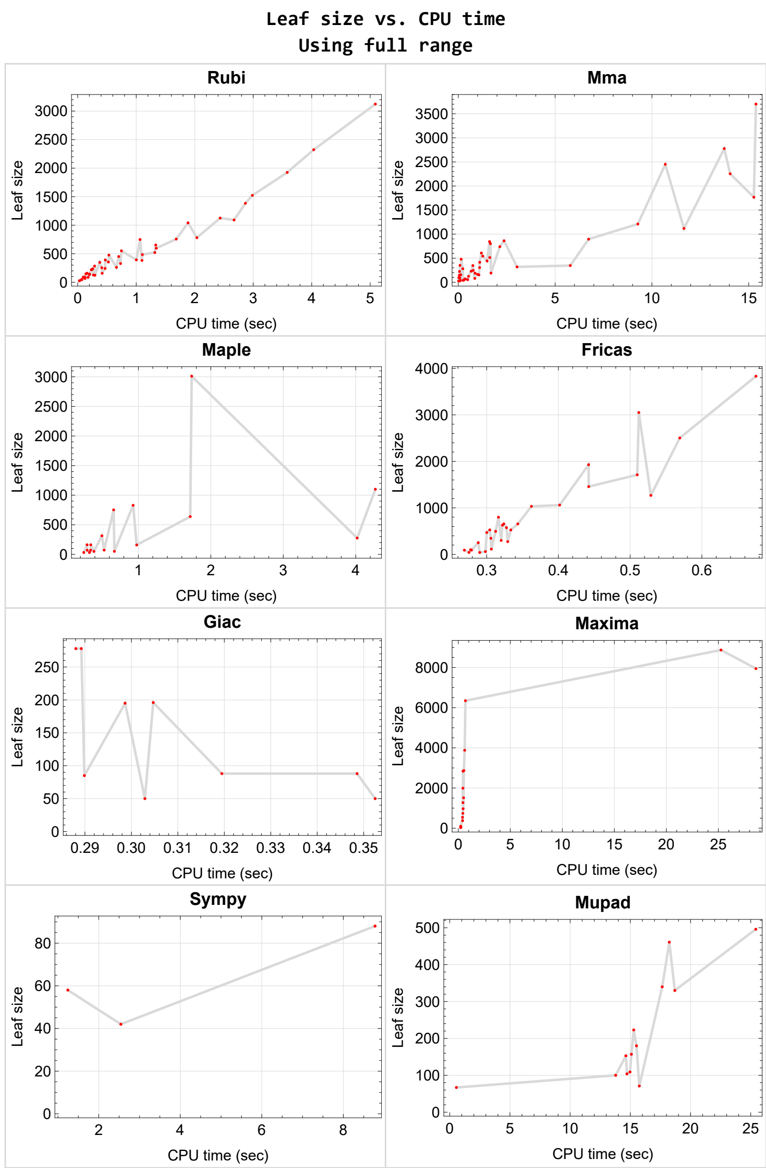


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{2, 4, 6, 7, 9, 11, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 34, 35, 39, 40, 44, 45, 49, 50, 54, 55, 59, 60, 64, 65, 69, 70, 71\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{25, 82\}$

Maple $\{72, 73, 75, 76, 78, 79, 81, 82\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

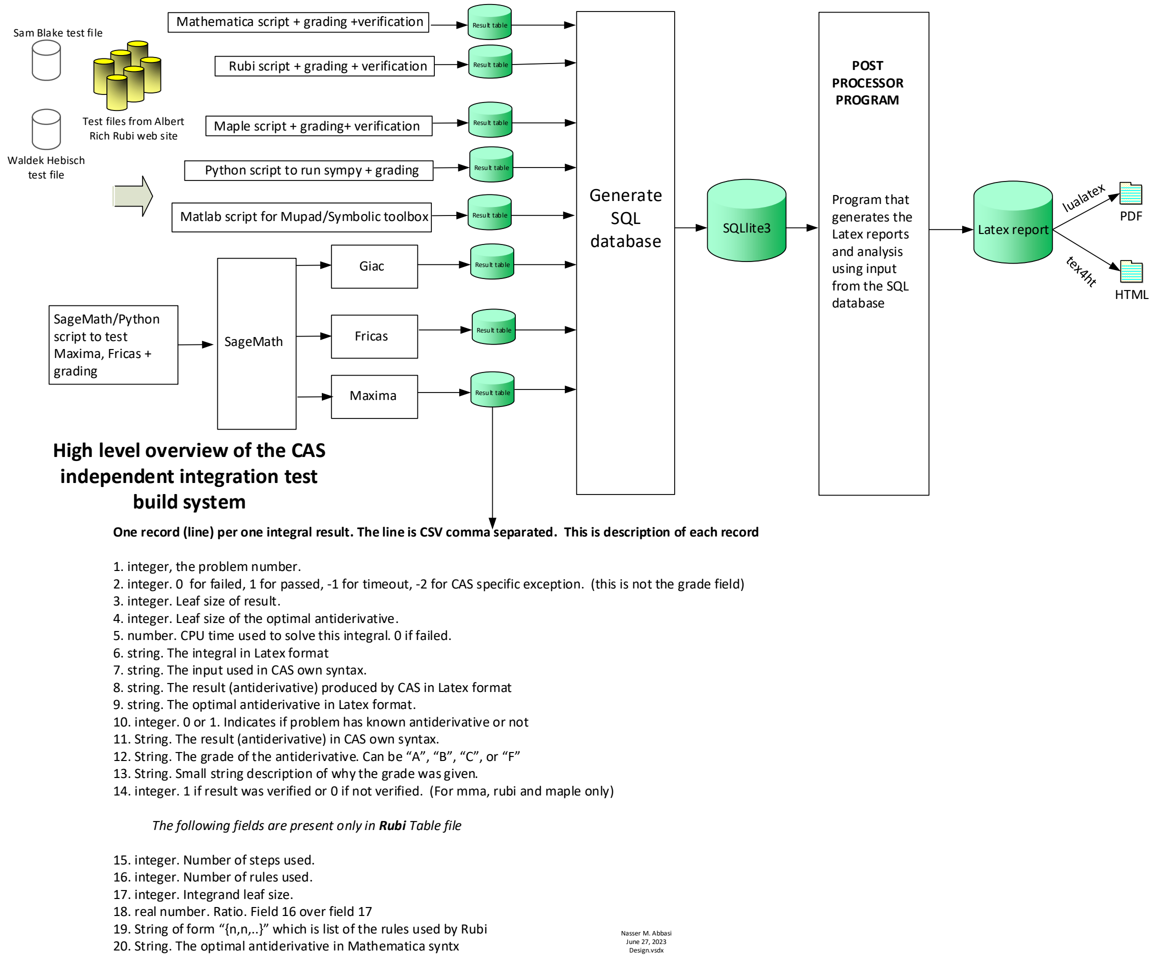
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 3, 5, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 53, 56, 57, 58, 61, 62, 63, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 3, 5, 8, 10, 12, 15, 16, 20, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 53, 56, 57, 58, 61, 62, 63, 66, 67, 68, 72, 73, 75, 76, 78, 81 }

B grade { 18, 79, 82 }

C grade { }

F normal fail { 74, 77, 80, 83 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 5, 12, 15, 20, 27, 53, 58, 63, 68 }

B grade { }

C grade { 72, 73, 75, 76, 78, 79, 81, 82 }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67, 74, 77, 80, 83 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 5, 15, 20, 53, 63, 72, 75, 78, 81 }

B grade { 1, 3, 8, 10, 12, 16, 18, 23, 25, 27, 58, 68, 73, 74, 76, 77, 79, 80, 82, 83 }

C grade { }

F normal fail { 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 5, 53, 58 }

B grade { 12, 15, 20, 27, 31, 32, 33, 36, 37, 38, 51, 52, 56, 57 }

C grade { }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

F(-1) timedout fail { 60, 69, 70 }

F(-2) exception fail { 25, 41, 42, 43, 46, 47, 48, 61, 62, 63, 66, 67, 68 }

Giac

A grade { 15, 27, 68 }

B grade { 5, 12, 20, 53, 58, 63 }

C grade { }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 5, 12, 15, 20, 27, 53, 58, 63, 68, 72, 75, 78, 81 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67, 73, 74, 76, 77, 79, 80, 82, 83 }

F(-2) exception fail { }

Sympy

A grade { 53, 58 }

B grade { 5 }

C grade { }

F normal fail { 1, 3, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 63, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	146	0	0	495	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	3.46	0.00	0.00	0.00
time (sec)	N/A	0.204	0.054	0.000	0.000	0.312	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	115	21	15	18	20
N.S.	1	1.00	1.12	1.00	7.19	1.31	0.94	1.12	1.25
time (sec)	N/A	0.021	1.180	0.191	0.360	0.280	2.144	0.379	13.236

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	95	0	0	346	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	3.76	0.00	0.00	0.00
time (sec)	N/A	0.117	0.027	0.000	0.000	0.306	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	115	21	15	18	20
N.S.	1	1.00	1.12	1.00	7.19	1.31	0.94	1.12	1.25
time (sec)	N/A	0.022	0.888	0.148	0.331	0.260	1.844	0.310	13.910

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	32	31	42	42	50	67
N.S.	1	1.00	1.00	1.23	1.19	1.62	1.62	1.92	2.58
time (sec)	N/A	0.030	0.017	0.247	0.220	0.275	2.543	0.303	0.553

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	108	18	14	18	20
N.S.	1	1.00	1.12	1.00	6.75	1.12	0.88	1.12	1.25
time (sec)	N/A	0.023	1.019	0.189	0.346	0.279	0.644	0.279	14.500

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	118	18	15	18	20
N.S.	1	1.00	1.12	1.00	7.38	1.12	0.94	1.12	1.25
time (sec)	N/A	0.023	0.872	0.152	0.334	0.261	0.420	0.398	13.262

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	242	229	0	0	799	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	3.30	0.00	0.00	0.00
time (sec)	N/A	0.468	0.664	0.000	0.000	0.316	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	256	42	17	20	22
N.S.	1	1.00	1.11	1.00	14.22	2.33	0.94	1.11	1.22
time (sec)	N/A	0.029	12.322	0.295	0.490	0.259	2.762	1.211	14.046

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	123	0	0	525	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	3.95	0.00	0.00	0.00
time (sec)	N/A	0.202	0.528	0.000	0.000	0.334	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	251	42	17	20	22
N.S.	1	1.00	1.11	1.00	13.94	2.33	0.94	1.11	1.22
time (sec)	N/A	0.028	9.074	0.245	0.509	0.274	2.185	1.012	13.183

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	50	96	91	0	88	100
N.S.	1	1.00	0.93	1.14	2.18	2.07	0.00	2.00	2.27
time (sec)	N/A	0.067	0.254	0.387	0.229	0.269	0.000	0.319	13.780

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	286	36	15	20	22
N.S.	1	1.00	1.11	1.00	15.89	2.00	0.83	1.11	1.22
time (sec)	N/A	0.027	29.803	0.245	0.495	0.272	2.025	0.376	13.583

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	289	36	17	20	22
N.S.	1	1.00	1.11	1.00	16.06	2.00	0.94	1.11	1.22
time (sec)	N/A	0.026	12.103	0.302	0.517	0.275	0.657	1.293	13.361

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	72	2838	100	0	85	496
N.S.	1	1.00	1.00	0.80	31.53	1.11	0.00	0.94	5.51
time (sec)	N/A	0.094	0.079	0.528	0.453	0.278	0.000	0.290	25.416

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	382	382	305	0	0	1457	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	3.81	0.00	0.00	0.00
time (sec)	N/A	1.098	1.101	0.000	0.000	0.442	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	252	20	15	20	22
N.S.	1	1.00	1.11	1.00	14.00	1.11	0.83	1.11	1.22
time (sec)	N/A	0.031	1.769	0.153	0.437	0.270	0.457	0.335	13.063

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	261	261	845	0	0	1060	0	0	0
N.S.	1	1.00	3.24	0.00	0.00	4.06	0.00	0.00	0.00
time (sec)	N/A	0.662	1.614	0.000	0.000	0.402	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	252	20	15	20	22
N.S.	1	1.00	1.11	1.00	14.00	1.11	0.83	1.11	1.22
time (sec)	N/A	0.033	1.537	0.181	0.438	0.272	0.369	0.327	13.057

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	70	7945	251	0	278	157
N.S.	1	1.00	1.02	1.06	120.38	3.80	0.00	4.21	2.38
time (sec)	N/A	0.125	0.338	0.294	28.612	0.288	0.000	0.288	15.088

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	249	19	15	20	22
N.S.	1	1.00	1.11	1.00	13.83	1.06	0.83	1.11	1.22
time (sec)	N/A	0.030	1.779	0.182	0.612	0.257	0.750	0.305	13.059

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	118	18	15	18	20
N.S.	1	1.00	1.12	1.00	7.38	1.12	0.94	1.12	1.25
time (sec)	N/A	0.016	0.068	0.002	0.440	0.265	0.443	0.428	0.002

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1092	1092	895	0	0	3050	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	2.79	0.00	0.00	0.00
time (sec)	N/A	2.673	6.730	0.000	0.000	0.512	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	1284	38	17	20	22
N.S.	1	1.00	1.11	1.00	71.33	2.11	0.94	1.11	1.22
time (sec)	N/A	0.027	6.946	0.207	1.158	0.267	1.233	0.389	12.906

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	596	596	1118	0	0	1928	0	0	0
N.S.	1	1.00	1.88	0.00	0.00	3.23	0.00	0.00	0.00
time (sec)	N/A	1.342	11.656	0.000	0.000	0.442	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	1261	38	17	20	22
N.S.	1	1.00	1.11	1.00	70.06	2.11	0.94	1.11	1.22
time (sec)	N/A	0.029	6.732	0.167	0.886	0.270	1.017	0.402	13.464

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	153	162	8871	525	0	195	340
N.S.	1	1.00	1.24	1.32	72.12	4.27	0.00	1.59	2.76
time (sec)	N/A	0.294	1.064	0.292	25.253	0.304	0.000	0.299	17.646

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	4629	38	17	20	22
N.S.	1	1.00	1.11	1.00	257.17	2.11	0.94	1.11	1.22
time (sec)	N/A	0.026	10.514	0.200	6.023	0.273	1.256	0.812	14.004

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	4550	44	19	20	22
N.S.	1	1.00	1.11	1.00	252.78	2.44	1.06	1.11	1.22
time (sec)	N/A	0.025	6.986	0.175	5.773	0.291	1.195	0.416	14.692

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	3521	44	19	20	22
N.S.	1	1.00	1.11	1.00	195.61	2.44	1.06	1.11	1.22
time (sec)	N/A	0.027	9.394	0.175	5.741	0.291	1.382	1.065	13.855

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	476	476	479	0	1512	0	0	0	0
N.S.	1	1.00	1.01	0.00	3.18	0.00	0.00	0.00	0.00
time (sec)	N/A	0.532	0.149	0.000	0.508	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	348	348	351	0	966	0	0	0	0
N.S.	1	1.00	1.01	0.00	2.78	0.00	0.00	0.00	0.00
time (sec)	N/A	0.376	0.087	0.000	0.466	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	220	223	0	540	0	0	0	0
N.S.	1	1.00	1.01	0.00	2.45	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.062	0.000	0.406	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	106	18	15	18	20
N.S.	1	1.00	1.11	0.89	5.89	1.00	0.83	1.00	1.11
time (sec)	N/A	0.021	3.043	0.527	0.666	0.263	1.609	0.323	13.804

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	110	18	17	18	20
N.S.	1	1.00	1.11	0.89	6.11	1.00	0.94	1.00	1.11
time (sec)	N/A	0.022	12.006	0.531	0.702	0.258	1.282	0.360	13.753

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	749	749	739	0	6347	0	0	0	0
N.S.	1	1.00	0.99	0.00	8.47	0.00	0.00	0.00	0.00
time (sec)	N/A	1.066	2.145	0.000	0.687	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	551	551	543	0	3879	0	0	0	0
N.S.	1	1.00	0.99	0.00	7.04	0.00	0.00	0.00	0.00
time (sec)	N/A	0.747	1.264	0.000	0.614	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	355	355	347	0	1991	0	0	0	0
N.S.	1	1.00	0.98	0.00	5.61	0.00	0.00	0.00	0.00
time (sec)	N/A	0.524	0.752	0.000	0.445	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	319	36	17	20	22
N.S.	1	1.00	1.10	0.90	15.95	1.80	0.85	1.00	1.10
time (sec)	N/A	0.024	109.927	0.707	0.892	0.270	8.294	0.456	13.429

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	321	36	19	20	22
N.S.	1	1.00	1.10	0.90	16.05	1.80	0.95	1.00	1.10
time (sec)	N/A	0.025	74.353	0.784	1.090	0.266	2.171	0.585	13.485

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1041	1041	802	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.886	1.661	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	781	781	608	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.037	1.192	0.000	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	521	521	414	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.318	1.106	0.000	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	241	19	17	20	22
N.S.	1	1.00	1.10	0.90	12.05	0.95	0.85	1.00	1.10
time (sec)	N/A	0.032	3.497	0.563	0.826	0.267	2.133	0.304	13.865

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	110	18	17	18	20
N.S.	1	1.00	1.11	0.89	6.11	1.00	0.94	1.00	1.11
time (sec)	N/A	0.019	0.074	0.002	0.702	0.266	1.270	0.365	0.002

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	3123	3123	3702	0	0	0	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.088	15.373	0.000	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2323	2323	2777	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.034	13.737	0.000	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1523	1523	1767	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.986	15.265	0.000	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	4405	38	19	20	22
N.S.	1	1.00	1.10	0.90	220.25	1.90	0.95	1.00	1.10
time (sec)	N/A	0.027	64.798	0.560	13.785	0.270	4.112	0.738	13.597

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	4406	44	20	20	22
N.S.	1	1.00	1.10	0.90	220.30	2.20	1.00	1.00	1.10
time (sec)	N/A	0.028	49.629	0.603	20.356	0.286	9.593	0.946	13.185

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	284	284	281	0	738	0	0	0	0
N.S.	1	1.00	0.99	0.00	2.60	0.00	0.00	0.00	0.00
time (sec)	N/A	0.288	0.233	0.000	0.439	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	155	0	374	0	0	0	0
N.S.	1	1.00	0.98	0.00	2.37	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	0.130	0.000	0.414	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	32	31	41	58	50	71
N.S.	1	1.00	1.00	1.23	1.19	1.58	2.23	1.92	2.73
time (sec)	N/A	0.030	0.073	0.325	0.232	0.290	1.242	0.352	15.745

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	112	25	19	18	20
N.S.	1	1.00	1.10	0.80	5.60	1.25	0.95	0.90	1.00
time (sec)	N/A	0.019	22.507	0.530	0.745	0.260	0.875	0.303	13.258

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	113	25	19	18	20
N.S.	1	1.00	1.10	0.80	5.65	1.25	0.95	0.90	1.00
time (sec)	N/A	0.019	22.476	0.541	0.781	0.271	3.856	0.362	13.139

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	451	451	443	0	2869	0	0	0	0
N.S.	1	1.00	0.98	0.00	6.36	0.00	0.00	0.00	0.00
time (sec)	N/A	0.700	1.487	0.000	0.535	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	255	247	0	1272	0	0	0	0
N.S.	1	1.00	0.97	0.00	4.99	0.00	0.00	0.00	0.00
time (sec)	N/A	0.412	0.772	0.000	0.449	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	51	50	91	88	88	109
N.S.	1	1.00	0.96	1.09	1.06	1.94	1.87	1.87	2.32
time (sec)	N/A	0.069	0.288	0.669	0.244	0.279	8.777	0.349	14.972

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	718	46	20	20	22
N.S.	1	1.00	1.09	0.82	32.64	2.09	0.91	0.91	1.00
time (sec)	N/A	0.027	72.660	0.795	1.580	0.283	1.961	0.370	13.541

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	46	20	20	22
N.S.	1	1.00	1.09	0.82	0.00	2.09	0.91	0.91	1.00
time (sec)	N/A	0.028	72.340	0.816	0.000	0.277	5.518	0.407	14.012

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	653	653	513	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.334	1.625	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	393	393	319	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.002	3.035	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	69	70	0	274	0	278	153
N.S.	1	1.00	1.01	1.03	0.00	4.03	0.00	4.09	2.25
time (sec)	N/A	0.118	0.369	0.342	0.000	0.329	0.000	0.289	14.629

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	243	27	20	20	22
N.S.	1	1.00	1.09	0.82	11.05	1.23	0.91	0.91	1.00
time (sec)	N/A	0.031	5.084	0.569	0.861	0.265	2.221	0.366	13.163

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	244	27	20	20	22
N.S.	1	1.00	1.09	0.82	11.09	1.23	0.91	0.91	1.00
time (sec)	N/A	0.031	5.167	0.483	1.091	0.271	6.474	0.405	13.336

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1925	1925	2254	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.582	14.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1125	1125	1210	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.435	9.273	0.000	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	163	162	0	574	0	196	330
N.S.	1	1.00	1.28	1.28	0.00	4.52	0.00	1.54	2.60
time (sec)	N/A	0.268	0.996	0.343	0.000	0.328	0.000	0.305	18.695

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	48	22	20	22
N.S.	1	1.00	1.09	0.82	0.00	2.18	1.00	0.91	1.00
time (sec)	N/A	0.027	44.623	0.565	0.000	0.280	5.915	1.040	13.754

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	48	22	20	22
N.S.	1	1.00	1.09	0.82	0.00	2.18	1.00	0.91	1.00
time (sec)	N/A	0.029	47.706	0.490	0.000	0.302	45.723	1.802	13.182

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	24
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.20
time (sec)	N/A	0.062	4.199	0.927	2.363	0.269	47.827	0.757	12.910

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	44	44	38	159	0	60	0	0	104
N.S.	1	1.00	0.86	3.61	0.00	1.36	0.00	0.00	2.36
time (sec)	N/A	0.064	0.110	0.977	0.000	0.298	0.000	0.000	14.717

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	149	149	188	829	0	470	0	0	0
N.S.	1	1.00	1.26	5.56	0.00	3.15	0.00	0.00	0.00
time (sec)	N/A	0.140	0.866	0.928	0.000	0.300	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	235	235	0	0	0	655	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	2.79	0.00	0.00	0.00
time (sec)	N/A	0.256	0.000	0.000	0.000	0.343	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	79	54	276	0	113	0	0	180
N.S.	1	1.00	0.68	3.49	0.00	1.43	0.00	0.00	2.28
time (sec)	N/A	0.125	0.487	4.020	0.000	0.307	0.000	0.000	15.510

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	221	221	347	1100	0	656	0	0	0
N.S.	1	1.00	1.57	4.98	0.00	2.97	0.00	0.00	0.00
time (sec)	N/A	0.242	5.784	4.271	0.000	0.324	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	390	390	0	0	0	1032	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	2.65	0.00	0.00	0.00
time (sec)	N/A	0.472	0.000	0.000	0.000	0.362	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	87	87	80	314	0	300	0	0	223
N.S.	1	1.00	0.92	3.61	0.00	3.45	0.00	0.00	2.56
time (sec)	N/A	0.178	0.853	0.497	0.000	0.320	0.000	0.000	15.280

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	328	328	861	752	0	1268	0	0	0
N.S.	1	1.00	2.62	2.29	0.00	3.87	0.00	0.00	0.00
time (sec)	N/A	0.732	2.362	0.659	0.000	0.529	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	485	485	0	0	0	1711	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	3.53	0.00	0.00	0.00
time (sec)	N/A	1.105	0.000	0.000	0.000	0.510	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	157	157	191	638	0	628	0	0	461
N.S.	1	1.00	1.22	4.06	0.00	4.00	0.00	0.00	2.94
time (sec)	N/A	0.418	1.687	1.716	0.000	0.322	0.000	0.000	18.234

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	757	757	2450	3010	0	2503	0	0	0
N.S.	1	1.00	3.24	3.98	0.00	3.31	0.00	0.00	0.00
time (sec)	N/A	1.684	10.692	1.736	0.000	0.569	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1384	1384	0	0	0	3831	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	2.77	0.00	0.00	0.00
time (sec)	N/A	2.867	0.000	0.000	0.000	0.675	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [23] had the largest ratio of [.66669999999999959]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	6	1.00	16	0.375
2	N/A	0	0	1.00	16	0.000
3	A	8	5	1.00	16	0.312
4	N/A	0	0	1.00	16	0.000
5	A	4	3	1.00	14	0.214
6	N/A	0	0	1.00	16	0.000
7	N/A	0	0	1.00	16	0.000
8	A	15	11	1.00	18	0.611
9	N/A	0	0	1.00	18	0.000
10	A	10	7	1.00	18	0.389
11	N/A	0	0	1.00	18	0.000
12	A	5	5	1.00	16	0.312
13	N/A	0	0	1.00	18	0.000
14	N/A	0	0	1.00	18	0.000
15	A	5	3	1.00	12	0.250
16	A	13	8	1.00	18	0.444
17	N/A	0	0	1.00	18	0.000
18	A	11	7	1.00	18	0.389
19	N/A	0	0	1.00	18	0.000
20	A	4	4	1.00	16	0.250
21	N/A	0	0	1.00	18	0.000
22	N/A	0	0	1.00	16	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	31	12	1.00	18	0.667
24	N/A	0	0	1.00	18	0.000
25	A	22	10	1.00	18	0.556
26	N/A	0	0	1.00	18	0.000
27	A	6	6	1.00	16	0.375
28	N/A	0	0	1.00	18	0.000
29	N/A	0	0	1.00	18	0.000
30	N/A	0	0	1.00	18	0.000
31	A	20	7	1.00	18	0.389
32	A	16	7	1.00	18	0.389
33	A	12	7	1.00	16	0.438
34	N/A	0	0	1.00	18	0.000
35	N/A	0	0	1.00	18	0.000
36	A	30	10	1.00	20	0.500
37	A	24	10	1.00	20	0.500
38	A	18	10	1.00	18	0.556
39	N/A	0	0	1.00	20	0.000
40	N/A	0	0	1.00	20	0.000
41	A	23	9	1.00	20	0.450
42	A	19	9	1.00	20	0.450
43	A	15	9	1.00	18	0.500
44	N/A	0	0	1.00	20	0.000
45	N/A	0	0	1.00	18	0.000
46	A	61	11	1.00	20	0.550
47	A	49	11	1.00	20	0.550
48	A	37	11	1.00	18	0.611
49	N/A	0	0	1.00	20	0.000
50	N/A	0	0	1.00	20	0.000
51	A	14	7	1.00	20	0.350
52	A	10	6	1.00	20	0.300
53	A	4	3	1.00	20	0.150
54	N/A	0	0	1.00	20	0.000
55	N/A	0	0	1.00	20	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	21	10	1.00	22	0.454
57	A	15	11	1.00	22	0.500
58	A	5	5	1.00	22	0.227
59	N/A	0	0	1.00	22	0.000
60	N/A	0	0	1.00	22	0.000
61	A	17	9	1.00	22	0.409
62	A	13	8	1.00	22	0.364
63	A	4	4	1.00	22	0.182
64	N/A	0	0	1.00	22	0.000
65	N/A	0	0	1.00	22	0.000
66	A	43	11	1.00	22	0.500
67	A	31	12	1.00	22	0.546
68	A	6	6	1.00	22	0.273
69	N/A	0	0	1.00	22	0.000
70	N/A	0	0	1.00	22	0.000
71	N/A	0	0	1.00	20	0.000
72	A	5	4	1.00	20	0.200
73	A	9	6	1.00	22	0.273
74	A	11	7	1.00	22	0.318
75	A	6	6	1.00	22	0.273
76	A	11	8	1.00	24	0.333
77	A	16	12	1.00	24	0.500
78	A	5	5	1.00	22	0.227
79	A	12	8	1.00	24	0.333
80	A	14	9	1.00	24	0.375
81	A	7	7	1.00	22	0.318
82	A	23	11	1.00	24	0.458
83	A	32	13	1.00	24	0.542

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^5(a+b\sec(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 143

$$\int x^5\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\frac{ax^6}{6}-\frac{ibx^4\arctan\left(e^{i(c+dx^2)}\right)}{d}+\frac{ibx^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx^2)}\right)}{d^2}-\frac{ibx^2\operatorname{PolyLog}\left(2,ie^{i(c+dx^2)}\right)}{d^2}-\frac{b\operatorname{PolyLog}\left(3,-ie^{i(c+dx^2)}\right)}{d^3}+\frac{b\operatorname{PolyLog}\left(3,ie^{i(c+dx^2)}\right)}{d^3}$$

```
[Out] 1/6*a*x^6-I*b*x^4*arctan(exp(I*(d*x^2+c)))/d+I*b*x^2*polylog(2,-I*exp(I*(d*
x^2+c)))/d^2-I*b*x^2*polylog(2,I*exp(I*(d*x^2+c)))/d^2-b*polylog(3,-I*exp(I
*(d*x^2+c)))/d^3+b*polylog(3,I*exp(I*(d*x^2+c)))/d^3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 4289, 4266, 2611, 2320, 6724}

$$\int x^5\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\frac{ax^6}{6}-\frac{ibx^4\arctan\left(e^{i(c+dx^2)}\right)}{d}-\frac{b\operatorname{PolyLog}\left(3,-ie^{i(dx^2+c)}\right)}{d^3}+\frac{b\operatorname{PolyLog}\left(3,ie^{i(dx^2+c)}\right)}{d^3}+\frac{ibx^2\operatorname{PolyLog}\left(2,-ie^{i(dx^2+c)}\right)}{d^2}-\frac{ibx^2\operatorname{PolyLog}\left(2,ie^{i(dx^2+c)}\right)}{d^2}$$

```
[In] Int[x^5*(a + b*Sec[c + d*x^2]),x]
```

```
[Out] (a*x^6)/6 - (I*b*x^4*ArcTan[E^(I*(c + d*x^2))])/d + (I*b*x^2*PolyLog[2, (-I
)*E^(I*(c + d*x^2))])/d^2 - (I*b*x^2*PolyLog[2, I*E^(I*(c + d*x^2))])/d^2 -
 (b*PolyLog[3, (-I)*E^(I*(c + d*x^2))])/d^3 + (b*PolyLog[3, I*E^(I*(c + d*x
^2))])/d^3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(ax^5 + bx^5 \sec\left(c+dx^2\right)\right) dx$$

$$= \frac{ax^6}{6} + b\int x^5 \sec\left(c+dx^2\right) dx$$

$$= \frac{ax^6}{6} + \frac{1}{2} b\text{Subst}\left(\int x^2 \sec(c+dx)\, dx, x, x^2\right)$$

$$= \frac{ax^6}{6} - \frac{ibx^4 \arctan\left(e^{i(c+dx^2)}\right)}{d} - \frac{b\text{Subst}\left(\int x \log\left(1-ie^{i(c+dx)}\right) dx, x, x^2\right)}{d} + \frac{b\text{Subst}\left(\int x \log\left(1+ie^{i(c+dx)}\right) dx, x, x^2\right)}{d}$$

$$= \frac{ax^6}{6} - \frac{ibx^4 \arctan\left(e^{i(c+dx^2)}\right)}{d} + \frac{ibx^2 \text{PolyLog}\left(2, -ie^{i(c+dx^2)}\right)}{d^2} - \frac{ibx^2 \text{PolyLog}\left(2, ie^{i(c+dx^2)}\right)}{d^2} - \frac{(ib)\text{Subst}\left(\int \text{PolyLog}\left(2, -ie^{i(c+dx)}\right) dx, x, x^2\right)}{d^2} + \frac{(ib)\text{Subst}\left(\int \text{PolyLog}\left(2, ie^{i(c+dx)}\right) dx, x, x^2\right)}{d^2}$$

$$= \frac{ax^6}{6} - \frac{ibx^4 \arctan\left(e^{i(c+dx^2)}\right)}{d} + \frac{ibx^2 \text{PolyLog}\left(2, -ie^{i(c+dx^2)}\right)}{d^2} - \frac{ibx^2 \text{PolyLog}\left(2, ie^{i(c+dx^2)}\right)}{d^2} - \frac{b\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x} dx, x, e^{i(c+dx^2)}\right)}{d^3} + \frac{b\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x} dx, x, e^{i(c+dx^2)}\right)}{d^3}$$

$$= \frac{ax^6}{6} - \frac{ibx^4 \arctan\left(e^{i(c+dx^2)}\right)}{d} + \frac{ibx^2 \text{PolyLog}\left(2, -ie^{i(c+dx^2)}\right)}{d^2} - \frac{ibx^2 \text{PolyLog}\left(2, ie^{i(c+dx^2)}\right)}{d^2} - \frac{b \text{PolyLog}\left(3, -ie^{i(c+dx^2)}\right)}{d^3} + \frac{b \text{PolyLog}\left(3, ie^{i(c+dx^2)}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02

$$\int x^5\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\frac{ax^6}{6}-\frac{ibx^4\arctan\left(e^{ic+idx^2}\right)}{d}+\frac{ibx^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx^2)}\right)}{d^2}-\frac{ibx^2\operatorname{PolyLog}\left(2,ie^{i(c+dx^2)}\right)}{d^2}-\frac{b\operatorname{PolyLog}\left(3,-ie^{i(c+dx^2)}\right)}{d^3}+\frac{b\operatorname{PolyLog}\left(3,ie^{i(c+dx^2)}\right)}{d^3}$$

```
[In] Integrate[x^5*(a + b*Sec[c + d*x^2]),x]
```

```
[Out] (a*x^6)/6 - (I*b*x^4*ArcTan[E^(I*c + I*d*x^2)])/d + (I*b*x^2*PolyLog[2, (-I
)*E^(I*(c + d*x^2))])/d^2 - (I*b*x^2*PolyLog[2, I*E^(I*(c + d*x^2))])/d^2 -
 (b*PolyLog[3, (-I)*E^(I*(c + d*x^2))])/d^3 + (b*PolyLog[3, I*E^(I*(c + d*x
^2))])/d^3
```

Maple [F]

$$\int x^5\left(a+b\sec\left(d\,x^2+c\right)\right)dx$$

```
[In] int(x^5*(a+b*sec(d*x^2+c)),x)
```

```
[Out] int(x^5*(a+b*sec(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(115) = 230.

Time = 0.31 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.46

$$\int x^5\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\frac{2\,ad^3x^6-6i\,bdx^2\mathrm{Li}_2(i\,\cos\left(dx^2+c\right)+\sin\left(dx^2+c\right))-6i\,bdx^2\mathrm{Li}_2(i\,\cos\left(dx^2+c\right)-\sin\left(dx^2+c\right))+6i\,bdx$$

```
[In] integrate(x^5*(a+b*sec(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*a*d^3*x^6 - 6*I*b*d*x^2*dilog(I*cos(d*x^2 + c) + sin(d*x^2 + c)) -
6*I*b*d*x^2*dilog(I*cos(d*x^2 + c) - sin(d*x^2 + c)) + 6*I*b*d*x^2*dilog(-I
*cos(d*x^2 + c) + sin(d*x^2 + c)) + 6*I*b*d*x^2*dilog(-I*cos(d*x^2 + c) - s
in(d*x^2 + c)) + 3*b*c^2*log(cos(d*x^2 + c) + I*sin(d*x^2 + c) + I) - 3*b*c
```

```
^2*log(cos(d*x^2 + c) - I*sin(d*x^2 + c) + I) + 3*b*c^2*log(-cos(d*x^2 + c)
 + I*sin(d*x^2 + c) + I) - 3*b*c^2*log(-cos(d*x^2 + c) - I*sin(d*x^2 + c) +
 I) + 3*(b*d^2*x^4 - b*c^2)*log(I*cos(d*x^2 + c) + sin(d*x^2 + c) + 1) - 3*
(b*d^2*x^4 - b*c^2)*log(I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1) + 3*(b*d^2*x
^4 - b*c^2)*log(-I*cos(d*x^2 + c) + sin(d*x^2 + c) + 1) - 3*(b*d^2*x^4 - b*
c^2)*log(-I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1) - 6*b*polylog(3, I*cos(d*x
^2 + c) + sin(d*x^2 + c)) + 6*b*polylog(3, I*cos(d*x^2 + c) - sin(d*x^2 + c
)) - 6*b*polylog(3, -I*cos(d*x^2 + c) + sin(d*x^2 + c)) + 6*b*polylog(3, -I
*cos(d*x^2 + c) - sin(d*x^2 + c)))/d^3
```

Sympy [F]

$$\int x^5(a+b\sec(c+dx^2))\,dx=\int x^5(a+b\sec(c+dx^2))\,dx$$

```
[In] integrate(x**5*(a+b*sec(d*x**2+c)),x)
[Out] Integral(x**5*(a + b*sec(c + d*x**2)), x)
```

Maxima [F]

$$\int x^5(a+b\sec(c+dx^2))\,dx=\int (b\sec(dx^2+c)+a)x^5\,dx$$

```
[In] integrate(x^5*(a+b*sec(d*x^2+c)),x, algorithm="maxima")
[Out] 1/6*a*x^6 + 2*b*integrate((x^5*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + x^5*sin(
2*d*x^2 + 2*c)*sin(d*x^2 + c) + x^5*cos(d*x^2 + c))/(cos(2*d*x^2 + 2*c)^2 +
 sin(2*d*x^2 + 2*c)^2 + 2*cos(2*d*x^2 + 2*c) + 1), x)
```

Giac [F]

$$\int x^5(a+b\sec(c+dx^2))\,dx=\int (b\sec(dx^2+c)+a)x^5\,dx$$

```
[In] integrate(x^5*(a+b*sec(d*x^2+c)),x, algorithm="giac")
[Out] integrate((b*sec(d*x^2 + c) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a + b \sec \left(c + dx^2\right)\right) \, dx = \int x^5 \left(a + \frac{b}{\cos \left(d\,x^2 + c\right)}\right) \, dx$$

```
[In] int(x^5*(a + b/cos(c + d*x^2)),x)
[Out] int(x^5*(a + b/cos(c + d*x^2)), x)
```

3.2 $\int x^4(a+b\sec(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^4(a+b\sec(c+dx^2))\,dx=\frac{ax^5}{5}+b\text{Int}\left(x^4\sec(c+dx^2),x\right)$$

```
[Out] 1/5*a*x^5+b*Unintegrable(x^4*sec(d*x^2+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^4(a+b\sec(c+dx^2))\,dx=\int x^4(a+b\sec(c+dx^2))\,dx$$

```
[In] Int[x^4*(a + b*Sec[c + d*x^2]),x]
```

```
[Out] (a*x^5)/5 + b*Defer[Int][x^4*Sec[c + d*x^2], x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(ax^4+bx^4\sec(c+dx^2)\right)dx\\&=\frac{ax^5}{5}+b\int x^4\sec(c+dx^2)\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\int x^4\left(a+b\sec\left(c+dx^2\right)\right)\,dx$$

```
[In] Integrate[x^4*(a + b*Sec[c + d*x^2]),x]
[Out] Integrate[x^4*(a + b*Sec[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^4\left(a+b\sec\left(d\,x^2+c\right)\right)dx$$

```
[In] int(x^4*(a+b*sec(d*x^2+c)),x)
[Out] int(x^4*(a+b*sec(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^4\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\int\left(b\sec\left(dx^2+c\right)+a\right)x^4\,dx$$

```
[In] integrate(x^4*(a+b*sec(d*x^2+c)),x, algorithm="fricas")
[Out] integral(b*x^4*sec(d*x^2 + c) + a*x^4, x)
```

Sympy [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^4\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\int x^4\left(a+b\sec\left(c+dx^2\right)\right)\,dx$$

```
[In] integrate(x**4*(a+b*sec(d*x**2+c)),x)
[Out] Integral(x**4*(a + b*sec(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 7.19

$$\int x^4 \left(a+b \sec \left(c+d x^2\right)\right) \, dx=\int (b \sec \left(d x^2+c\right)+a) x^4 \, dx$$

```
[In] integrate(x^4*(a+b*sec(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 2*b*integrate((x^4*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + x^4*sin(
2*d*x^2 + 2*c)*sin(d*x^2 + c) + x^4*cos(d*x^2 + c))/(cos(2*d*x^2 + 2*c)^2 +
 sin(2*d*x^2 + 2*c)^2 + 2*cos(2*d*x^2 + 2*c) + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4 \left(a+b \sec \left(c+d x^2\right)\right) \, dx=\int (b \sec \left(d x^2+c\right)+a) x^4 \, dx$$

```
[In] integrate(x^4*(a+b*sec(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^2 + c) + a)*x^4, x)
```

Mupad [N/A]

Not integrable

Time = 13.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^4 \left(a+b \sec \left(c+d x^2\right)\right) \, dx=\int x^4 \left(a+\frac{b}{\cos \left(d\,x^2+c\right)}\right) \,dx$$

```
[In] int(x^4*(a + b/cos(c + d*x^2)),x)
```

```
[Out] int(x^4*(a + b/cos(c + d*x^2)), x)
```

3.3 $\int x^3(a+b\sec(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int x^3(a+b\sec(c+dx^2))\,dx=\frac{ax^4}{4}-\frac{ibx^2\arctan\left(e^{i(c+dx^2)}\right)}{d}+\frac{ib\,\text{PolyLog}\left(2,-ie^{i(c+dx^2)}\right)}{2d^2}-\frac{ib\,\text{PolyLog}\left(2,ie^{i(c+dx^2)}\right)}{2d^2}$$

```
[Out] 1/4*a*x^4-I*b*x^2*arctan(exp(I*(d*x^2+c)))/d+1/2*I*b*polylog(2,-I*exp(I*(d*
x^2+c)))/d^2-1/2*I*b*polylog(2,I*exp(I*(d*x^2+c)))/d^2
```

Rubi [A] (verified)

Time = 0.12 (sec), antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {14, 4289, 4266, 2317, 2438}

$$\int x^3(a+b\sec(c+dx^2))\,dx=\frac{ax^4}{4}-\frac{ibx^2\arctan\left(e^{i(c+dx^2)}\right)}{d}+\frac{ib\,\text{PolyLog}\left(2,-ie^{i(dx^2+c)}\right)}{2d^2}-\frac{ib\,\text{PolyLog}\left(2,ie^{i(dx^2+c)}\right)}{2d^2}$$

```
[In] Int[x^3*(a + b*Sec[c + d*x^2]),x]
```

```
[Out] (a*x^4)/4 - (I*b*x^2*ArcTan[E^(I*(c + d*x^2))])/d + ((I/2)*b*PolyLog[2, (-I
)*E^(I*(c + d*x^2))])/d^2 - ((I/2)*b*PolyLog[2, I*E^(I*(c + d*x^2))])/d^2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^3+bx^3\sec\left(c+dx^2\right)\right)\,dx \\
&= \frac{ax^4}{4}+b\int x^3\sec\left(c+dx^2\right)\,dx \\
&= \frac{ax^4}{4}+\frac{1}{2}b\text{Subst}\left(\int x\sec(c+dx)\,dx,x,x^2\right) \\
&= \frac{ax^4}{4}-\frac{ibx^2\arctan\left(e^{i(c+dx^2)}\right)}{d}-\frac{b\text{Subst}\left(\int\log\left(1-ie^{i(c+dx)}\right)\,dx,x,x^2\right)}{2d} \\
&\quad+\frac{b\text{Subst}\left(\int\log\left(1+ie^{i(c+dx)}\right)\,dx,x,x^2\right)}{2d}
\end{aligned}$$

$$= \frac{ax^4}{4} - \frac{ibx^2 \arctan\left(e^{i(c+dx^2)}\right)}{d} + \frac{(ib)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{2d^2} - \frac{(ib)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{2d^2}$$

$$= \frac{ax^4}{4} - \frac{ibx^2 \arctan\left(e^{i(c+dx^2)}\right)}{d} + \frac{ib\,\text{PolyLog}\left(2, -ie^{i(c+dx^2)}\right)}{2d^2} - \frac{ib\,\text{PolyLog}\left(2, ie^{i(c+dx^2)}\right)}{2d^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int x^3 \left(a + b\sec\left(c + dx^2\right)\right)\,dx = \frac{ax^4}{4} - \frac{ibx^2 \arctan\left(e^{ic+idx^2}\right)}{d} + \frac{ib\,\text{PolyLog}\left(2, -ie^{i(c+dx^2)}\right)}{2d^2} - \frac{ib\,\text{PolyLog}\left(2, ie^{i(c+dx^2)}\right)}{2d^2}$$

```
[In] Integrate[x^3*(a + b*Sec[c + d*x^2]),x]
[Out] (a*x^4)/4 - (I*b*x^2*ArcTan[E^(I*c + I*d*x^2)])/d + ((I/2)*b*PolyLog[2, (-I
)*E^(I*(c + d*x^2))])/d^2 - ((I/2)*b*PolyLog[2, I*E^(I*(c + d*x^2))])/d^2
```

Maple [F]

$$\int x^3 \left(a + b\sec\left(d\,x^2 + c\right)\right) dx$$

```
[In] int(x^3*(a+b*sec(d*x^2+c)),x)
[Out] int(x^3*(a+b*sec(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(68) = 136.

Time = 0.31 (sec) , antiderivative size = 346, normalized size of antiderivative = 3.76

$$\int x^3 \left(a + b\sec\left(c + dx^2\right)\right)\,dx = \frac{ad^2x^4 - bc\log\left(\cos\left(dx^2 + c\right) + i\,\sin\left(dx^2 + c\right) + i\right) + bc\log\left(\cos\left(dx^2 + c\right) - i\,\sin\left(dx^2 + c\right) + i\right) - bc\log\left(-\right.}{}$$

```
[In] integrate(x^3*(a+b*sec(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^2*x^4 - b*c*log(cos(d*x^2 + c) + I*sin(d*x^2 + c) + I) + b*c*log(c
os(d*x^2 + c) - I*sin(d*x^2 + c) + I) - b*c*log(-cos(d*x^2 + c) + I*sin(d*x
^2 + c) + I) + b*c*log(-cos(d*x^2 + c) - I*sin(d*x^2 + c) + I) - I*b*dilog(
I*cos(d*x^2 + c) + sin(d*x^2 + c)) - I*b*dilog(I*cos(d*x^2 + c) - sin(d*x^2
 + c)) + I*b*dilog(-I*cos(d*x^2 + c) + sin(d*x^2 + c)) + I*b*dilog(-I*cos(d
*x^2 + c) - sin(d*x^2 + c)) + (b*d*x^2 + b*c)*log(I*cos(d*x^2 + c) + sin(d*
x^2 + c) + 1) - (b*d*x^2 + b*c)*log(I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1)
+ (b*d*x^2 + b*c)*log(-I*cos(d*x^2 + c) + sin(d*x^2 + c) + 1) - (b*d*x^2 +
b*c)*log(-I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1))/d^2
```

Sympy [F]

$$\int x^3\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\int x^3\left(a+b\sec\left(c+dx^2\right)\right)\,dx$$

```
[In] integrate(x**3*(a+b*sec(d*x**2+c)),x)
```

```
[Out] Integral(x**3*(a + b*sec(c + d*x**2)), x)
```

Maxima [F]

$$\int x^3\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\int\left(b\sec\left(dx^2+c\right)+a\right)x^3\,dx$$

```
[In] integrate(x^3*(a+b*sec(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 2*b*integrate((x^3*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + x^3*sin(
2*d*x^2 + 2*c)*sin(d*x^2 + c) + x^3*cos(d*x^2 + c))/(cos(2*d*x^2 + 2*c)^2 +
 sin(2*d*x^2 + 2*c)^2 + 2*cos(2*d*x^2 + 2*c) + 1), x)
```

Giac [F]

$$\int x^3\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\int\left(b\sec\left(dx^2+c\right)+a\right)x^3\,dx$$

```
[In] integrate(x^3*(a+b*sec(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^2 + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \sec \left(c + dx^2\right)\right) \, dx = \int x^3 \left(a + \frac{b}{\cos \left(d\,x^2 + c\right)}\right) \, dx$$

```
[In] int(x^3*(a + b/cos(c + d*x^2)),x)
[Out] int(x^3*(a + b/cos(c + d*x^2)), x)
```

3.4 $\int x^2(a+b\sec(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^2\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\frac{ax^3}{3}+b\text{Int}\left(x^2\sec\left(c+dx^2\right),x\right)$$

```
[Out] 1/3*a*x^3+b*Unintegrable(x^2*sec(d*x^2+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\int x^2\left(a+b\sec\left(c+dx^2\right)\right)\,dx$$

```
[In] Int[x^2*(a + b*Sec[c + d*x^2]),x]
```

```
[Out] (a*x^3)/3 + b*Defer[Int][x^2*Sec[c + d*x^2], x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(ax^2+bx^2\sec\left(c+dx^2\right)\right)\,dx\\&=\frac{ax^3}{3}+b\int x^2\sec\left(c+dx^2\right)\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\int x^2\left(a+b\sec\left(c+dx^2\right)\right)\,dx$$

```
[In] Integrate[x^2*(a + b*Sec[c + d*x^2]),x]
[Out] Integrate[x^2*(a + b*Sec[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\sec\left(d\,x^2+c\right)\right)dx$$

```
[In] int(x^2*(a+b*sec(d*x^2+c)),x)
[Out] int(x^2*(a+b*sec(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^2\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\int\left(b\sec\left(dx^2+c\right)+a\right)x^2\,dx$$

```
[In] integrate(x^2*(a+b*sec(d*x^2+c)),x, algorithm="fricas")
[Out] integral(b*x^2*sec(d*x^2 + c) + a*x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^2\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\int x^2\left(a+b\sec\left(c+dx^2\right)\right)\,dx$$

```
[In] integrate(x**2*(a+b*sec(d*x**2+c)),x)
[Out] Integral(x**2*(a + b*sec(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 7.19

$$\int x^2(a + b\sec(c + dx^2))\ dx = \int (b\sec(dx^2 + c) + a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*sec(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 2*b*integrate((x^2*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + x^2*sin(
2*d*x^2 + 2*c)*sin(d*x^2 + c) + x^2*cos(d*x^2 + c))/(cos(2*d*x^2 + 2*c)^2 +
 sin(2*d*x^2 + 2*c)^2 + 2*cos(2*d*x^2 + 2*c) + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2(a + b\sec(c + dx^2))\ dx = \int (b\sec(dx^2 + c) + a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*sec(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^2 + c) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 13.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^2(a + b\sec(c + dx^2))\ dx = \int x^2\left(a + \frac{b}{\cos(d\,x^2 + c)}\right)\,dx$$

```
[In] int(x^2*(a + b/cos(c + d*x^2)),x)
```

```
[Out] int(x^2*(a + b/cos(c + d*x^2)), x)
```

3.5 $\int x(a + b\sec(c + dx^2))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int x\left(a + b\sec\left(c + dx^2\right)\right)\,dx = \frac{ax^2}{2} + \frac{b\operatorname{arctanh}(\sin(c + dx^2))}{2d}$$

```
[Out] 1/2*a*x^2+1/2*b*arctanh(sin(d*x^2+c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {14, 4289, 3855}

$$\int x\left(a + b\sec\left(c + dx^2\right)\right)\,dx = \frac{ax^2}{2} + \frac{b\operatorname{arctanh}(\sin(c + dx^2))}{2d}$$

```
[In] Int[x*(a + b*Sec[c + d*x^2]),x]
```

```
[Out] (a*x^2)/2 + (b*ArcTanh[Sin[c + d*x^2]])/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax + bx\sec\left(c+dx^2\right)\right)\,dx \\
&= \frac{ax^2}{2} + b\int x\sec\left(c+dx^2\right)\,dx \\
&= \frac{ax^2}{2} + \frac{1}{2}b\text{Subst}\left(\int \sec(c+dx)\,dx, x, x^2\right) \\
&= \frac{ax^2}{2} + \frac{b\text{arctanh}(\sin\left(c+dx^2\right))}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x\left(a+b\sec\left(c+dx^2\right)\right)\,dx = \frac{ax^2}{2} + \frac{b\text{arctanh}(\sin\left(c+dx^2\right))}{2d}$$

```
[In] Integrate[x*(a + b*Sec[c + d*x^2]),x]
[Out] (a*x^2)/2 + (b*ArcTanh[Sin[c + d*x^2]])/(2*d)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{b\ln\left(\sec\left(d\,x^{2}+c\right)+\tan\left(d\,x^{2}+c\right)\right)}{2d}$	32
derivativedivides	$\frac{\left(d\,x^{2}+c\right)a+b\ln\left(\sec\left(d\,x^{2}+c\right)+\tan\left(d\,x^{2}+c\right)\right)}{2d}$	36
default	$\frac{\left(d\,x^{2}+c\right)a+b\ln\left(\sec\left(d\,x^{2}+c\right)+\tan\left(d\,x^{2}+c\right)\right)}{2d}$	36
parallelrisch	$\frac{ad\,x^{2}-b\ln\left(\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)-1\right)+b\ln\left(\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)+1\right)}{2d}$	46
norman	$\frac{a\,x^{2}}{2}-\frac{b\ln\left(\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{b\ln\left(\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)+1\right)}{2d}$	48
risch	$\frac{a\,x^{2}}{2}+\frac{b\ln\left(\mathrm{e}^{i\left(d\,x^{2}+c\right)}+i\right)}{2d}-\frac{b\ln\left(\mathrm{e}^{i\left(d\,x^{2}+c\right)}-i\right)}{2d}$	50

```
[In] int(x*(a+b*sec(d*x^2+c)),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x^2+1/2*b/d*ln(sec(d*x^2+c)+tan(d*x^2+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int x\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\frac{2\,adx^2+b\log\left(\sin\left(dx^2+c\right)+1\right)-b\log\left(-\sin\left(dx^2+c\right)+1\right)}{4\,d}$$

```
[In] integrate(x*(a+b*sec(d*x^2+c)),x, algorithm="fricas")
[Out] 1/4*(2*a*d*x^2 + b*log(sin(d*x^2 + c) + 1) - b*log(-sin(d*x^2 + c) + 1))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 2.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int x\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\begin{cases}\frac{a(c+dx^2)+b\log\left(\tan\left(c+dx^2\right)+\sec\left(c+dx^2\right)\right)}{2d} & \text{for } d\neq 0\\ \frac{x^2(a+b\sec(c))}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(a+b*sec(d*x**2+c)),x)
[Out] Piecewise(((a*(c + d*x**2) + b*log(tan(c + d*x**2) + sec(c + d*x**2)))/(2*d
), Ne(d, 0)), (x**2*(a + b*sec(c))/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int x\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\frac{1}{2}\,ax^2+\frac{b\log\left(\sec\left(dx^2+c\right)+\tan\left(dx^2+c\right)\right)}{2\,d}$$

```
[In] integrate(x*(a+b*sec(d*x^2+c)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/2*b*log(sec(d*x^2 + c) + tan(d*x^2 + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(22) = 44.

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int x\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\frac{(dx^2+c)a+b\log\left(\left|\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+1\right|\right)-b\log\left(\left|\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)-1\right|\right)}{2\,d}$$

```
[In] integrate(x*(a+b*sec(d*x^2+c)),x, algorithm="giac")
[Out] 1/2*((d*x^2 + c)*a + b*log(abs(tan(1/2*d*x^2 + 1/2*c) + 1)) - b*log(abs(tan
(1/2*d*x^2 + 1/2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.58

$$\int x\left(a+b\sec\left(c+dx^2\right)\right)\,dx=\frac{a\,x^2}{2}+\frac{b\,\ln\left(-b\,x\,2\mathrm{i}-2\,b\,x\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{2\,d}-\frac{b\,\ln\left(b\,x\,2\mathrm{i}-2\,b\,x\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{2\,d}$$

```
[In] int(x*(a + b/cos(c + d*x^2)),x)
[Out] (a*x^2)/2 + (b*log(- b*x*2i - 2*b*x*exp(d*x^2*1i)*exp(c*1i)))/(2*d) - (b*lo
g(b*x*2i - 2*b*x*exp(d*x^2*1i)*exp(c*1i)))/(2*d)
```

3.6 $\int \frac{a+b\sec(c+dx^2)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\sec(c+dx^2)}{x}\,dx = a\log(x)+b\text{Int}\left(\frac{\sec(c+dx^2)}{x},x\right)$$

```
[Out] a*ln(x)+b*Unintegrable(sec(d*x^2+c)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sec(c+dx^2)}{x}\,dx = \int \frac{a+b\sec(c+dx^2)}{x}\,dx$$

```
[In] Int[(a + b*Sec[c + d*x^2])/x,x]
```

```
[Out] a*Log[x] + b*Defer[Int][Sec[c + d*x^2]/x, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a}{x}+\frac{b\sec(c+dx^2)}{x}\right)dx\\ &= a\log(x)+b\int \frac{\sec(c+dx^2)}{x}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\sec(c + dx^2)}{x}\,dx = \int \frac{a + b\sec(c + dx^2)}{x}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x^2])/x,x]
[Out] Integrate[(a + b*Sec[c + d*x^2])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a + b\sec(d\,x^2 + c)}{x}dx$$

```
[In] int((a+b*sec(d*x^2+c))/x,x)
[Out] int((a+b*sec(d*x^2+c))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\sec(c + dx^2)}{x}\,dx = \int \frac{b\sec(dx^2 + c) + a}{x}\,dx$$

```
[In] integrate((a+b*sec(d*x^2+c))/x,x, algorithm="fricas")
[Out] integral((b*sec(d*x^2 + c) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a + b\sec(c + dx^2)}{x}\,dx = \int \frac{a + b\sec(c + dx^2)}{x}\,dx$$

```
[In] integrate((a+b*sec(d*x**2+c))/x,x)
[Out] Integral((a + b*sec(c + d*x**2))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 6.75

$$\int \frac{a+b\sec(c+dx^2)}{x}\,dx = \int \frac{b\sec(dx^2+c)+a}{x}\,dx$$

```
[In] integrate((a+b*sec(d*x^2+c))/x,x, algorithm="maxima")
```

```
[Out] 2*b*integrate((cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + sin(2*d*x^2 + 2*c)*sin(d
*x^2 + c) + cos(d*x^2 + c))/(x*cos(2*d*x^2 + 2*c)^2 + x*sin(2*d*x^2 + 2*c)^
2 + 2*x*cos(2*d*x^2 + 2*c) + x), x) + a*log(x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\sec(c+dx^2)}{x}\,dx = \int \frac{b\sec(dx^2+c)+a}{x}\,dx$$

```
[In] integrate((a+b*sec(d*x^2+c))/x,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^2 + c) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 14.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a+b\sec(c+dx^2)}{x}\,dx = \int \frac{a+\frac{b}{\cos(d\,x^2+c)}}{x}\,dx$$

```
[In] int((a + b/cos(c + d*x^2))/x,x)
```

```
[Out] int((a + b/cos(c + d*x^2))/x, x)
```

3.7 $\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx = -\frac{a}{x} + b\text{Int}\left(\frac{\sec(c+dx^2)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(sec(d*x^2+c)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx = \int \frac{a+b\sec(c+dx^2)}{x^2}\,dx$$

```
[In] Int[(a + b*Sec[c + d*x^2])/x^2,x]
```

```
[Out] -(a/x) + b*Defer[Int][Sec[c + d*x^2]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a}{x^2} + \frac{b\sec(c+dx^2)}{x^2}\right)dx \\ &= -\frac{a}{x} + b\int \frac{\sec(c+dx^2)}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx=\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x^2])/x^2,x]
[Out] Integrate[(a + b*Sec[c + d*x^2])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec(d\,x^2+c)}{x^2}dx$$

```
[In] int((a+b*sec(d*x^2+c))/x^2,x)
[Out] int((a+b*sec(d*x^2+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx=\int \frac{b\sec(dx^2+c)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sec(d*x^2+c))/x^2,x, algorithm="fricas")
[Out] integral((b*sec(d*x^2 + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx=\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx$$

```
[In] integrate((a+b*sec(d*x**2+c))/x**2,x)
[Out] Integral((a + b*sec(c + d*x**2))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 7.38

$$\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx = \int \frac{b\sec(dx^2+c)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sec(d*x^2+c))/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*integrate((cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + sin(2*d*x^2 + 2*c)*sin(d
*x^2 + c) + cos(d*x^2 + c))/(x^2*cos(2*d*x^2 + 2*c)^2 + x^2*sin(2*d*x^2 + 2
*c)^2 + 2*x^2*cos(2*d*x^2 + 2*c) + x^2), x) - a/x
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx = \int \frac{b\sec(dx^2+c)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sec(d*x^2+c))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^2 + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 13.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx = \int \frac{a+\frac{b}{\cos(d\,x^2+c)}}{x^2}\,dx$$

```
[In] int((a + b/cos(c + d*x^2))/x^2,x)
```

```
[Out] int((a + b/cos(c + d*x^2))/x^2, x)
```

3.8 $\int x^5(a+b\sec(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 242

$$\int x^5\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=-\frac{ib^2x^4}{2d}+\frac{a^2x^6}{6}-\frac{2iabx^4\arctan\left(e^{i(c+dx^2)}\right)}{d}+\frac{b^2x^2\log\left(1+e^{2i(c+dx^2)}\right)}{d^2}+\frac{2iabx^2\,\mathrm{PolyLog}\left(2,-ie^{i(c+dx^2)}\right)}{d^2}-\frac{2iabx^2\,\mathrm{PolyLog}\left(2,ie^{i(c+dx^2)}\right)}{d^2}-\frac{ib^2\,\mathrm{PolyLog}\left(2,-e^{2i(c+dx^2)}\right)}{2d^3}-\frac{2ab\,\mathrm{PolyLog}\left(3,-ie^{i(c+dx^2)}\right)}{d^3}+\frac{2ab\,\mathrm{PolyLog}\left(3,ie^{i(c+dx^2)}\right)}{d^3}+\frac{b^2x^4\tan\left(c+dx^2\right)}{2d}$$

```
[Out] -1/2*I*b^2*x^4/d+1/6*a^2*x^6-2*I*a*b*x^4*arctan(exp(I*(d*x^2+c)))/d+b^2*x^2
*ln(1+exp(2*I*(d*x^2+c)))/d^2+2*I*a*b*x^2*polylog(2,-I*exp(I*(d*x^2+c)))/d^
2-2*I*a*b*x^2*polylog(2,I*exp(I*(d*x^2+c)))/d^2-1/2*I*b^2*polylog(2,-exp(2*
I*(d*x^2+c)))/d^3-2*a*b*polylog(3,-I*exp(I*(d*x^2+c)))/d^3+2*a*b*polylog(3,
I*exp(I*(d*x^2+c)))/d^3+1/2*b^2*x^4*tan(d*x^2+c)/d
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {4289, 4275, 4266, 2611, 2320, 6724, 4269, 3800, 2221, 2317, 2438}

$$\int x^5 \left(a+b \sec \left(c+dx^2\right)\right)^2 dx = \frac{a^2x^6}{6} - \frac{2iabx^4 \arctan \left(e^{i(c+dx^2)}\right)}{d} - \frac{2ab \operatorname{PolyLog}\left(3,-ie^{i(dx^2+c)}\right)}{d^3} + \frac{2ab \operatorname{PolyLog}\left(3,ie^{i(dx^2+c)}\right)}{d^3} + \frac{2iabx^2 \operatorname{PolyLog}\left(2,-ie^{i(dx^2+c)}\right)}{d^2} - \frac{2iabx^2 \operatorname{PolyLog}\left(2,ie^{i(dx^2+c)}\right)}{d^2} - \frac{ib^2 \operatorname{PolyLog}\left(2,-e^{2i(dx^2+c)}\right)}{2d^3} + \frac{b^2x^2 \log \left(1+e^{2i(c+dx^2)}\right)}{d^2} + \frac{b^2x^4 \tan \left(c+dx^2\right)}{2d} - \frac{ib^2x^4}{2d}$$

```
[In] Int[x^5*(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] ((-1/2*I)*b^2*x^4)/d + (a^2*x^6)/6 - ((2*I)*a*b*x^4*ArcTan[E^(I*(c + d*x^2)
)])/d + (b^2*x^2*Log[1 + E^((2*I)*(c + d*x^2))])/d^2 + ((2*I)*a*b*x^2*PolyL
og[2, (-I)*E^(I*(c + d*x^2))])/d^2 - ((2*I)*a*b*x^2*PolyLog[2, I*E^(I*(c +
d*x^2))])/d^2 - ((I/2)*b^2*PolyLog[2, -E^((2*I)*(c + d*x^2))])/d^3 - (2*a*b
*PolyLog[3, (-I)*E^(I*(c + d*x^2))])/d^3 + (2*a*b*PolyLog[3, I*E^(I*(c + d*
x^2))])/d^3 + (b^2*x^4*Tan[c + d*x^2])/(2*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2(a+b\sec(c+dx))^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x^2+2abx^2\sec(c+dx)+b^2x^2\sec^2(c+dx)\right)\,dx,x,x^2\right)\\
&= \frac{a^2x^6}{6}+(ab)\text{Subst}\left(\int x^2\sec(c+dx)\,dx,x,x^2\right)+\frac{1}{2}b^2\text{Subst}\left(\int x^2\sec^2(c+dx)\,dx,x,x^2\right)\\
&= \frac{a^2x^6}{6}-\frac{2iabx^4\arctan\left(e^{i(c+dx^2)}\right)}{d}+\frac{b^2x^4\tan\left(c+dx^2\right)}{2d}\\
&\quad-\frac{(2ab)\text{Subst}\left(\int x\log\left(1-ie^{i(c+dx)}\right)\,dx,x,x^2\right)}{d}\\
&\quad+\frac{(2ab)\text{Subst}\left(\int x\log\left(1+ie^{i(c+dx)}\right)\,dx,x,x^2\right)}{d}\\
&\quad-\frac{b^2\text{Subst}\left(\int x\tan(c+dx)\,dx,x,x^2\right)}{d}
\end{aligned}$$

$$= -\frac{ib^2x^4}{2d} + \frac{a^2x^6}{6} - \frac{2iabx^4\arctan\left(e^{i(c+dx^2)}\right)}{d} + \frac{2iabx^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx^2)}\right)}{d^2} - \frac{2iabx^2\operatorname{PolyLog}\left(2,ie^{i(c+dx^2)}\right)}{d^2} + \frac{b^2x^4\tan\left(c+dx^2\right)}{2d} - \frac{(2iab)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)\,dx,x,x^2\right)}{d^2} + \frac{(2iab)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)\,dx,x,x^2\right)}{d^2} + \frac{(2ib^2)\operatorname{Subst}\left(\int \frac{e^{2i(c+dx)}x}{1+e^{2i(c+dx)}}\,dx,x,x^2\right)}{d}$$

$$= -\frac{ib^2x^4}{2d} + \frac{a^2x^6}{6} - \frac{2iabx^4\arctan\left(e^{i(c+dx^2)}\right)}{d} + \frac{b^2x^2\log\left(1+e^{2i(c+dx^2)}\right)}{d^2} + \frac{2iabx^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx^2)}\right)}{d^2} - \frac{2iabx^2\operatorname{PolyLog}\left(2,ie^{i(c+dx^2)}\right)}{d^2} + \frac{b^2x^4\tan\left(c+dx^2\right)}{2d} - \frac{(2ab)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx,x,e^{i(c+dx^2)}\right)}{d^3} + \frac{(2ab)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x}\,dx,x,e^{i(c+dx^2)}\right)}{d^3} - \frac{b^2\operatorname{Subst}\left(\int \log\left(1+e^{2i(c+dx)}\right)\,dx,x,x^2\right)}{d^2}$$

$$= -\frac{ib^2x^4}{2d} + \frac{a^2x^6}{6} - \frac{2iabx^4\arctan\left(e^{i(c+dx^2)}\right)}{d} + \frac{b^2x^2\log\left(1+e^{2i(c+dx^2)}\right)}{d^2} + \frac{2iabx^2\operatorname{PolyLog}\left(2,-ie^{i(c+dx^2)}\right)}{d^2} - \frac{2iabx^2\operatorname{PolyLog}\left(2,ie^{i(c+dx^2)}\right)}{d^2} - \frac{2ab\operatorname{PolyLog}\left(3,-ie^{i(c+dx^2)}\right)}{d^3} + \frac{2ab\operatorname{PolyLog}\left(3,ie^{i(c+dx^2)}\right)}{d^3} + \frac{b^2x^4\tan\left(c+dx^2\right)}{2d} + \frac{(ib^2)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2i(c+dx^2)}\right)}{2d^3}$$

$$
\begin{aligned}
&= -\frac{ib^2x^4}{2d} + \frac{a^2x^6}{6} - \frac{2iabx^4 \arctan\left(e^{i(c+dx^2)}\right)}{d} + \frac{b^2x^2 \log\left(1+e^{2i(c+dx^2)}\right)}{d^2} \\
&+ \frac{2iabx^2 \operatorname{PolyLog}\left(2, -ie^{i(c+dx^2)}\right)}{d^2} - \frac{2iabx^2 \operatorname{PolyLog}\left(2, ie^{i(c+dx^2)}\right)}{d^2} \\
&- \frac{ib^2 \operatorname{PolyLog}\left(2, -e^{2i(c+dx^2)}\right)}{2d^3} - \frac{2ab \operatorname{PolyLog}\left(3, -ie^{i(c+dx^2)}\right)}{d^3} \\
&+ \frac{2ab \operatorname{PolyLog}\left(3, ie^{i(c+dx^2)}\right)}{d^3} + \frac{b^2x^4 \tan\left(c+dx^2\right)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$
\int x^5\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx = \frac{-3ib^2d^2x^4 + a^2d^3x^6 - 12iabd^2x^4 \arctan\left(e^{i(c+dx^2)}\right) + 6b^2dx^2 \log\left(1+e^{2i(c+dx^2)}\right) + 12iabdx^2 \operatorname{PolyLog}\left(2, -\right.}{}
$$

```
[In] Integrate[x^5*(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] ((-3*I)*b^2*d^2*x^4 + a^2*d^3*x^6 - (12*I)*a*b*d^2*x^4*ArcTan[E^(I*(c + d*x
^2))] + 6*b^2*d*x^2*Log[1 + E^((2*I)*(c + d*x^2))] + (12*I)*a*b*d*x^2*PolyL
og[2, (-I)*E^(I*(c + d*x^2))] - (12*I)*a*b*d*x^2*PolyLog[2, I*E^(I*(c + d*x
^2))] - (3*I)*b^2*PolyLog[2, -E^((2*I)*(c + d*x^2))] - 12*a*b*PolyLog[3, (-
I)*E^(I*(c + d*x^2))] + 12*a*b*PolyLog[3, I*E^(I*(c + d*x^2))] + 3*b^2*d^2*
x^4*Tan[c + d*x^2])/(6*d^3)
```

Maple [F]

$$
\int x^5\left(a+b\sec\left(d\,x^2+c\right)\right)^2dx
$$

```
[In] int(x^5*(a+b*sec(d*x^2+c))^2,x)
```

```
[Out] int(x^5*(a+b*sec(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 799 vs. 2(199) = 398.

Time = 0.32 (sec) , antiderivative size = 799, normalized size of antiderivative = 3.30

$$\int x^5 \left(a + b \sec \left(c + dx^2\right)\right)^2 \, dx = \frac{a^2d^3x^6\cos\left(dx^2+c\right) + 3\,b^2d^2x^4\sin\left(dx^2+c\right) - 6\,ab\cos\left(dx^2+c\right)\operatorname{polylog}(3, i\,\cos\left(dx^2+c\right) + \sin\left(dx^2+c\right)}{}$$

```
[In] integrate(x^5*(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*d^3*x^6*cos(d*x^2 + c) + 3*b^2*d^2*x^4*sin(d*x^2 + c) - 6*a*b*cos(
d*x^2 + c)*polylog(3, I*cos(d*x^2 + c) + sin(d*x^2 + c)) + 6*a*b*cos(d*x^2
+ c)*polylog(3, I*cos(d*x^2 + c) - sin(d*x^2 + c)) - 6*a*b*cos(d*x^2 + c)*p
olylog(3, -I*cos(d*x^2 + c) + sin(d*x^2 + c)) + 6*a*b*cos(d*x^2 + c)*polylo
g(3, -I*cos(d*x^2 + c) - sin(d*x^2 + c)) - 3*(2*I*a*b*d*x^2 - I*b^2)*cos(d*
x^2 + c)*dilog(I*cos(d*x^2 + c) + sin(d*x^2 + c)) - 3*(2*I*a*b*d*x^2 + I*b^
2)*cos(d*x^2 + c)*dilog(I*cos(d*x^2 + c) - sin(d*x^2 + c)) - 3*(-2*I*a*b*d*
x^2 + I*b^2)*cos(d*x^2 + c)*dilog(-I*cos(d*x^2 + c) + sin(d*x^2 + c)) - 3*(
-2*I*a*b*d*x^2 - I*b^2)*cos(d*x^2 + c)*dilog(-I*cos(d*x^2 + c) - sin(d*x^2
+ c)) + 3*(a*b*c^2 - b^2*c)*cos(d*x^2 + c)*log(cos(d*x^2 + c) + I*sin(d*x^2
 + c) + I) - 3*(a*b*c^2 + b^2*c)*cos(d*x^2 + c)*log(cos(d*x^2 + c) - I*sin(
d*x^2 + c) + I) + 3*(a*b*d^2*x^4 + b^2*d*x^2 - a*b*c^2 + b^2*c)*cos(d*x^2 +
 c)*log(I*cos(d*x^2 + c) + sin(d*x^2 + c) + 1) - 3*(a*b*d^2*x^4 - b^2*d*x^2
 - a*b*c^2 - b^2*c)*cos(d*x^2 + c)*log(I*cos(d*x^2 + c) - sin(d*x^2 + c) +
1) + 3*(a*b*d^2*x^4 + b^2*d*x^2 - a*b*c^2 + b^2*c)*cos(d*x^2 + c)*log(-I*co
s(d*x^2 + c) + sin(d*x^2 + c) + 1) - 3*(a*b*d^2*x^4 - b^2*d*x^2 - a*b*c^2 -
 b^2*c)*cos(d*x^2 + c)*log(-I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1) + 3*(a*b
*c^2 - b^2*c)*cos(d*x^2 + c)*log(-cos(d*x^2 + c) + I*sin(d*x^2 + c) + I) -
3*(a*b*c^2 + b^2*c)*cos(d*x^2 + c)*log(-cos(d*x^2 + c) - I*sin(d*x^2 + c) +
 I))/(d^3*cos(d*x^2 + c))
```

Sympy [F]

$$\int x^5 \left(a + b \sec \left(c + dx^2\right)\right)^2 \, dx = \int x^5 \left(a + b \sec \left(c + dx^2\right)\right)^2 \, dx$$

```
[In] integrate(x**5*(a+b*sec(d*x**2+c))**2,x)
```

```
[Out] Integral(x**5*(a + b*sec(c + d*x**2))**2, x)
```

Maxima [F]

$$\int x^5 \left(a + b \sec \left(c + dx^2\right)\right)^2 \, dx = \int \left(b \sec \left(dx^2 + c\right) + a\right)^2 x^5 \, dx$$

```
[In] integrate(x^5*(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*x^6 + (b^2*x^4*sin(2*d*x^2 + 2*c) + (d*cos(2*d*x^2 + 2*c)^2 + d*sin
(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d)*integrate(4*(a*b*d*x^5*cos(
2*d*x^2 + 2*c)*cos(d*x^2 + c) + a*b*d*x^5*cos(d*x^2 + c) + (a*b*d*x^5*sin(d
*x^2 + c) - b^2*x^3)*sin(2*d*x^2 + 2*c))/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*
d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d), x))/(d*cos(2*d*x^2 + 2*c)^2 +
 d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d)
```

Giac [F]

$$\int x^5 \left(a + b \sec \left(c + dx^2\right)\right)^2 \, dx = \int \left(b \sec \left(dx^2 + c\right) + a\right)^2 x^5 \, dx$$

```
[In] integrate(x^5*(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^2 + c) + a)^2*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a + b \sec \left(c + dx^2\right)\right)^2 \, dx = \int x^5 \left(a + \frac{b}{\cos \left(d\,x^2 + c\right)}\right)^2 \, dx$$

```
[In] int(x^5*(a + b/cos(c + d*x^2))^2,x)
```

```
[Out] int(x^5*(a + b/cos(c + d*x^2))^2, x)
```

3.9 $\int x^4(a+b\sec(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^4\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\text{Int}\left(x^4\left(a+b\sec\left(c+dx^2\right)\right)^2,x\right)$$

```
[Out] Unintegrable(x^4*(a+b*sec(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^4\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\int x^4\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx$$

```
[In] Int[x^4*(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] Defer[Int][x^4*(a + b*Sec[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral}=\int x^4\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx$$

Mathematica [N/A]

Not integrable

Time = 12.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4 \left(a+b \sec \left(c+d x^2\right)\right)^2 \, dx=\int x^4 \left(a+b \sec \left(c+d x^2\right)\right)^2 \, dx$$

```
[In] Integrate[x^4*(a + b*Sec[c + d*x^2])^2,x]
[Out] Integrate[x^4*(a + b*Sec[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^{4} \left(a+b \sec \left(d \,x^{2}+c \right)\right)^{2}d x$$

```
[In] int(x^4*(a+b*sec(d*x^2+c))^2,x)
[Out] int(x^4*(a+b*sec(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^4 \left(a+b \sec \left(c+d x^2\right)\right)^2 \, dx=\int { {\left(b \sec \left(d x^{2} + c\right) + a\right)}^{2} x^{4} \,d x }$$

```
[In] integrate(x^4*(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(b^2*x^4*sec(d*x^2 + c)^2 + 2*a*b*x^4*sec(d*x^2 + c) + a^2*x^4, x)
```

Sympy [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^4 \left(a+b \sec \left(c+d x^2\right)\right)^2 \, dx=\int x^{4} \left(a + b \sec {\left(c + d x^{2} \right)}\right)^{2}\, dx$$

```
[In] integrate(x**4*(a+b*sec(d*x**2+c))**2,x)
[Out] Integral(x**4*(a + b*sec(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 256, normalized size of antiderivative = 14.22

$$\int x^4 (a + b \sec (c + dx^2))^2 \, dx = \int (b \sec (dx^2 + c) + a)^2 x^4 \, dx$$

```
[In] integrate(x^4*(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 + (b^2*x^3*sin(2*d*x^2 + 2*c) + (d*cos(2*d*x^2 + 2*c)^2 + d*sin
(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d)*integrate((4*a*b*d*x^4*cos(
2*d*x^2 + 2*c)*cos(d*x^2 + c) + 4*a*b*d*x^4*cos(d*x^2 + c) + (4*a*b*d*x^4*s
in(d*x^2 + c) - 3*b^2*x^2)*sin(2*d*x^2 + 2*c))/(d*cos(2*d*x^2 + 2*c)^2 + d*
sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d), x))/(d*cos(2*d*x^2 + 2*
c)^2 + d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d)
```

Giac [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4 (a + b \sec (c + dx^2))^2 \, dx = \int (b \sec (dx^2 + c) + a)^2 x^4 \, dx$$

```
[In] integrate(x^4*(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^2 + c) + a)^2*x^4, x)
```

Mupad [N/A]

Not integrable

Time = 14.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^4 (a + b \sec (c + dx^2))^2 \, dx = \int x^4 \left(a + \frac{b}{\cos (d x^2 + c)} \right)^2 \, dx$$

```
[In] int(x^4*(a + b/cos(c + d*x^2))^2,x)
```

```
[Out] int(x^4*(a + b/cos(c + d*x^2))^2, x)
```

3.10 $\int x^3(a+b\sec(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 133

$$\int x^3(a+b\sec(c+dx^2))^2\,dx = \frac{a^2x^4}{4} - \frac{2iabx^2\arctan\left(e^{i(c+dx^2)}\right)}{d} + \frac{b^2\log(\cos(c+dx^2))}{2d^2} + \frac{iab\,\text{PolyLog}\left(2,-ie^{i(c+dx^2)}\right)}{d^2} - \frac{iab\,\text{PolyLog}\left(2,ie^{i(c+dx^2)}\right)}{d^2} + \frac{b^2x^2\tan(c+dx^2)}{2d}$$

```
[Out] 1/4*a^2*x^4-2*I*a*b*x^2*arctan(exp(I*(d*x^2+c)))/d+1/2*b^2*ln(cos(d*x^2+c))
/d^2+I*a*b*polylog(2,-I*exp(I*(d*x^2+c)))/d^2-I*a*b*polylog(2,I*exp(I*(d*x^
2+c)))/d^2+1/2*b^2*x^2*tan(d*x^2+c)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {4289, 4275, 4266, 2317, 2438, 4269, 3556}

$$\int x^3(a+b\sec(c+dx^2))^2\,dx = \frac{a^2x^4}{4} - \frac{2iabx^2\arctan\left(e^{i(c+dx^2)}\right)}{d} + \frac{iab\,\text{PolyLog}\left(2,-ie^{i(dx^2+c)}\right)}{d^2} - \frac{iab\,\text{PolyLog}\left(2,ie^{i(dx^2+c)}\right)}{d^2} + \frac{b^2\log(\cos(c+dx^2))}{2d^2} + \frac{b^2x^2\tan(c+dx^2)}{2d}$$

```
[In] Int[x^3*(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] (a^2*x^4)/4 - ((2*I)*a*b*x^2*ArcTan[E^(I*(c + d*x^2))])/d + (b^2*Log[Cos[c
+ d*x^2]])/(2*d^2) + (I*a*b*PolyLog[2, (-I)*E^(I*(c + d*x^2))])/d^2 - (I*a*
b*PolyLog[2, I*E^(I*(c + d*x^2))])/d^2 + (b^2*x^2*Tan[c + d*x^2])/(2*d)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
```

```
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(a+b\sec(c+dx))^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x+2abx\sec(c+dx)+b^2x\sec^2(c+dx)\right)\,dx,x,x^2\right)\\
&= \frac{a^2x^4}{4}+(ab)\text{Subst}\left(\int x\sec(c+dx)\,dx,x,x^2\right)+\frac{1}{2}b^2\text{Subst}\left(\int x\sec^2(c+dx)\,dx,x,x^2\right)\\
&= \frac{a^2x^4}{4}-\frac{2iabx^2\arctan\left(e^{i(c+dx^2)}\right)}{d}+\frac{b^2x^2\tan\left(c+dx^2\right)}{2d}\\
&\quad-\frac{(ab)\text{Subst}\left(\int\log\left(1-ie^{i(c+dx)}\right)\,dx,x,x^2\right)}{d}\\
&\quad+\frac{(ab)\text{Subst}\left(\int\log\left(1+ie^{i(c+dx)}\right)\,dx,x,x^2\right)}{d}-\frac{b^2\text{Subst}\left(\int\tan(c+dx)\,dx,x,x^2\right)}{2d}\\
&= \frac{a^2x^4}{4}-\frac{2iabx^2\arctan\left(e^{i(c+dx^2)}\right)}{d}+\frac{b^2\log\left(\cos\left(c+dx^2\right)\right)}{2d^2}+\frac{b^2x^2\tan\left(c+dx^2\right)}{2d}\\
&\quad+\frac{(iab)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{i(c+dx^2)}\right)}{d^2}-\frac{(iab)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{i(c+dx^2)}\right)}{d^2}\\
&= \frac{a^2x^4}{4}-\frac{2iabx^2\arctan\left(e^{i(c+dx^2)}\right)}{d}+\frac{b^2\log\left(\cos\left(c+dx^2\right)\right)}{2d^2}\\
&\quad+\frac{iab\,\text{PolyLog}\left(2,-ie^{i(c+dx^2)}\right)}{d^2}-\frac{iab\,\text{PolyLog}\left(2,ie^{i(c+dx^2)}\right)}{d^2}+\frac{b^2x^2\tan\left(c+dx^2\right)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$
\int x^3\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx
$$

$$
=\frac{a^2d^2x^4-8iabdx^2\arctan\left(e^{i(c+dx^2)}\right)+2b^2\log\left(\cos\left(c+dx^2\right)\right)+4iab\,\text{PolyLog}\left(2,-ie^{i(c+dx^2)}\right)-4iab\,\text{PolyL}}{4d^2}
$$

```
[In] Integrate[x^3*(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] (a^2*d^2*x^4 - (8*I)*a*b*d*x^2*ArcTan[E^(I*(c + d*x^2))] + 2*b^2*Log[Cos[c
+ d*x^2]] + (4*I)*a*b*PolyLog[2, (-I)*E^(I*(c + d*x^2))] - (4*I)*a*b*PolyLo
g[2, I*E^(I*(c + d*x^2))] + 2*b^2*d*x^2*Tan[c + d*x^2])/(4*d^2)
```

Maple [F]

$$\int x^{3} \left(a +b \sec \left(d \,x^{2}+c \right)\right)^{2}d x$$

```
[In] int(x^3*(a+b*sec(d*x^2+c))^2,x)
[Out] int(x^3*(a+b*sec(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(109) = 218.

Time = 0.33 (sec) , antiderivative size = 525, normalized size of antiderivative = 3.95

$$\int x^3 \left(a+b\sec\left(c+dx^2\right)\right)^2\,dx$$
$$= \frac{a^2d^2x^4\cos\left(dx^2+c\right)+2\,b^2dx^2\sin\left(dx^2+c\right)-2i\,ab\cos\left(dx^2+c\right)\mathrm{Li}_2(i\,\cos\left(dx^2+c\right)+\sin\left(dx^2+c\right))-2i\,a}{}$$

```
[In] integrate(x^3*(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
[Out] 1/4*(a^2*d^2*x^4*cos(d*x^2 + c) + 2*b^2*d*x^2*sin(d*x^2 + c) - 2*I*a*b*cos(
d*x^2 + c)*dilog(I*cos(d*x^2 + c) + sin(d*x^2 + c)) - 2*I*a*b*cos(d*x^2 + c
)*dilog(I*cos(d*x^2 + c) - sin(d*x^2 + c)) + 2*I*a*b*cos(d*x^2 + c)*dilog(-
I*cos(d*x^2 + c) + sin(d*x^2 + c)) + 2*I*a*b*cos(d*x^2 + c)*dilog(-I*cos(d*
x^2 + c) - sin(d*x^2 + c)) - (2*a*b*c - b^2)*cos(d*x^2 + c)*log(cos(d*x^2 +
 c) + I*sin(d*x^2 + c) + I) + (2*a*b*c + b^2)*cos(d*x^2 + c)*log(cos(d*x^2
+ c) - I*sin(d*x^2 + c) + I) + 2*(a*b*d*x^2 + a*b*c)*cos(d*x^2 + c)*log(I*c
os(d*x^2 + c) + sin(d*x^2 + c) + 1) - 2*(a*b*d*x^2 + a*b*c)*cos(d*x^2 + c)*
log(I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1) + 2*(a*b*d*x^2 + a*b*c)*cos(d*x^
2 + c)*log(-I*cos(d*x^2 + c) + sin(d*x^2 + c) + 1) - 2*(a*b*d*x^2 + a*b*c)*
cos(d*x^2 + c)*log(-I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1) - (2*a*b*c - b^2
)*cos(d*x^2 + c)*log(-cos(d*x^2 + c) + I*sin(d*x^2 + c) + I) + (2*a*b*c + b
^2)*cos(d*x^2 + c)*log(-cos(d*x^2 + c) - I*sin(d*x^2 + c) + I))/(d^2*cos(d*
x^2 + c))
```

Sympy [F]

$$\int x^3 \left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\int x^3 \left(a+b\sec\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x**3*(a+b*sec(d*x**2+c))**2,x)
```

```
[Out] Integral(x**3*(a + b*sec(c + d*x**2))**2, x)
```

Maxima [F]

$$\int x^3 \left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\int \left(b\sec\left(dx^2+c\right)+a\right)^2 x^3\,dx$$

```
[In] integrate(x^3*(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 + 1/4*(4*b^2*d*x^2*sin(2*d*x^2 + 2*c) + 16*(a*b*d^3*cos(2*d*x^2
 + 2*c)^2 + a*b*d^3*sin(2*d*x^2 + 2*c)^2 + 2*a*b*d^3*cos(2*d*x^2 + 2*c) + a
*b*d^3)*integrate((x^3*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + x^3*sin(2*d*x^2
+ 2*c)*sin(d*x^2 + c) + x^3*cos(d*x^2 + c))/(d*cos(2*d*x^2 + 2*c)^2 + d*sin
(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d), x) + (b^2*cos(2*d*x^2 + 2*
c)^2 + b^2*sin(2*d*x^2 + 2*c)^2 + 2*b^2*cos(2*d*x^2 + 2*c) + b^2)*log(cos(2
*d*x^2 + 2*c)^2 + sin(2*d*x^2 + 2*c)^2 + 2*cos(2*d*x^2 + 2*c) + 1))/(d^2*co
s(2*d*x^2 + 2*c)^2 + d^2*sin(2*d*x^2 + 2*c)^2 + 2*d^2*cos(2*d*x^2 + 2*c) +
d^2)
```

Giac [F]

$$\int x^3 \left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\int \left(b\sec\left(dx^2+c\right)+a\right)^2 x^3\,dx$$

```
[In] integrate(x^3*(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^2 + c) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \sec \left(c + dx^2\right)\right)^2 \, dx = \int x^3 \left(a + \frac{b}{\cos \left(d \, x^2 + c\right)}\right)^2 \, dx$$

```
[In] int(x^3*(a + b/cos(c + d*x^2))^2,x)
[Out] int(x^3*(a + b/cos(c + d*x^2))^2, x)
```

3.11 $\int x^2(a+b\sec(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^2(a+b\sec(c+dx^2))^2\,dx=\operatorname{Int}\left(x^2(a+b\sec(c+dx^2))^2,x\right)$$

```
[Out] Unintegrable(x^2*(a+b*sec(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2(a+b\sec(c+dx^2))^2\,dx=\int x^2(a+b\sec(c+dx^2))^2\,dx$$

```
[In] Int[x^2*(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] Defer[Int][x^2*(a + b*Sec[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral}=\int x^2(a+b\sec(c+dx^2))^2\,dx$$

Mathematica [N/A]

Not integrable

Time = 9.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\int x^2\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx$$

```
[In] Integrate[x^2*(a + b*Sec[c + d*x^2])^2,x]
[Out] Integrate[x^2*(a + b*Sec[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^{2}\left(a+b\sec\left(d\,x^{2}+c\right)\right)^{2}dx$$

```
[In] int(x^2*(a+b*sec(d*x^2+c))^2,x)
[Out] int(x^2*(a+b*sec(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^2\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\int\left(b\sec\left(dx^2+c\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*sec(d*x^2 + c)^2 + 2*a*b*x^2*sec(d*x^2 + c) + a^2*x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^2\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\int x^2\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x**2*(a+b*sec(d*x**2+c))**2,x)
[Out] Integral(x**2*(a + b*sec(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 251, normalized size of antiderivative = 13.94

$$\int x^2\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\int\left(b\sec\left(dx^2+c\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + (b^2*x*sin(2*d*x^2 + 2*c) + (d*cos(2*d*x^2 + 2*c)^2 + d*sin(2
*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d)*integrate((4*a*b*d*x^2*cos(2*
d*x^2 + 2*c)*cos(d*x^2 + c) + 4*a*b*d*x^2*cos(d*x^2 + c) + (4*a*b*d*x^2*sin
(d*x^2 + c) - b^2)*sin(2*d*x^2 + 2*c))/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*
x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d), x))/(d*cos(2*d*x^2 + 2*c)^2 + d
*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d)
```

Giac [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\int\left(b\sec\left(dx^2+c\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^2 + c) + a)^2*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 13.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^2\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\int x^2\left(a+\frac{b}{\cos\left(d\,x^2+c\right)}\right)^2\,dx$$

```
[In] int(x^2*(a + b/cos(c + d*x^2))^2,x)
```

```
[Out] int(x^2*(a + b/cos(c + d*x^2))^2, x)
```

3.12 $\int x(a+b\sec(c+dx^2))^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int x\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\frac{a^2x^2}{2}+\frac{ab\,\text{arctanh}(\sin\left(c+dx^2\right))}{d}+\frac{b^2\tan\left(c+dx^2\right)}{2d}$$

```
[Out] 1/2*a^2*x^2+a*b*arctanh(sin(d*x^2+c))/d+1/2*b^2*tan(d*x^2+c)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4289, 3858, 3855, 3852, 8}

$$\int x\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\frac{a^2x^2}{2}+\frac{ab\,\text{arctanh}(\sin\left(c+dx^2\right))}{d}+\frac{b^2\tan\left(c+dx^2\right)}{2d}$$

```
[In] Int[x*(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] (a^2*x^2)/2 + (a*b*ArcTanh[Sin[c + d*x^2]])/d + (b^2*Tan[c + d*x^2])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (a+b\sec(c+dx))^2\,dx, x, x^2\right)\\
&= \frac{a^2x^2}{2} + (ab)\text{Subst}\left(\int \sec(c+dx)\,dx, x, x^2\right) + \frac{1}{2}b^2\text{Subst}\left(\int \sec^2(c+dx)\,dx, x, x^2\right)\\
&= \frac{a^2x^2}{2} + \frac{ab\,\text{arctanh}(\sin(c+dx^2))}{d} - \frac{b^2\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx^2)\right)}{2d}\\
&= \frac{a^2x^2}{2} + \frac{ab\,\text{arctanh}(\sin(c+dx^2))}{d} + \frac{b^2\tan(c+dx^2)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int x\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx = \frac{a^2dx^2 + 2ab\,\text{arctanh}(\sin(c+dx^2)) + b^2\tan(c+dx^2)}{2d}$$

```
[In] Integrate[x*(a + b*Sec[c + d*x^2])^2,x]
[Out] (a^2*d*x^2 + 2*a*b*ArcTanh[Sin[c + d*x^2]] + b^2*Tan[c + d*x^2])/(2*d)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

method	result
parts	$\frac{a^{2} x^{2}}{2}+\frac{b^{2} \tan \left(d \,x^{2}+c \right)}{2 d}+\frac{b a \ln \left(\sec \left(d \,x^{2}+c \right)+\tan \left(d \,x^{2}+c \right)\right)}{d}$
derivativedivides	$\frac{a^{2} \left(d \,x^{2}+c \right)+2 b a \ln \left(\sec \left(d \,x^{2}+c \right)+\tan \left(d \,x^{2}+c \right)\right)+b^{2} \tan \left(d \,x^{2}+c \right)}{2 d}$
default	$\frac{a^{2} \left(d \,x^{2}+c \right)+2 b a \ln \left(\sec \left(d \,x^{2}+c \right)+\tan \left(d \,x^{2}+c \right)\right)+b^{2} \tan \left(d \,x^{2}+c \right)}{2 d}$
risch	$\frac{a^{2} x^{2}}{2}+\frac{i b^{2}}{d \left(1+{\mathrm e}^{2 i \left(d \,x^{2}+c \right)}\right)}+\frac{b a \ln \left({\mathrm e}^{i \left(d \,x^{2}+c \right)}+i \right)}{d}-\frac{b a \ln \left({\mathrm e}^{i \left(d \,x^{2}+c \right)}-i \right)}{d}$
parallelrisch	$\frac{a^{2} d \,x^{2} \cos \left(d \,x^{2}+c \right)-2 b a \ln \left(\tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)-1\right) \cos \left(d \,x^{2}+c \right)+2 b a \ln \left(\tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)+1\right) \cos \left(d \,x^{2}+c \right)+b^{2} \sin \left(d \,x^{2}+c \right)}{2 d \cos \left(d \,x^{2}+c \right)}$
norman	$\frac{-\frac{a^{2} x^{2}}{2}+\frac{a^{2} x^{2} \tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)^{2}}{2}-\frac{b^{2} \tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)}{d}}{\tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)^{2}-1}+\frac{b a \ln \left(\tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b a \ln \left(\tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)-1\right)}{d}$

```
[In] int(x*(a+b*sec(d*x^2+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*a^2*x^2+1/2*b^2*tan(d*x^2+c)/d+b*a/d*ln(sec(d*x^2+c)+tan(d*x^2+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(40) = 80.

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.07

$$\int x\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx = \frac{a^2dx^2\cos\left(dx^2+c\right)+ab\cos\left(dx^2+c\right)\log\left(\sin\left(dx^2+c\right)+1\right)-ab\cos\left(dx^2+c\right)\log\left(-\sin\left(dx^2+c\right)+1\right)+}{2\,d\cos\left(dx^2+c\right)}$$

```
[In] integrate(x*(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
[Out] 1/2*(a^2*d*x^2*cos(d*x^2 + c) + a*b*cos(d*x^2 + c)*log(sin(d*x^2 + c) + 1)
- a*b*cos(d*x^2 + c)*log(-sin(d*x^2 + c) + 1) + b^2*sin(d*x^2 + c))/(d*cos(
d*x^2 + c))
```

Sympy [F]

$$\int x\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\int x\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x*(a+b*sec(d*x**2+c))**2,x)
[Out] Integral(x*(a + b*sec(c + d*x**2))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(40) = 80$.

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.18

$$\begin{aligned}\int x\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx\\ &=\frac{1}{2}a^2x^2+\frac{ab\log\left(\sec\left(dx^2+c\right)+\tan\left(dx^2+c\right)\right)}{d}\\ &\quad+\frac{b^2\sin\left(2\,dx^2+2\,c\right)}{d\cos\left(2\,dx^2+2\,c\right)^2+d\sin\left(2\,dx^2+2\,c\right)^2+2\,d\cos\left(2\,dx^2+2\,c\right)+d}\end{aligned}$$

```
[In] integrate(x*(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
[Out] 1/2*a^2*x^2 + a*b*log(sec(d*x^2 + c) + tan(d*x^2 + c))/d + b^2*sin(2*d*x^2
+ 2*c)/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 +
 2*c) + d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(40) = 80$.

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.00

$$\int x\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\frac{(dx^2+c)a^2+2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+1\right|\right)-2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b^2\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)^2-1}}{2\,d}$$

```
[In] integrate(x*(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
[Out] 1/2*((d*x^2 + c)*a^2 + 2*a*b*log(abs(tan(1/2*d*x^2 + 1/2*c) + 1)) - 2*a*b*l
og(abs(tan(1/2*d*x^2 + 1/2*c) - 1)) - 2*b^2*tan(1/2*d*x^2 + 1/2*c)/(tan(1/2
*d*x^2 + 1/2*c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.27

$$\int x\left(a+b\sec\left(c+dx^2\right)\right)^2\,dx=\frac{a^2\,x^2}{2}+\frac{b^2\,1\mathrm{i}}{d\,\left(\mathrm{e}^{2\mathrm{i}\,d\,x^2+c\,2\mathrm{i}}+1\right)}+\frac{a\,b\,\ln\left(-a\,b\,x\,4\mathrm{i}-4\,a\,b\,x\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{d}-\frac{a\,b\,\ln\left(a\,b\,x\,4\mathrm{i}-4\,a\,b\,x\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{d}$$

```
[In] int(x*(a + b/cos(c + d*x^2))^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*1i)/(d*(exp(c*2i + d*x^2*2i) + 1)) + (a*b*log(- a*b*x*4i
 - 4*a*b*x*exp(d*x^2*1i)*exp(c*1i)))/d - (a*b*log(a*b*x*4i - 4*a*b*x*exp(d*
x^2*1i)*exp(c*1i)))/d
```

3.13 $\int \frac{(a+b\sec(c+dx^2))^2}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\sec(c+dx^2))^2}{x}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx^2))^2}{x},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x^2+c))^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx^2))^2}{x}\,dx = \int \frac{(a+b\sec(c+dx^2))^2}{x}\,dx$$

```
[In] Int[(a + b*Sec[c + d*x^2])^2/x,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x^2])^2/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\sec(c+dx^2))^2}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 29.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\sec(c+dx^2))^2}{x}\,dx=\int \frac{(a+b\sec(c+dx^2))^2}{x}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x^2])^2/x,x]
[Out] Integrate[(a + b*Sec[c + d*x^2])^2/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(d\,x^2+c))^2}{x}dx$$

```
[In] int((a+b*sec(d*x^2+c))^2/x,x)
[Out] int((a+b*sec(d*x^2+c))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\sec(c+dx^2))^2}{x}\,dx=\int \frac{(b\sec(dx^2+c)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*sec(d*x^2+c))^2/x,x, algorithm="fricas")
[Out] integral((b^2*sec(d*x^2 + c)^2 + 2*a*b*sec(d*x^2 + c) + a^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\sec(c+dx^2))^2}{x}\,dx=\int \frac{(a+b\sec(c+dx^2))^2}{x}\,dx$$

```
[In] integrate((a+b*sec(d*x**2+c))**2/x,x)
[Out] Integral((a + b*sec(c + d*x**2))**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 286, normalized size of antiderivative = 15.89

$$\int \frac{(a + b\sec(c + dx^2))^2}{x} dx = \int \frac{(b\sec(dx^2 + c) + a)^2}{x} dx$$

```
[In] integrate((a+b*sec(d*x^2+c))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + (b^2*sin(2*d*x^2 + 2*c) + (d*x^2*cos(2*d*x^2 + 2*c)^2 + d*x^2*
sin(2*d*x^2 + 2*c)^2 + 2*d*x^2*cos(2*d*x^2 + 2*c) + d*x^2)*integrate(2*(2*a
*b*d*x^2*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + 2*a*b*d*x^2*cos(d*x^2 + c) + (
2*a*b*d*x^2*sin(d*x^2 + c) + b^2)*sin(2*d*x^2 + 2*c))/(d*x^3*cos(2*d*x^2 +
2*c)^2 + d*x^3*sin(2*d*x^2 + 2*c)^2 + 2*d*x^3*cos(2*d*x^2 + 2*c) + d*x^3),
x))/(d*x^2*cos(2*d*x^2 + 2*c)^2 + d*x^2*sin(2*d*x^2 + 2*c)^2 + 2*d*x^2*cos(
2*d*x^2 + 2*c) + d*x^2)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a + b\sec(c + dx^2))^2}{x} dx = \int \frac{(b\sec(dx^2 + c) + a)^2}{x} dx$$

```
[In] integrate((a+b*sec(d*x^2+c))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^2 + c) + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 13.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a + b\sec(c + dx^2))^2}{x} dx = \int \frac{\left(a + \frac{b}{\cos(d\,x^2+c)}\right)^2}{x} dx$$

```
[In] int((a + b/cos(c + d*x^2))^2/x,x)
```

```
[Out] int((a + b/cos(c + d*x^2))^2/x, x)
```

3.14 $\int \frac{(a+b\sec(c+dx^2))^2}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\sec(c+dx^2))^2}{x^2}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx^2))^2}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x^2+c))^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx^2))^2}{x^2}\,dx = \int \frac{(a+b\sec(c+dx^2))^2}{x^2}\,dx$$

```
[In] Int[(a + b*Sec[c + d*x^2])^2/x^2,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x^2])^2/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\sec(c+dx^2))^2}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 12.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\sec(c+dx^2))^2}{x^2}\,dx=\int \frac{(a+b\sec(c+dx^2))^2}{x^2}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x^2])^2/x^2,x]
[Out] Integrate[(a + b*Sec[c + d*x^2])^2/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(d\,x^2+c))^2}{x^2}dx$$

```
[In] int((a+b*sec(d*x^2+c))^2/x^2,x)
[Out] int((a+b*sec(d*x^2+c))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\sec(c+dx^2))^2}{x^2}\,dx=\int \frac{(b\sec(dx^2+c)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sec(d*x^2+c))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*sec(d*x^2 + c)^2 + 2*a*b*sec(d*x^2 + c) + a^2)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\sec(c+dx^2))^2}{x^2}\,dx=\int \frac{(a+b\sec(c+dx^2))^2}{x^2}\,dx$$

```
[In] integrate((a+b*sec(d*x**2+c))**2/x**2,x)
[Out] Integral((a + b*sec(c + d*x**2))**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 289, normalized size of antiderivative = 16.06

$$\int \frac{(a+b\sec(c+dx^2))^2}{x^2}\,dx = \int \frac{(b\sec(dx^2+c)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sec(d*x^2+c))^2/x^2,x, algorithm="maxima")
```

```
[Out] -a^2/x + (b^2*sin(2*d*x^2 + 2*c) + (d*x^3*cos(2*d*x^2 + 2*c)^2 + d*x^3*sin(
2*d*x^2 + 2*c)^2 + 2*d*x^3*cos(2*d*x^2 + 2*c) + d*x^3)*integrate((4*a*b*d*x
^2*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + 4*a*b*d*x^2*cos(d*x^2 + c) + (4*a*b*
d*x^2*sin(d*x^2 + c) + 3*b^2)*sin(2*d*x^2 + 2*c))/(d*x^4*cos(2*d*x^2 + 2*c)
^2 + d*x^4*sin(2*d*x^2 + 2*c)^2 + 2*d*x^4*cos(2*d*x^2 + 2*c) + d*x^4), x))/
(d*x^3*cos(2*d*x^2 + 2*c)^2 + d*x^3*sin(2*d*x^2 + 2*c)^2 + 2*d*x^3*cos(2*d*
x^2 + 2*c) + d*x^3)
```

Giac [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\sec(c+dx^2))^2}{x^2}\,dx = \int \frac{(b\sec(dx^2+c)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sec(d*x^2+c))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^2 + c) + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 13.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\sec(c+dx^2))^2}{x^2}\,dx = \int \frac{\left(a+\frac{b}{\cos(d\,x^2+c)}\right)^2}{x^2}\,dx$$

```
[In] int((a + b/cos(c + d*x^2))^2/x^2,x)
```

```
[Out] int((a + b/cos(c + d*x^2))^2/x^2, x)
```

3.15 $\int x \sec^7(a+bx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int x\sec^7(a+bx^2)\,dx=\frac{5\operatorname{arctanh}(\sin(a+bx^2))}{32b}+\frac{5\sec(a+bx^2)\tan(a+bx^2)}{32b}+\frac{5\sec^3(a+bx^2)\tan(a+bx^2)}{48b}+\frac{\sec^5(a+bx^2)\tan(a+bx^2)}{12b}$$

```
[Out] 5/32*arctanh(sin(b*x^2+a))/b+5/32*sec(b*x^2+a)*tan(b*x^2+a)/b+5/48*sec(b*x^
2+a)^3*tan(b*x^2+a)/b+1/12*sec(b*x^2+a)^5*tan(b*x^2+a)/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4289, 3853, 3855}

$$\int x\sec^7(a+bx^2)\,dx=\frac{5\operatorname{arctanh}(\sin(a+bx^2))}{32b}+\frac{\tan(a+bx^2)\sec^5(a+bx^2)}{12b}+\frac{5\tan(a+bx^2)\sec^3(a+bx^2)}{48b}+\frac{5\tan(a+bx^2)\sec(a+bx^2)}{32b}$$

```
[In] Int[x*Sec[a + b*x^2]^7,x]
```

```
[Out] (5*ArcTanh[Sin[a + b*x^2]])/(32*b) + (5*Sec[a + b*x^2]*Tan[a + b*x^2])/(32*
b) + (5*Sec[a + b*x^2]^3*Tan[a + b*x^2])/(48*b) + (Sec[a + b*x^2]^5*Tan[a +
 b*x^2])/(12*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sec^7(a+bx)\,dx, x, x^2\right)\\
&= \frac{\sec^5\left(a+bx^2\right)\tan\left(a+bx^2\right)}{12b} + \frac{5}{12}\text{Subst}\left(\int \sec^5(a+bx)\,dx, x, x^2\right)\\
&= \frac{5\sec^3\left(a+bx^2\right)\tan\left(a+bx^2\right)}{48b} + \frac{\sec^5\left(a+bx^2\right)\tan\left(a+bx^2\right)}{12b}\\
&\quad + \frac{5}{16}\text{Subst}\left(\int \sec^3(a+bx)\,dx, x, x^2\right)\\
&= \frac{5\sec\left(a+bx^2\right)\tan\left(a+bx^2\right)}{32b} + \frac{5\sec^3\left(a+bx^2\right)\tan\left(a+bx^2\right)}{48b}\\
&\quad + \frac{\sec^5\left(a+bx^2\right)\tan\left(a+bx^2\right)}{12b} + \frac{5}{32}\text{Subst}\left(\int \sec(a+bx)\,dx, x, x^2\right)\\
&= \frac{5\text{arctanh}\left(\sin\left(a+bx^2\right)\right)}{32b} + \frac{5\sec\left(a+bx^2\right)\tan\left(a+bx^2\right)}{32b}\\
&\quad + \frac{5\sec^3\left(a+bx^2\right)\tan\left(a+bx^2\right)}{48b} + \frac{\sec^5\left(a+bx^2\right)\tan\left(a+bx^2\right)}{12b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int x\sec^7\left(a+bx^2\right)\,dx=\frac{5\operatorname{arctanh}(\sin\left(a+bx^2\right))}{32b}+\frac{5\sec\left(a+bx^2\right)\tan\left(a+bx^2\right)}{32b}+\frac{5\sec^3\left(a+bx^2\right)\tan\left(a+bx^2\right)}{48b}+\frac{\sec^5\left(a+bx^2\right)\tan\left(a+bx^2\right)}{12b}$$

[In] Integrate[x*Sec[a + b*x^2]^7,x]

[Out] (5*ArcTanh[Sin[a + b*x^2]])/(32*b) + (5*Sec[a + b*x^2]*Tan[a + b*x^2])/(32*b) + (5*Sec[a + b*x^2]^3*Tan[a + b*x^2])/(48*b) + (Sec[a + b*x^2]^5*Tan[a + b*x^2])/(12*b)

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{-\left(-\frac{\sec\left(x^2b+a\right)^5}{6}-\frac{5\sec\left(x^2b+a\right)^3}{24}-\frac{5\sec\left(x^2b+a\right)}{16}\right)\tan\left(x^2b+a\right)+\frac{5\ln\left(\sec\left(x^2b+a\right)+\tan\left(x^2b+a\right)\right)}{16}}{2b}$
default	$\frac{-\left(-\frac{\sec\left(x^2b+a\right)^5}{6}-\frac{5\sec\left(x^2b+a\right)^3}{24}-\frac{5\sec\left(x^2b+a\right)}{16}\right)\tan\left(x^2b+a\right)+\frac{5\ln\left(\sec\left(x^2b+a\right)+\tan\left(x^2b+a\right)\right)}{16}}{2b}$
risch	$-\frac{i\left(15\,\mathrm{e}^{11i\left(x^2b+a\right)}+85\,\mathrm{e}^{9i\left(x^2b+a\right)}+198\,\mathrm{e}^{7i\left(x^2b+a\right)}-198\,\mathrm{e}^{5i\left(x^2b+a\right)}-85\,\mathrm{e}^{3i\left(x^2b+a\right)}-15\,\mathrm{e}^{i\left(x^2b+a\right)}\right)}{48b\left(\mathrm{e}^{2i\left(x^2b+a\right)}+1\right)^6}-\frac{5\ln\left(\mathrm{e}^{i\left(x^2b+a\right.}\right.}{32b}$
parallelrisch	$\frac{\left(-225\cos\left(2x^2b+2a\right)-90\cos\left(4x^2b+4a\right)-15\cos\left(6x^2b+6a\right)-150\right)\ln\left(\tan\left(\frac{a}{2}+\frac{x^2b}{2}\right)-1\right)+\left(225\cos\left(2x^2b+2a\right)+90\cos\left(4x^2\right.\right.}{96b(10+\cos(6x^2b+6a)+6\cos(4x^2b+}$

[In] int(x*sec(b*x^2+a)^7,x,method=_RETURNVERBOSE)

[Out] 1/2/b*(-(-1/6*sec(b*x^2+a)^5-5/24*sec(b*x^2+a)^3-5/16*sec(b*x^2+a))*tan(b*x^2+a)+5/16*ln(sec(b*x^2+a)+tan(b*x^2+a)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int x\sec^7(a+bx^2)\,dx = \frac{15\cos(bx^2+a)^6\log(\sin(bx^2+a)+1) - 15\cos(bx^2+a)^6\log(-\sin(bx^2+a)+1) + 2\left(15\cos(bx^2+a\right.}{192\,b\cos(bx^2+a)^6}$$

```
[In] integrate(x*sec(b*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/192*(15*cos(b*x^2 + a)^6*log(sin(b*x^2 + a) + 1) - 15*cos(b*x^2 + a)^6*lo
g(-sin(b*x^2 + a) + 1) + 2*(15*cos(b*x^2 + a)^4 + 10*cos(b*x^2 + a)^2 + 8)*
sin(b*x^2 + a))/(b*cos(b*x^2 + a)^6)
```

Sympy [F]

$$\int x\sec^7(a+bx^2)\,dx = \int x\sec^7(a+bx^2)\,dx$$

```
[In] integrate(x*sec(b*x**2+a)**7,x)
```

```
[Out] Integral(x*sec(a + b*x**2)**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2838 vs. 2(82) = 164.

Time = 0.45 (sec) , antiderivative size = 2838, normalized size of antiderivative = 31.53

$$\int x\sec^7(a+bx^2)\,dx = \text{Too large to display}$$

```
[In] integrate(x*sec(b*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/192*(4*(15*sin(11*b*x^2 + 11*a) + 85*sin(9*b*x^2 + 9*a) + 198*sin(7*b*x^2
 + 7*a) - 198*sin(5*b*x^2 + 5*a) - 85*sin(3*b*x^2 + 3*a) - 15*sin(b*x^2 + a
))*cos(12*b*x^2 + 12*a) - 60*(6*sin(10*b*x^2 + 10*a) + 15*sin(8*b*x^2 + 8*a
) + 20*sin(6*b*x^2 + 6*a) + 15*sin(4*b*x^2 + 4*a) + 6*sin(2*b*x^2 + 2*a))*c
os(11*b*x^2 + 11*a) + 24*(85*sin(9*b*x^2 + 9*a) + 198*sin(7*b*x^2 + 7*a) -
198*sin(5*b*x^2 + 5*a) - 85*sin(3*b*x^2 + 3*a) - 15*sin(b*x^2 + a))*cos(10*
b*x^2 + 10*a) - 340*(15*sin(8*b*x^2 + 8*a) + 20*sin(6*b*x^2 + 6*a) + 15*sin
(4*b*x^2 + 4*a) + 6*sin(2*b*x^2 + 2*a))*cos(9*b*x^2 + 9*a) + 60*(198*sin(7*
```

```
b*x^2 + 7*a) - 198*sin(5*b*x^2 + 5*a) - 85*sin(3*b*x^2 + 3*a) - 15*sin(b*x^
2 + a))*cos(8*b*x^2 + 8*a) - 792*(20*sin(6*b*x^2 + 6*a) + 15*sin(4*b*x^2 +
4*a) + 6*sin(2*b*x^2 + 2*a))*cos(7*b*x^2 + 7*a) - 80*(198*sin(5*b*x^2 + 5*a
) + 85*sin(3*b*x^2 + 3*a) + 15*sin(b*x^2 + a))*cos(6*b*x^2 + 6*a) + 2376*(5
*sin(4*b*x^2 + 4*a) + 2*sin(2*b*x^2 + 2*a))*cos(5*b*x^2 + 5*a) - 300*(17*si
n(3*b*x^2 + 3*a) + 3*sin(b*x^2 + a))*cos(4*b*x^2 + 4*a) - 15*(2*(6*cos(10*b
*x^2 + 10*a) + 15*cos(8*b*x^2 + 8*a) + 20*cos(6*b*x^2 + 6*a) + 15*cos(4*b*x
^2 + 4*a) + 6*cos(2*b*x^2 + 2*a) + 1)*cos(12*b*x^2 + 12*a) + cos(12*b*x^2 +
 12*a)^2 + 12*(15*cos(8*b*x^2 + 8*a) + 20*cos(6*b*x^2 + 6*a) + 15*cos(4*b*x
^2 + 4*a) + 6*cos(2*b*x^2 + 2*a) + 1)*cos(10*b*x^2 + 10*a) + 36*cos(10*b*x^
2 + 10*a)^2 + 30*(20*cos(6*b*x^2 + 6*a) + 15*cos(4*b*x^2 + 4*a) + 6*cos(2*b
*x^2 + 2*a) + 1)*cos(8*b*x^2 + 8*a) + 225*cos(8*b*x^2 + 8*a)^2 + 40*(15*cos
(4*b*x^2 + 4*a) + 6*cos(2*b*x^2 + 2*a) + 1)*cos(6*b*x^2 + 6*a) + 400*cos(6*
b*x^2 + 6*a)^2 + 30*(6*cos(2*b*x^2 + 2*a) + 1)*cos(4*b*x^2 + 4*a) + 225*cos
(4*b*x^2 + 4*a)^2 + 36*cos(2*b*x^2 + 2*a)^2 + 2*(6*sin(10*b*x^2 + 10*a) + 1
5*sin(8*b*x^2 + 8*a) + 20*sin(6*b*x^2 + 6*a) + 15*sin(4*b*x^2 + 4*a) + 6*si
n(2*b*x^2 + 2*a))*sin(12*b*x^2 + 12*a) + sin(12*b*x^2 + 12*a)^2 + 12*(15*si
n(8*b*x^2 + 8*a) + 20*sin(6*b*x^2 + 6*a) + 15*sin(4*b*x^2 + 4*a) + 6*sin(2*
b*x^2 + 2*a))*sin(10*b*x^2 + 10*a) + 36*sin(10*b*x^2 + 10*a)^2 + 30*(20*sin
(6*b*x^2 + 6*a) + 15*sin(4*b*x^2 + 4*a) + 6*sin(2*b*x^2 + 2*a))*sin(8*b*x^2
 + 8*a) + 225*sin(8*b*x^2 + 8*a)^2 + 120*(5*sin(4*b*x^2 + 4*a) + 2*sin(2*b*
x^2 + 2*a))*sin(6*b*x^2 + 6*a) + 400*sin(6*b*x^2 + 6*a)^2 + 225*sin(4*b*x^2
 + 4*a)^2 + 180*sin(4*b*x^2 + 4*a)*sin(2*b*x^2 + 2*a) + 36*sin(2*b*x^2 + 2*
a)^2 + 12*cos(2*b*x^2 + 2*a) + 1)*log((cos(b*x^2 + 2*a)^2 + cos(a)^2 - 2*co
s(a)*sin(b*x^2 + 2*a) + sin(b*x^2 + 2*a)^2 + 2*cos(b*x^2 + 2*a)*sin(a) + si
n(a)^2)/(cos(b*x^2 + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x^2 + 2*a) + sin(b*
x^2 + 2*a)^2 - 2*cos(b*x^2 + 2*a)*sin(a) + sin(a)^2)) - 4*(15*cos(11*b*x^2
+ 11*a) + 85*cos(9*b*x^2 + 9*a) + 198*cos(7*b*x^2 + 7*a) - 198*cos(5*b*x^2
+ 5*a) - 85*cos(3*b*x^2 + 3*a) - 15*cos(b*x^2 + a))*sin(12*b*x^2 + 12*a) +
60*(6*cos(10*b*x^2 + 10*a) + 15*cos(8*b*x^2 + 8*a) + 20*cos(6*b*x^2 + 6*a)
+ 15*cos(4*b*x^2 + 4*a) + 6*cos(2*b*x^2 + 2*a) + 1)*sin(11*b*x^2 + 11*a) -
24*(85*cos(9*b*x^2 + 9*a) + 198*cos(7*b*x^2 + 7*a) - 198*cos(5*b*x^2 + 5*a)
 - 85*cos(3*b*x^2 + 3*a) - 15*cos(b*x^2 + a))*sin(10*b*x^2 + 10*a) + 340*(1
5*cos(8*b*x^2 + 8*a) + 20*cos(6*b*x^2 + 6*a) + 15*cos(4*b*x^2 + 4*a) + 6*co
s(2*b*x^2 + 2*a) + 1)*sin(9*b*x^2 + 9*a) - 60*(198*cos(7*b*x^2 + 7*a) - 198
*cos(5*b*x^2 + 5*a) - 85*cos(3*b*x^2 + 3*a) - 15*cos(b*x^2 + a))*sin(8*b*x^
2 + 8*a) + 792*(20*cos(6*b*x^2 + 6*a) + 15*cos(4*b*x^2 + 4*a) + 6*cos(2*b*x
^2 + 2*a) + 1)*sin(7*b*x^2 + 7*a) + 80*(198*cos(5*b*x^2 + 5*a) + 85*cos(3*b
*x^2 + 3*a) + 15*cos(b*x^2 + a))*sin(6*b*x^2 + 6*a) - 792*(15*cos(4*b*x^2 +
 4*a) + 6*cos(2*b*x^2 + 2*a) + 1)*sin(5*b*x^2 + 5*a) + 300*(17*cos(3*b*x^2
+ 3*a) + 3*cos(b*x^2 + a))*sin(4*b*x^2 + 4*a) - 340*(6*cos(2*b*x^2 + 2*a) +
 1)*sin(3*b*x^2 + 3*a) + 2040*cos(3*b*x^2 + 3*a)*sin(2*b*x^2 + 2*a) + 360*c
os(b*x^2 + a)*sin(2*b*x^2 + 2*a) - 360*cos(2*b*x^2 + 2*a)*sin(b*x^2 + a) -
60*sin(b*x^2 + a))/(b*cos(12*b*x^2 + 12*a)^2 + 36*b*cos(10*b*x^2 + 10*a)^2
+ 225*b*cos(8*b*x^2 + 8*a)^2 + 400*b*cos(6*b*x^2 + 6*a)^2 + 225*b*cos(4*b*x
```

```
^2 + 4*a)^2 + 36*b*cos(2*b*x^2 + 2*a)^2 + b*sin(12*b*x^2 + 12*a)^2 + 36*b*s
in(10*b*x^2 + 10*a)^2 + 225*b*sin(8*b*x^2 + 8*a)^2 + 400*b*sin(6*b*x^2 + 6*
a)^2 + 225*b*sin(4*b*x^2 + 4*a)^2 + 180*b*sin(4*b*x^2 + 4*a)*sin(2*b*x^2 +
2*a) + 36*b*sin(2*b*x^2 + 2*a)^2 + 2*(6*b*cos(10*b*x^2 + 10*a) + 15*b*cos(8
*b*x^2 + 8*a) + 20*b*cos(6*b*x^2 + 6*a) + 15*b*cos(4*b*x^2 + 4*a) + 6*b*cos
(2*b*x^2 + 2*a) + b)*cos(12*b*x^2 + 12*a) + 12*(15*b*cos(8*b*x^2 + 8*a) + 2
0*b*cos(6*b*x^2 + 6*a) + 15*b*cos(4*b*x^2 + 4*a) + 6*b*cos(2*b*x^2 + 2*a) +
 b)*cos(10*b*x^2 + 10*a) + 30*(20*b*cos(6*b*x^2 + 6*a) + 15*b*cos(4*b*x^2 +
 4*a) + 6*b*cos(2*b*x^2 + 2*a) + b)*cos(8*b*x^2 + 8*a) + 40*(15*b*cos(4*b*x
^2 + 4*a) + 6*b*cos(2*b*x^2 + 2*a) + b)*cos(6*b*x^2 + 6*a) + 30*(6*b*cos(2*
b*x^2 + 2*a) + b)*cos(4*b*x^2 + 4*a) + 12*b*cos(2*b*x^2 + 2*a) + 2*(6*b*sin
(10*b*x^2 + 10*a) + 15*b*sin(8*b*x^2 + 8*a) + 20*b*sin(6*b*x^2 + 6*a) + 15*
b*sin(4*b*x^2 + 4*a) + 6*b*sin(2*b*x^2 + 2*a))*sin(12*b*x^2 + 12*a) + 12*(1
5*b*sin(8*b*x^2 + 8*a) + 20*b*sin(6*b*x^2 + 6*a) + 15*b*sin(4*b*x^2 + 4*a)
+ 6*b*sin(2*b*x^2 + 2*a))*sin(10*b*x^2 + 10*a) + 30*(20*b*sin(6*b*x^2 + 6*a
) + 15*b*sin(4*b*x^2 + 4*a) + 6*b*sin(2*b*x^2 + 2*a))*sin(8*b*x^2 + 8*a) +
120*(5*b*sin(4*b*x^2 + 4*a) + 2*b*sin(2*b*x^2 + 2*a))*sin(6*b*x^2 + 6*a) +
b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int x\sec^7\left(a+bx^2\right)\,dx = -\frac{\frac{2\left(15\,\sin\left(bx^2+a\right)^5-40\,\sin\left(bx^2+a\right)^3+33\,\sin\left(bx^2+a\right)\right)}{\left(\sin\left(bx^2+a\right)^2-1\right)^3}-15\,\log\left(\sin\left(bx^2+a\right)+1\right)+15\,\log\left(-\sin\left(bx^2+a\right)+1\right)}{192\,b}$$

```
[In] integrate(x*sec(b*x^2+a)^7,x, algorithm="giac")
```

```
[Out] -1/192*(2*(15*sin(b*x^2 + a)^5 - 40*sin(b*x^2 + a)^3 + 33*sin(b*x^2 + a))/(
sin(b*x^2 + a)^2 - 1)^3 - 15*log(sin(b*x^2 + a) + 1) + 15*log(-sin(b*x^2 +
a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 25.42 (sec) , antiderivative size = 496, normalized size of antiderivative = 5.51

$$\int x\sec^7\left(a+bx^2\right)\,dx=\frac{5\,\ln\left(-\frac{x\,5\mathrm{i}}{8}-\frac{5\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x^2\,1\mathrm{i}}}{8}\right)}{32\,b}-\frac{5\,\ln\left(\frac{x\,5\mathrm{i}}{8}-\frac{5\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x^2\,1\mathrm{i}}}{8}\right)}{32\,b}$$
$$+\frac{\mathrm{e}^{3\mathrm{i}\,b\,x^2+a\,3\mathrm{i}}\,8\mathrm{i}}{3\,b\,\left(5\,\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}+10\,\mathrm{e}^{4\mathrm{i}\,b\,x^2+a\,4\mathrm{i}}+10\,\mathrm{e}^{6\mathrm{i}\,b\,x^2+a\,6\mathrm{i}}+5\,\mathrm{e}^{8\mathrm{i}\,b\,x^2+a\,8\mathrm{i}}+\mathrm{e}^{10\mathrm{i}\,b\,x^2+a\,10\mathrm{i}}+1\right)}$$
$$-\frac{\mathrm{e}^{1\mathrm{i}\,b\,x^2+a\,1\mathrm{i}}\,1\mathrm{i}}{6\,b\,\left(3\,\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}+3\,\mathrm{e}^{4\mathrm{i}\,b\,x^2+a\,4\mathrm{i}}+\mathrm{e}^{6\mathrm{i}\,b\,x^2+a\,6\mathrm{i}}+1\right)}-\frac{\mathrm{e}^{1\mathrm{i}\,b\,x^2+a\,1\mathrm{i}}\,5\mathrm{i}}{16\,b\,\left(\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}+1\right)}$$
$$+\frac{\mathrm{e}^{5\mathrm{i}\,b\,x^2+a\,5\mathrm{i}}\,16\mathrm{i}}{3\,b\,\left(6\,\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}+15\,\mathrm{e}^{4\mathrm{i}\,b\,x^2+a\,4\mathrm{i}}+20\,\mathrm{e}^{6\mathrm{i}\,b\,x^2+a\,6\mathrm{i}}+15\,\mathrm{e}^{8\mathrm{i}\,b\,x^2+a\,8\mathrm{i}}+6\,\mathrm{e}^{10\mathrm{i}\,b\,x^2+a\,10\mathrm{i}}+\mathrm{e}^{12\mathrm{i}\,b\,x^2+a\,12\mathrm{i}}+1\right)}$$
$$+\frac{\mathrm{e}^{1\mathrm{i}\,b\,x^2+a\,1\mathrm{i}}\,1\mathrm{i}}{b\,\left(4\,\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}+6\,\mathrm{e}^{4\mathrm{i}\,b\,x^2+a\,4\mathrm{i}}+4\,\mathrm{e}^{6\mathrm{i}\,b\,x^2+a\,6\mathrm{i}}+\mathrm{e}^{8\mathrm{i}\,b\,x^2+a\,8\mathrm{i}}+1\right)}$$
$$-\frac{\mathrm{e}^{1\mathrm{i}\,b\,x^2+a\,1\mathrm{i}}\,5\mathrm{i}}{24\,b\,\left(2\,\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}+\mathrm{e}^{4\mathrm{i}\,b\,x^2+a\,4\mathrm{i}}+1\right)}$$

```
[In] int(x/cos(a + b*x^2)^7,x)
```

```
[Out] (5*log(- (x*5i)/8 - (5*x*exp(a*1i)*exp(b*x^2*1i))/8))/(32*b) - (5*log((x*5i
)/8 - (5*x*exp(a*1i)*exp(b*x^2*1i))/8))/(32*b) + (exp(a*3i + b*x^2*3i)*8i)/
(3*b*(5*exp(a*2i + b*x^2*2i) + 10*exp(a*4i + b*x^2*4i) + 10*exp(a*6i + b*x^
2*6i) + 5*exp(a*8i + b*x^2*8i) + exp(a*10i + b*x^2*10i) + 1)) - (exp(a*1i +
 b*x^2*1i)*1i)/(6*b*(3*exp(a*2i + b*x^2*2i) + 3*exp(a*4i + b*x^2*4i) + exp(
a*6i + b*x^2*6i) + 1)) - (exp(a*1i + b*x^2*1i)*5i)/(16*b*(exp(a*2i + b*x^2*
2i) + 1)) + (exp(a*5i + b*x^2*5i)*16i)/(3*b*(6*exp(a*2i + b*x^2*2i) + 15*ex
p(a*4i + b*x^2*4i) + 20*exp(a*6i + b*x^2*6i) + 15*exp(a*8i + b*x^2*8i) + 6*
exp(a*10i + b*x^2*10i) + exp(a*12i + b*x^2*12i) + 1)) + (exp(a*1i + b*x^2*1
i)*1i)/(b*(4*exp(a*2i + b*x^2*2i) + 6*exp(a*4i + b*x^2*4i) + 4*exp(a*6i + b
*x^2*6i) + exp(a*8i + b*x^2*8i) + 1)) - (exp(a*1i + b*x^2*1i)*5i)/(24*b*(2*
exp(a*2i + b*x^2*2i) + exp(a*4i + b*x^2*4i) + 1))
```

3.16 $\int \frac{x^5}{a+b\sec(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 382

$$\int \frac{x^5}{a+b\sec(c+dx^2)}\,dx = \frac{x^6}{6a} + \frac{ibx^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{ibx^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

```
[Out] 1/6*x^6/a+1/2*I*b*x^4*ln(1+a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a/d/(-a
^2+b^2)^(1/2)-1/2*I*b*x^4*ln(1+a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a/d
/(-a^2+b^2)^(1/2)+b*x^2*polylog(2,-a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))
/a/d^2/(-a^2+b^2)^(1/2)-b*x^2*polylog(2,-a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(
1/2)))/a/d^2/(-a^2+b^2)^(1/2)+I*b*polylog(3,-a*exp(I*(d*x^2+c))/(b-(-a^2+b^
2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)-I*b*polylog(3,-a*exp(I*(d*x^2+c))/(b+(-a^
2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4289, 4276, 3402, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{x^5}{a+b\sec(c+dx^2)}\,dx = \frac{ib\,\text{PolyLog}\left(3,-\frac{ae^{i\left(dx^2+c\right)}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{ib\,\text{PolyLog}\left(3,-\frac{ae^{i\left(dx^2+c\right)}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} + \frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{i\left(dx^2+c\right)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{i\left(dx^2+c\right)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{ibx^4\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{b^2-a^2}}\right)}{2ad\sqrt{b^2-a^2}} - \frac{ibx^4\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{\sqrt{b^2-a^2}+b}\right)}{2ad\sqrt{b^2-a^2}} + \frac{x^6}{6a}$$

```
[In] Int[x^5/(a + b*Sec[c + d*x^2]),x]
```

```
[Out] x^6/(6*a) + ((I/2)*b*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2
])])/(a*Sqrt[-a^2 + b^2]*d) - ((I/2)*b*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(b
 + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (b*x^2*PolyLog[2, -((a*E^(I
*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) - (b*x^2*
PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 +
 b^2]*d^2) + (I*b*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])
)])/(a*Sqrt[-a^2 + b^2]*d^3) - (I*b*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(b +
 Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+b\sec(c+dx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x^2}{a}-\frac{bx^2}{a(b+a\cos(c+dx))}\right)\,dx, x, x^2\right)\end{aligned}$$

$$=\frac{x^6}{6a}-\frac{b\text{Subst}\left(\int \frac{x^2}{b+a\cos(c+dx)}\,dx,x,x^2\right)}{2a}$$

$$=\frac{x^6}{6a}-\frac{b\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx,x,x^2\right)}{a}$$

$$=\frac{x^6}{6a}-\frac{b\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,x^2\right)}{\sqrt{-a^2+b^2}}+\frac{b\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,x^2\right)}{\sqrt{-a^2+b^2}}$$

$$=\frac{x^6}{6a}+\frac{ibx^4\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}-\frac{ibx^4\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}-\frac{(ib)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a\sqrt{-a^2+b^2}d}+\frac{(ib)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a\sqrt{-a^2+b^2}d}$$

$$=\frac{x^6}{6a}+\frac{ibx^4\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}-\frac{ibx^4\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}+\frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{b\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{b\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$=\frac{x^6}{6a}+\frac{ibx^4\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}-\frac{ibx^4\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}+\frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{(ib)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i\left(c+dx^2\right)}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{(ib)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i\left(c+dx^2\right)}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$= \frac{x^6}{6a} + \frac{ibx^4 \log \left(1 + \frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{ibx^4 \log \left(1 + \frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{ib \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{ib \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{a + b\sec(c+dx^2)} dx = \frac{\sqrt{-a^2+b^2}d^3x^6 + 3ibd^2x^4 \log\left(1 - \frac{ae^{i(c+dx^2)}}{-b+\sqrt{-a^2+b^2}}\right) - 3ibd^2x^4 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right) + 6bdx^2 \operatorname{PolyLog}\left(2, \frac{a\cdots}{-b\cdots}\right.}{6a\sqrt{\cdots}}$$

```
[In] Integrate[x^5/(a + b*Sec[c + d*x^2]),x]
```

```
[Out] (Sqrt[-a^2 + b^2]*d^3*x^6 + (3*I)*b*d^2*x^4*Log[1 - (a*E^(I*(c + d*x^2)))/(
-b + Sqrt[-a^2 + b^2])] - (3*I)*b*d^2*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(b
+ Sqrt[-a^2 + b^2])] + 6*b*d*x^2*PolyLog[2, (a*E^(I*(c + d*x^2)))/(-b + Sqr
t[-a^2 + b^2])] - 6*b*d*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b + Sqrt[-a
^2 + b^2]))] + (6*I)*b*PolyLog[3, (a*E^(I*(c + d*x^2)))/(-b + Sqrt[-a^2 + b
^2])] - (6*I)*b*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2]))]
)/(6*a*Sqrt[-a^2 + b^2]*d^3)
```

Maple [F]

$$\int \frac{x^5}{a + b\sec(d\,x^2 + c)} dx$$

```
[In] int(x^5/(a+b*sec(d*x^2+c)),x)
```

```
[Out] int(x^5/(a+b*sec(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1457 vs. $2(330) = 660$.

Time = 0.44 (sec) , antiderivative size = 1457, normalized size of antiderivative = 3.81

$$\int \frac{x^5}{a + b\sec(c + dx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(a+b*sec(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*(a^2 - b^2)*d^3*x^6 - 6*a*b*d*x^2*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*
cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c
))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + 6*a*b*d*x^2*sqrt(-(a^2 - b^2)/a^2)*
dilog(-(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin
(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - 6*a*b*d*x^2*sqrt(-(a^2 -
b^2)/a^2)*dilog(-(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c)
 - I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + 6*a*b*d*x^2*sqr
t(-(a^2 - b^2)/a^2)*dilog(-(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) - (a*cos(
d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + 3*I*a
*b*c^2*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^2 + c) + 2*I*a*sin(d*x^2 + c)
 + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - 3*I*a*b*c^2*sqrt(-(a^2 - b^2)/a^2)*l
og(2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) +
 2*b) + 3*I*a*b*c^2*sqrt(-(a^2 - b^2)/a^2)*log(-2*a*cos(d*x^2 + c) + 2*I*a*
sin(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) - 3*I*a*b*c^2*sqrt(-(a^2
 - b^2)/a^2)*log(-2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^
2 - b^2)/a^2) - 2*b) - 6*I*a*b*sqrt(-(a^2 - b^2)/a^2)*polylog(3, -(b*cos(d*
x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqr
t(-(a^2 - b^2)/a^2))/a) + 6*I*a*b*sqrt(-(a^2 - b^2)/a^2)*polylog(3, -(b*cos
(d*x^2 + c) + I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*
sqrt(-(a^2 - b^2)/a^2))/a) + 6*I*a*b*sqrt(-(a^2 - b^2)/a^2)*polylog(3, -(b*
cos(d*x^2 + c) - I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c
))*sqrt(-(a^2 - b^2)/a^2))/a) - 6*I*a*b*sqrt(-(a^2 - b^2)/a^2)*polylog(3, -
(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2
+ c))*sqrt(-(a^2 - b^2)/a^2))/a) + 3*(-I*a*b*d^2*x^4 + I*a*b*c^2)*sqrt(-(a^
2 - b^2)/a^2)*log((b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c
) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) + 3*(I*a*b*d^2*x^4 -
 I*a*b*c^2)*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c) + I*b*sin(d*x^2 +
c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a)
 + 3*(I*a*b*d^2*x^4 - I*a*b*c^2)*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 +
c) - I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^
2 - b^2)/a^2) + a)/a) + 3*(-I*a*b*d^2*x^4 + I*a*b*c^2)*sqrt(-(a^2 - b^2)/a^
2)*log((b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin
(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a))/((a^3 - a*b^2)*d^3)
```

Sympy [F]

$$\int \frac{x^5}{a+b\sec\left(c+dx^2\right)}\,dx = \int \frac{x^5}{a+b\sec\left(c+dx^2\right)}\,dx$$

```
[In] integrate(x**5/(a+b*sec(d*x**2+c)),x)
[Out] Integral(x**5/(a + b*sec(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^5}{a+b\sec\left(c+dx^2\right)}\,dx = \int \frac{x^5}{b\sec\left(dx^2+c\right)+a}\,dx$$

```
[In] integrate(x^5/(a+b*sec(d*x^2+c)),x, algorithm="maxima")
[Out] 1/6*(x^6 - 12*a*b*integrate((a*x^5*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + 2*b*
x^5*cos(d*x^2 + c)^2 + a*x^5*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^5*si
n(d*x^2 + c)^2 + a*x^5*cos(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*
cos(d*x^2 + c)^2 + a^3*sin(2*d*x^2 + 2*c)^2 + 4*a^2*b*sin(2*d*x^2 + 2*c)*si
n(d*x^2 + c) + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c) + a^3 + 2*
(2*a^2*b*cos(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a
```

Giac [F]

$$\int \frac{x^5}{a+b\sec\left(c+dx^2\right)}\,dx = \int \frac{x^5}{b\sec\left(dx^2+c\right)+a}\,dx$$

```
[In] integrate(x^5/(a+b*sec(d*x^2+c)),x, algorithm="giac")
[Out] integrate(x^5/(b*sec(d*x^2 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{a+b\sec\left(c+dx^2\right)}\,dx = \int \frac{x^5}{a+\frac{b}{\cos\left(d\,x^2+c\right)}}\,dx$$

```
[In] int(x^5/(a + b/cos(c + d*x^2)),x)
[Out] int(x^5/(a + b/cos(c + d*x^2)), x)
```

3.17 $\int \frac{x^4}{a+b\sec(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{a+b\sec(c+dx^2)}\,dx = \text{Int}\left(\frac{x^4}{a+b\sec(c+dx^2)},x\right)$$

```
[Out] Unintegrable(x^4/(a+b*sec(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^4}{a+b\sec(c+dx^2)}\,dx$$

```
[In] Int[x^4/(a + b*Sec[c + d*x^2]),x]
```

```
[Out] Defer[Int][x^4/(a + b*Sec[c + d*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4}{a+b\sec(c+dx^2)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^4}{a+b\sec(c+dx^2)}\,dx$$

```
[In] Integrate[x^4/(a + b*Sec[c + d*x^2]),x]
[Out] Integrate[x^4/(a + b*Sec[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a+b\sec(d\,x^2+c)}dx$$

```
[In] int(x^4/(a+b*sec(d*x^2+c)),x)
[Out] int(x^4/(a+b*sec(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^4}{b\sec(dx^2+c)+a}\,dx$$

```
[In] integrate(x^4/(a+b*sec(d*x^2+c)),x, algorithm="fricas")
[Out] integral(x^4/(b*sec(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^4}{a+b\sec(c+dx^2)}\,dx$$

```
[In] integrate(x**4/(a+b*sec(d*x**2+c)),x)
[Out] Integral(x**4/(a + b*sec(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 252, normalized size of antiderivative = 14.00

$$\int \frac{x^4}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^4}{b\sec(dx^2+c)+a}\,dx$$

```
[In] integrate(x^4/(a+b*sec(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/5*(x^5 - 10*a*b*integrate((a*x^4*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + 2*b*
x^4*cos(d*x^2 + c)^2 + a*x^4*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^4*si
n(d*x^2 + c)^2 + a*x^4*cos(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*
cos(d*x^2 + c)^2 + a^3*sin(2*d*x^2 + 2*c)^2 + 4*a^2*b*sin(2*d*x^2 + 2*c)*si
n(d*x^2 + c) + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c) + a^3 + 2*
(2*a^2*b*cos(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^4}{b\sec(dx^2+c)+a}\,dx$$

```
[In] integrate(x^4/(a+b*sec(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*sec(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 13.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^4}{a+\frac{b}{\cos(d\,x^2+c)}}\,dx$$

```
[In] int(x^4/(a + b/cos(c + d*x^2)),x)
```

```
[Out] int(x^4/(a + b/cos(c + d*x^2)), x)
```

3.18 $\int \frac{x^3}{a+b\sec(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 261

$$\int \frac{x^3}{a+b\sec(c+dx^2)}\,dx=\frac{x^4}{4a}+\frac{ibx^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}-\frac{ibx^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}+\frac{b\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2}-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2}$$

```
[Out] 1/4*x^4/a+1/2*I*b*x^2*ln(1+a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a/d/(-a
^2+b^2)^(1/2)-1/2*I*b*x^2*ln(1+a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a/d
/(-a^2+b^2)^(1/2)+1/2*b*polylog(2,-a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))
/a/d^2/(-a^2+b^2)^(1/2)-1/2*b*polylog(2,-a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(
1/2)))/a/d^2/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {4289, 4276, 3402, 2296, 2221, 2317, 2438}

$$\int \frac{x^3}{a+b\sec(c+dx^2)}\,dx=\frac{b\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{2ad^2\sqrt{b^2-a^2}}-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{2ad^2\sqrt{b^2-a^2}}+\frac{ibx^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{2ad\sqrt{b^2-a^2}}-\frac{ibx^2\log\left(1+\frac{ae^{i(c+dx^2)}}{\sqrt{b^2-a^2}+b}\right)}{2ad\sqrt{b^2-a^2}}+\frac{x^4}{4a}$$

```
[In] Int[x^3/(a + b*Sec[c + d*x^2]),x]
```

```
[Out] x^4/(4*a) + ((I/2)*b*x^2*Log[1 + (a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2
])])/(a*Sqrt[-a^2 + b^2]*d) - ((I/2)*b*x^2*Log[1 + (a*E^(I*(c + d*x^2)))/(b
 + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (b*PolyLog[2, -((a*E^(I*(c
+ d*x^2)))/(b - Sqrt[-a^2 + b^2]))])/(2*a*Sqrt[-a^2 + b^2]*d^2) - (b*PolyLo
g[2, -((a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2]))])/(2*a*Sqrt[-a^2 + b^2
]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
```

```
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+b\sec(c+dx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x}{a}-\frac{bx}{a(b+a\cos(c+dx))}\right)\,dx, x, x^2\right)\\
&= \frac{x^4}{4a}-\frac{b\text{Subst}\left(\int \frac{x}{b+a\cos(c+dx)}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^4}{4a}-\frac{b\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, x^2\right)}{a}\\
&= \frac{x^4}{4a}-\frac{b\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{-a^2+b^2}}+\frac{b\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{-a^2+b^2}}\\
&= \frac{x^4}{4a}+\frac{ibx^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}-\frac{ibx^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}\\
&\quad-\frac{(ib)\text{Subst}\left(\int \log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{2a\sqrt{-a^2+b^2}d}\\
&\quad+\frac{(ib)\text{Subst}\left(\int \log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{2a\sqrt{-a^2+b^2}d}
\end{aligned}$$

$$= \frac{x^4}{4a} + \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{2a\sqrt{-a^2+b^2}d^2} + \frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{2a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{x^4}{4a} + \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{b\,\text{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2} - \frac{b\,\text{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 845 vs. 2(261) = 522.

Time = 1.61 (sec) , antiderivative size = 845, normalized size of antiderivative = 3.24

$$\int \frac{x^3}{a + b\sec(c + dx^2)}\,dx$$

$$= \frac{(b + a\cos(c + dx^2))\left(x^4 - \frac{2b\left(2(c+dx^2)\text{arctanh}\left(\frac{(a+b)\cot\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right) - 2\left(c+\arccos\left(-\frac{b}{a}\right)\right)\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right) \cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[x^3/(a + b*Sec[c + d*x^2]),x]
```

```
[Out] ((b + a*Cos[c + d*x^2])*(x^4 - (2*b*(2*(c + d*x^2)*ArcTanh[((a + b)*Cot[(c
+ d*x^2)/2])/Sqrt[a^2 - b^2]] - 2*(c + ArcCos[-(b/a)])*ArcTanh[((a - b)*Tan
[(c + d*x^2)/2])/Sqrt[a^2 - b^2]] + (ArcCos[-(b/a)] - (2*I)*ArcTanh[((a + b
)*Cot[(c + d*x^2)/2])/Sqrt[a^2 - b^2]] + (2*I)*ArcTanh[((a - b)*Tan[(c + d*
x^2)/2])/Sqrt[a^2 - b^2]])*Log[Sqrt[a^2 - b^2]/(Sqrt[2]*Sqrt[a]*E^((I/2)*(c
 + d*x^2))*Sqrt[b + a*Cos[c + d*x^2]])] + (ArcCos[-(b/a)] + (2*I)*(ArcTanh[
((a + b)*Cot[(c + d*x^2)/2])/Sqrt[a^2 - b^2]] - ArcTanh[((a - b)*Tan[(c + d
*x^2)/2])/Sqrt[a^2 - b^2]]))*Log[(Sqrt[a^2 - b^2]*E^((I/2)*(c + d*x^2)))/(S
qrt[2]*Sqrt[a]*Sqrt[b + a*Cos[c + d*x^2]])] - (ArcCos[-(b/a)] - (2*I)*ArcTa
```

```
nh[((a - b)*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])*Log[((a + b)*(a - b - I*S
qrt[a^2 - b^2])*(1 + I*Tan[(c + d*x^2)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan
[(c + d*x^2)/2]))] - (ArcCos[-(b/a)] + (2*I)*ArcTanh[((a - b)*Tan[(c + d*x^
2)/2])/Sqrt[a^2 - b^2]])*Log[((a + b)*((-I)*a + I*b + Sqrt[a^2 - b^2])*(I +
 Tan[(c + d*x^2)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^2)/2]))] + I
*(PolyLog[2, ((b - I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x
^2)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^2)/2]))] - PolyLog[2, ((b
 + I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x^2)/2]))/(a*(a +
 b + Sqrt[a^2 - b^2]*Tan[(c + d*x^2)/2]))])))/(Sqrt[a^2 - b^2]*d^2))*Sec[c
+ d*x^2])/(4*a*(a + b*Sec[c + d*x^2]))
```

Maple [F]

$$\int \frac{x^3}{a + b\sec\left(d\,x^2 + c\right)}dx$$

[In] int(x^3/(a+b*sec(d*x^2+c)),x)

[Out] int(x^3/(a+b*sec(d*x^2+c)),x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1060 vs. $2(221) = 442$.

Time = 0.40 (sec) , antiderivative size = 1060, normalized size of antiderivative = 4.06

$$\int \frac{x^3}{a + b\sec\left(c + dx^2\right)}\,dx = \text{Too large to display}$$

[In] integrate(x^3/(a+b*sec(d*x^2+c)),x, algorithm="fricas")

```
[Out] 1/4*((a^2 - b^2)*d^2*x^4 - I*a*b*c*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^2
 + c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) + I*a*b*c*
sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*
sqrt(-(a^2 - b^2)/a^2) + 2*b) - I*a*b*c*sqrt(-(a^2 - b^2)/a^2)*log(-2*a*cos
(d*x^2 + c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) + I*
a*b*c*sqrt(-(a^2 - b^2)/a^2)*log(-2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c)
 + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) - a*b*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b
*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 +
c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + a*b*sqrt(-(a^2 - b^2)/a^2)*dilog(-
(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2
+ c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - a*b*sqrt(-(a^2 - b^2)/a^2)*dilog
(-(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^
2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + a*b*sqrt(-(a^2 - b^2)/a^2)*dil
og(-(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d*
```

```
x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - (I*a*b*d*x^2 + I*a*b*c)*sqrt
(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^
2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) - (-I*a*b*d*x^2
 - I*a*b*c)*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c) + I*b*sin(d*x^2 +
c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a)
 - (-I*a*b*d*x^2 - I*a*b*c)*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c) -
I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b
^2)/a^2) + a)/a) - (I*a*b*d*x^2 + I*a*b*c)*sqrt(-(a^2 - b^2)/a^2)*log((b*co
s(d*x^2 + c) - I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))
*sqrt(-(a^2 - b^2)/a^2) + a)/a))/((a^3 - a*b^2)*d^2)
```

Sympy [F]

$$\int \frac{x^3}{a+b\sec(c+dx^2)}\,dx=\int \frac{x^3}{a+b\sec(c+dx^2)}\,dx$$

```
[In] integrate(x**3/(a+b*sec(d*x**2+c)),x)
[Out] Integral(x**3/(a + b*sec(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3}{a+b\sec(c+dx^2)}\,dx=\int \frac{x^3}{b\sec(dx^2+c)+a}\,dx$$

```
[In] integrate(x^3/(a+b*sec(d*x^2+c)),x, algorithm="maxima")
[Out] 1/4*(x^4 - 8*a*b*integrate((a*x^3*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + 2*b*x
^3*cos(d*x^2 + c)^2 + a*x^3*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^3*sin
(d*x^2 + c)^2 + a*x^3*cos(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*c
os(d*x^2 + c)^2 + a^3*sin(2*d*x^2 + 2*c)^2 + 4*a^2*b*sin(2*d*x^2 + 2*c)*sin
(d*x^2 + c) + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c) + a^3 + 2*(
2*a^2*b*cos(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a
```

Giac [F]

$$\int \frac{x^3}{a+b\sec(c+dx^2)}\,dx=\int \frac{x^3}{b\sec(dx^2+c)+a}\,dx$$

```
[In] integrate(x^3/(a+b*sec(d*x^2+c)),x, algorithm="giac")
[Out] integrate(x^3/(b*sec(d*x^2 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\sec(c+dx^2)}\,dx=\int \frac{x^3}{a+\frac{b}{\cos(d\,x^2+c)}}\,dx$$

```
[In] int(x^3/(a + b/cos(c + d*x^2)),x)
[Out] int(x^3/(a + b/cos(c + d*x^2)), x)
```

3.19 $\int \frac{x^2}{a+b\sec(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{a+b\sec(c+dx^2)}\,dx = \operatorname{Int}\left(\frac{x^2}{a+b\sec(c+dx^2)},x\right)$$

```
[Out] Unintegrable(x^2/(a+b*sec(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^2}{a+b\sec(c+dx^2)}\,dx$$

```
[In] Int[x^2/(a + b*Sec[c + d*x^2]),x]
```

```
[Out] Defer[Int][x^2/(a + b*Sec[c + d*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{a+b\sec(c+dx^2)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^2}{a+b\sec(c+dx^2)}\,dx$$

```
[In] Integrate[x^2/(a + b*Sec[c + d*x^2]),x]
[Out] Integrate[x^2/(a + b*Sec[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\sec(d\,x^2+c)}dx$$

```
[In] int(x^2/(a+b*sec(d*x^2+c)),x)
[Out] int(x^2/(a+b*sec(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^2}{b\sec(dx^2+c)+a}\,dx$$

```
[In] integrate(x^2/(a+b*sec(d*x^2+c)),x, algorithm="fricas")
[Out] integral(x^2/(b*sec(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^2}{a+b\sec(c+dx^2)}\,dx$$

```
[In] integrate(x**2/(a+b*sec(d*x**2+c)),x)
[Out] Integral(x**2/(a + b*sec(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 252, normalized size of antiderivative = 14.00

$$\int \frac{x^2}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^2}{b\sec(dx^2+c)+a}\,dx$$

```
[In] integrate(x^2/(a+b*sec(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(x^3 - 6*a*b*integrate((a*x^2*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + 2*b*x
^2*cos(d*x^2 + c)^2 + a*x^2*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^2*sin
(d*x^2 + c)^2 + a*x^2*cos(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*c
os(d*x^2 + c)^2 + a^3*sin(2*d*x^2 + 2*c)^2 + 4*a^2*b*sin(2*d*x^2 + 2*c)*sin
(d*x^2 + c) + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c) + a^3 + 2*(
2*a^2*b*cos(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^2}{b\sec(dx^2+c)+a}\,dx$$

```
[In] integrate(x^2/(a+b*sec(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sec(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 13.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{a+b\sec(c+dx^2)}\,dx = \int \frac{x^2}{a+\frac{b}{\cos(d\,x^2+c)}}\,dx$$

```
[In] int(x^2/(a + b/cos(c + d*x^2)),x)
```

```
[Out] int(x^2/(a + b/cos(c + d*x^2)), x)
```

3.20 $\int \frac{x}{a+b\sec(c+dx^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{x}{a+b\sec(c+dx^2)}\,dx=\frac{x^2}{2a}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}$$

```
[Out] 1/2*x^2/a-b*arctanh((a-b)^(1/2)*tan(1/2*d*x^2+1/2*c)/(a+b)^(1/2))/a/d/(a-b)
^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4289, 3868, 2738, 214}

$$\int \frac{x}{a+b\sec(c+dx^2)}\,dx=\frac{x^2}{2a}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[In] Int[x/(a + b*Sec[c + d*x^2]),x]
```

```
[Out] x^2/(2*a) - (b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x^2)/2])/Sqrt[a + b]])/(a*Sq
rt[a - b]*Sqrt[a + b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b\sec(c+dx)}\,dx, x, x^2\right)\\
&= \frac{x^2}{2a} - \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^2}{2a} - \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{ad}\\
&= \frac{x^2}{2a} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{x}{a+b\sec\left(c+dx^2\right)}\,dx = \frac{\frac{c}{d}+x^2+\frac{2b\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}}{2a}$$

```
[In] Integrate[x/(a + b*Sec[c + d*x^2]),x]
```

```
[Out] (c/d + x^2 + (2*b*ArcTanh[((-a + b)*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])/(
Sqrt[a^2 - b^2]*d))/(2*a)
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}}{2d}$	70
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}}{2d}$	70
risch	$\frac{x^{2}}{2a}+\frac{b\ln\left(\mathrm{e}^{i\left(d\,x^{2}+c\right)}-\frac{ia^{2}-ib^{2}-b\sqrt{a^{2}-b^{2}}}{\sqrt{a^{2}-b^{2}}\,a}\right)}{2\sqrt{a^{2}-b^{2}}\,da}-\frac{b\ln\left(\mathrm{e}^{i\left(d\,x^{2}+c\right)}+\frac{ia^{2}-ib^{2}+b\sqrt{a^{2}-b^{2}}}{\sqrt{a^{2}-b^{2}}\,a}\right)}{2\sqrt{a^{2}-b^{2}}\,da}$	160

```
[In] int(x/(a+b*sec(d*x^2+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(2/a*arctan(tan(1/2*d*x^2+1/2*c))-2*b/a/((a-b)*(a+b))^(1/2)*arctanh((
a-b)*tan(1/2*d*x^2+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 3.80

$$\int \frac{x}{a+b\sec(c+dx^2)}\,dx = \left[\frac{2\,(a^2-b^2)dx^2+\sqrt{a^2-b^2}b\log\left(\frac{2\,ab\cos(dx^2+c)-(a^2-2\,b^2)\cos(dx^2+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx^2+c)+a)\sin(dx^2+c)+2\,a^2-b^2}{a^2\cos(dx^2+c)^2+2\,ab\cos(dx^2+c)+b^2}\right)}{4\,(a^3-ab^2)d},\right.$$

```
[In] integrate(x/(a+b*sec(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2 - b^2)*d*x^2 + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*x^2 + c) - (
a^2 - 2*b^2)*cos(d*x^2 + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x^2 + c) + a)*si
n(d*x^2 + c) + 2*a^2 - b^2)/(a^2*cos(d*x^2 + c)^2 + 2*a*b*cos(d*x^2 + c) +
b^2)))/((a^3 - a*b^2)*d), 1/2*((a^2 - b^2)*d*x^2 - sqrt(-a^2 + b^2)*b*arcta
n(-sqrt(-a^2 + b^2)*(b*cos(d*x^2 + c) + a)/((a^2 - b^2)*sin(d*x^2 + c))))/(
(a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{x}{a+b\sec(c+dx^2)}\,dx = \int \frac{x}{a+b\sec(c+dx^2)}\,dx$$

```
[In] integrate(x/(a+b*sec(d*x**2+c)),x)
[Out] Integral(x/(a + b*sec(c + d*x**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7945 vs. $2(55) = 110$.

Time = 28.61 (sec) , antiderivative size = 7945, normalized size of antiderivative = 120.38

$$\int \frac{x}{a+b\sec(c+dx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(a+b*sec(d*x^2+c)),x, algorithm="maxima")
[Out] 1/2*(sqrt(-a^2 + b^2)*d*x^2 - b*arctan2(2*(4*(a^6 - a^4*b^2)*cos(d*x^2 + 2*
c)^4*cos(c)*sin(c) - 4*(a^6 - a^4*b^2)*cos(c)*sin(d*x^2 + 2*c)^4*sin(c) + 4
*(3*(a^5*b - a^3*b^3)*cos(c)^2*sin(c) + (a^5*b - a^3*b^3)*sin(c)^3)*cos(d*x
^2 + 2*c)^3 - 4*((a^5*b - a^3*b^3)*cos(c)^3 + 3*(a^5*b - a^3*b^3)*cos(c)*si
n(c)^2 + ((a^6 - a^4*b^2)*cos(c)^2 - (a^6 - a^4*b^2)*sin(c)^2)*cos(d*x^2 +
2*c))*sin(d*x^2 + 2*c)^3 - 4*((a^6 - 5*a^4*b^2 + 4*a^2*b^4)*cos(c)^3*sin(c)
 + (a^6 - 5*a^4*b^2 + 4*a^2*b^4)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + 4*((
a^6 - 5*a^4*b^2 + 4*a^2*b^4)*cos(c)^3*sin(c) + (a^6 - 5*a^4*b^2 + 4*a^2*b^4
)*cos(c)*sin(c)^3 - 3*((a^5*b - a^3*b^3)*cos(c)^2*sin(c) - (a^5*b - a^3*b^3
)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - 4*((a^5*b - 3*a^3*b^3 +
2*a*b^5)*cos(c)^4*sin(c) + 2*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(c)^2*sin(c)^
3 + (a^5*b - 3*a^3*b^3 + 2*a*b^5)*sin(c)^5)*cos(d*x^2 + 2*c) + 4*((a^5*b -
3*a^3*b^3 + 2*a*b^5)*cos(c)^5 + 2*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(c)^3*si
n(c)^2 + (a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(c)*sin(c)^4 - ((a^6 - a^4*b^2)*c
os(c)^2 - (a^6 - a^4*b^2)*sin(c)^2)*cos(d*x^2 + 2*c)^3 - 3*((a^5*b - a^3*b^
3)*cos(c)^3 - (a^5*b - a^3*b^3)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^2 + ((a^6
 - 5*a^4*b^2 + 4*a^2*b^4)*cos(c)^4 - (a^6 - 5*a^4*b^2 + 4*a^2*b^4)*sin(c)^4
)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c) + (a^5*cos(c)*sin(d*x^2 + 2*c)^5 - a^5
*cos(d*x^2 + 2*c)^5*sin(c) - 4*a^4*b*cos(d*x^2 + 2*c)^4*cos(c)*sin(c) - (a^
5*cos(d*x^2 + 2*c)*sin(c) - 4*a^4*b*cos(c)*sin(c))*sin(d*x^2 + 2*c)^4 + 2*(
3*(a^5 - 2*a^3*b^2)*cos(c)^2*sin(c) + (a^5 - 2*a^3*b^2)*sin(c)^3)*cos(d*x^2
 + 2*c)^3 + 2*(a^5*cos(d*x^2 + 2*c)^2*cos(c) - (a^5 - 2*a^3*b^2)*cos(c)^3 -
 3*(a^5 - 2*a^3*b^2)*cos(c)*sin(c)^2 + 2*(a^4*b*cos(c)^2 - a^4*b*sin(c)^2)*
cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^3 + 4*((3*a^4*b - 4*a^2*b^3)*cos(c)^3*si
n(c) + (3*a^4*b - 4*a^2*b^3)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c)^2 - 2*(a^5*c
```

os(d*x^2 + 2*c)^3*sin(c) + 2*(3*a^4*b - 4*a^2*b^3)*cos(c)^3*sin(c) + 2*(3*a
^4*b - 4*a^2*b^3)*cos(c)*sin(c)^3 + 3*((a^5 - 2*a^3*b^2)*cos(c)^2*sin(c) -
(a^5 - 2*a^3*b^2)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - ((a^5 -
8*a^3*b^2 + 8*a*b^4)*cos(c)^4*sin(c) + 2*(a^5 - 8*a^3*b^2 + 8*a*b^4)*cos(c)
^2*sin(c)^3 + (a^5 - 8*a^3*b^2 + 8*a*b^4)*sin(c)^5)*cos(d*x^2 + 2*c) + (a^5
*cos(d*x^2 + 2*c)^4*cos(c) + (a^5 - 8*a^3*b^2 + 8*a*b^4)*cos(c)^5 + 2*(a^5
- 8*a^3*b^2 + 8*a*b^4)*cos(c)^3*sin(c)^2 + (a^5 - 8*a^3*b^2 + 8*a*b^4)*cos(
c)*sin(c)^4 + 4*(a^4*b*cos(c)^2 - a^4*b*sin(c)^2)*cos(d*x^2 + 2*c)^3 - 6*((
a^5 - 2*a^3*b^2)*cos(c)^3 - (a^5 - 2*a^3*b^2)*cos(c)*sin(c)^2)*cos(d*x^2 +
2*c)^2 - 4*((3*a^4*b - 4*a^2*b^3)*cos(c)^4 - (3*a^4*b - 4*a^2*b^3)*sin(c)^4
)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c))*sqrt(-a^2 + b^2))/(a^6*cos(d*x^2 + 2*
c)^6 + 6*a^5*b*cos(d*x^2 + 2*c)^5*cos(c) + a^6*sin(d*x^2 + 2*c)^6 + 6*a^5*b
*sin(d*x^2 + 2*c)^5*sin(c) - (a^6 - 18*a^4*b^2 + 48*a^2*b^4 - 32*b^6)*cos(c
)^6 - 3*(a^6 - 18*a^4*b^2 + 48*a^2*b^4 - 32*b^6)*cos(c)^4*sin(c)^2 - 3*(a^6
 - 18*a^4*b^2 + 48*a^2*b^4 - 32*b^6)*cos(c)^2*sin(c)^4 - (a^6 - 18*a^4*b^2
+ 48*a^2*b^4 - 32*b^6)*sin(c)^6 - 3*(5*(a^6 - 2*a^4*b^2)*cos(c)^2 + (a^6 -
2*a^4*b^2)*sin(c)^2)*cos(d*x^2 + 2*c)^4 + 3*(a^6*cos(d*x^2 + 2*c)^2 + 2*a^5
*b*cos(d*x^2 + 2*c)*cos(c) - (a^6 - 2*a^4*b^2)*cos(c)^2 - 5*(a^6 - 2*a^4*b^
2)*sin(c)^2)*sin(d*x^2 + 2*c)^4 - 4*(5*(3*a^5*b - 4*a^3*b^3)*cos(c)^3 + 3*(
3*a^5*b - 4*a^3*b^3)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 + 4*(3*a^5*b*cos(d
*x^2 + 2*c)^2*sin(c) - 6*(a^6 - 2*a^4*b^2)*cos(d*x^2 + 2*c)*cos(c)*sin(c) -
 3*(3*a^5*b - 4*a^3*b^3)*cos(c)^2*sin(c) - 5*(3*a^5*b - 4*a^3*b^3)*sin(c)^3
)*sin(d*x^2 + 2*c)^3 + 3*(5*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^4 + 6*(a^6
 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^2*sin(c)^2 + (a^6 - 8*a^4*b^2 + 8*a^2*b^4)
*sin(c)^4)*cos(d*x^2 + 2*c)^2 + 3*(a^6*cos(d*x^2 + 2*c)^4 + 4*a^5*b*cos(d*x
^2 + 2*c)^3*cos(c) + (a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^4 + 6*(a^6 - 8*a^
4*b^2 + 8*a^2*b^4)*cos(c)^2*sin(c)^2 + 5*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*sin(
c)^4 - 6*((a^6 - 2*a^4*b^2)*cos(c)^2 + (a^6 - 2*a^4*b^2)*sin(c)^2)*cos(d*x^
2 + 2*c)^2 - 4*((3*a^5*b - 4*a^3*b^3)*cos(c)^3 + 3*(3*a^5*b - 4*a^3*b^3)*co
s(c)*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 + 6*((5*a^5*b - 20*a^3*
b^3 + 16*a*b^5)*cos(c)^5 + 2*(5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*cos(c)^3*sin
(c)^2 + (5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*cos(c)*sin(c)^4)*cos(d*x^2 + 2*c)
 + 6*(a^5*b*cos(d*x^2 + 2*c)^4*sin(c) - 4*(a^6 - 2*a^4*b^2)*cos(d*x^2 + 2*c
)^3*cos(c)*sin(c) + (5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*cos(c)^4*sin(c) + 2*(
5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*cos(c)^2*sin(c)^3 + (5*a^5*b - 20*a^3*b^3
+ 16*a*b^5)*sin(c)^5 - 2*(3*(3*a^5*b - 4*a^3*b^3)*cos(c)^2*sin(c) + (3*a^5*
b - 4*a^3*b^3)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + 4*((a^6 - 8*a^4*b^2 + 8*a^2*b
^4)*cos(c)^3*sin(c) + (a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)*sin(c)^3)*cos(d*
x^2 + 2*c))*sin(d*x^2 + 2*c) + 2*(3*a^5*cos(d*x^2 + 2*c)^5*cos(c) + 3*a^5*s
in(d*x^2 + 2*c)^5*sin(c) + (3*a^4*b - 16*a^2*b^3 + 16*b^5)*cos(c)^6 + 3*(3*
a^4*b - 16*a^2*b^3 + 16*b^5)*cos(c)^4*sin(c)^2 + 3*(3*a^4*b - 16*a^2*b^3 +
16*b^5)*cos(c)^2*sin(c)^4 + (3*a^4*b - 16*a^2*b^3 + 16*b^5)*sin(c)^6 + 3*(5
*a^4*b*cos(c)^2 + a^4*b*sin(c)^2)*cos(d*x^2 + 2*c)^4 + 3*(a^5*cos(d*x^2 + 2
*c)*cos(c) + a^4*b*cos(c)^2 + 5*a^4*b*sin(c)^2)*sin(d*x^2 + 2*c)^4 - 2*(5*(
a^5 - 4*a^3*b^2)*cos(c)^3 + 3*(a^5 - 4*a^3*b^2)*cos(c)*sin(c)^2)*cos(d*x^2

```
+ 2*c)^3 + 2*(3*a^5*cos(d*x^2 + 2*c)^2*sin(c) + 12*a^4*b*cos(d*x^2 + 2*c)*c
os(c)*sin(c) - 3*(a^5 - 4*a^3*b^2)*cos(c)^2*sin(c) - 5*(a^5 - 4*a^3*b^2)*si
n(c)^3)*sin(d*x^2 + 2*c)^3 - 6*(5*(a^4*b - 2*a^2*b^3)*cos(c)^4 + 6*(a^4*b -
 2*a^2*b^3)*cos(c)^2*sin(c)^2 + (a^4*b - 2*a^2*b^3)*sin(c)^4)*cos(d*x^2 + 2
*c)^2 + 6*(a^5*cos(d*x^2 + 2*c)^3*cos(c) - (a^4*b - 2*a^2*b^3)*cos(c)^4 - 6
*(a^4*b - 2*a^2*b^3)*cos(c)^2*sin(c)^2 - 5*(a^4*b - 2*a^2*b^3)*sin(c)^4 + 3
*(a^4*b*cos(c)^2 + a^4*b*sin(c)^2)*cos(d*x^2 + 2*c)^2 - ((a^5 - 4*a^3*b^2)*
cos(c)^3 + 3*(a^5 - 4*a^3*b^2)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2
 + 2*c)^2 + 3*((a^5 - 12*a^3*b^2 + 16*a*b^4)*cos(c)^5 + 2*(a^5 - 12*a^3*b^2
 + 16*a*b^4)*cos(c)^3*sin(c)^2 + (a^5 - 12*a^3*b^2 + 16*a*b^4)*cos(c)*sin(c
)^4)*cos(d*x^2 + 2*c) + 3*(a^5*cos(d*x^2 + 2*c)^4*sin(c) + 8*a^4*b*cos(d*x^
2 + 2*c)^3*cos(c)*sin(c) + (a^5 - 12*a^3*b^2 + 16*a*b^4)*cos(c)^4*sin(c) +
2*(a^5 - 12*a^3*b^2 + 16*a*b^4)*cos(c)^2*sin(c)^3 + (a^5 - 12*a^3*b^2 + 16*
a*b^4)*sin(c)^5 - 2*(3*(a^5 - 4*a^3*b^2)*cos(c)^2*sin(c) + (a^5 - 4*a^3*b^2
)*sin(c)^3)*cos(d*x^2 + 2*c)^2 - 16*((a^4*b - 2*a^2*b^3)*cos(c)^3*sin(c) +
(a^4*b - 2*a^2*b^3)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c))*sq
rt(-a^2 + b^2)), (a^6*cos(d*x^2 + 2*c)^6 + 6*a^5*b*cos(d*x^2 + 2*c)^5*cos(c
) + a^6*sin(d*x^2 + 2*c)^6 + 6*a^5*b*sin(d*x^2 + 2*c)^5*sin(c) + (a^6 - 8*a
^4*b^2 + 8*a^2*b^4)*cos(c)^6 + 3*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^4*sin
(c)^2 + 3*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^2*sin(c)^4 + (a^6 - 8*a^4*b^
2 + 8*a^2*b^4)*sin(c)^6 - (5*(a^6 - 4*a^4*b^2)*cos(c)^2 + (a^6 - 4*a^4*b^2)
*sin(c)^2)*cos(d*x^2 + 2*c)^4 + (3*a^6*cos(d*x^2 + 2*c)^2 + 6*a^5*b*cos(d*x
^2 + 2*c)*cos(c) - (a^6 - 4*a^4*b^2)*cos(c)^2 - 5*(a^6 - 4*a^4*b^2)*sin(c)^
2)*sin(d*x^2 + 2*c)^4 - 4*(5*(a^5*b - 2*a^3*b^3)*cos(c)^3 + 3*(a^5*b - 2*a^
3*b^3)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 + 4*(3*a^5*b*cos(d*x^2 + 2*c)^2*
sin(c) - 2*(a^6 - 4*a^4*b^2)*cos(d*x^2 + 2*c)*cos(c)*sin(c) - 3*(a^5*b - 2*
a^3*b^3)*cos(c)^2*sin(c) - 5*(a^5*b - 2*a^3*b^3)*sin(c)^3)*sin(d*x^2 + 2*c)
^3 - (5*(a^6 + 4*a^4*b^2 - 8*a^2*b^4)*cos(c)^4 + 6*(a^6 + 4*a^4*b^2 - 8*a^2
*b^4)*cos(c)^2*sin(c)^2 + (a^6 + 4*a^4*b^2 - 8*a^2*b^4)*sin(c)^4)*cos(d*x^2
 + 2*c)^2 + (3*a^6*cos(d*x^2 + 2*c)^4 + 12*a^5*b*cos(d*x^2 + 2*c)^3*cos(c)
- (a^6 + 4*a^4*b^2 - 8*a^2*b^4)*cos(c)^4 - 6*(a^6 + 4*a^4*b^2 - 8*a^2*b^4)*
cos(c)^2*sin(c)^2 - 5*(a^6 + 4*a^4*b^2 - 8*a^2*b^4)*sin(c)^4 - 6*((a^6 - 4*
a^4*b^2)*cos(c)^2 + (a^6 - 4*a^4*b^2)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 12*((a
^5*b - 2*a^3*b^3)*cos(c)^3 + 3*(a^5*b - 2*a^3*b^3)*cos(c)*sin(c)^2)*cos(d*x
^2 + 2*c))*sin(d*x^2 + 2*c)^2 - 2*((5*a^5*b - 8*a*b^5)*cos(c)^5 + 2*(5*a^5*
b - 8*a*b^5)*cos(c)^3*sin(c)^2 + (5*a^5*b - 8*a*b^5)*cos(c)*sin(c)^4)*cos(d
*x^2 + 2*c) + 2*(3*a^5*b*cos(d*x^2 + 2*c)^4*sin(c) - 4*(a^6 - 4*a^4*b^2)*co
s(d*x^2 + 2*c)^3*cos(c)*sin(c) - (5*a^5*b - 8*a*b^5)*cos(c)^4*sin(c) - 2*(5
*a^5*b - 8*a*b^5)*cos(c)^2*sin(c)^3 - (5*a^5*b - 8*a*b^5)*sin(c)^5 - 6*(3*(
a^5*b - 2*a^3*b^3)*cos(c)^2*sin(c) + (a^5*b - 2*a^3*b^3)*sin(c)^3)*cos(d*x^
2 + 2*c)^2 - 4*((a^6 + 4*a^4*b^2 - 8*a^2*b^4)*cos(c)^3*sin(c) + (a^6 + 4*a^
4*b^2 - 8*a^2*b^4)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c) + 4*
(a^5*cos(d*x^2 + 2*c)^5*cos(c) + a^5*sin(d*x^2 + 2*c)^5*sin(c) - (a^4*b - 2
*a^2*b^3)*cos(c)^6 - 3*(a^4*b - 2*a^2*b^3)*cos(c)^4*sin(c)^2 - 3*(a^4*b - 2
*a^2*b^3)*cos(c)^2*sin(c)^4 - (a^4*b - 2*a^2*b^3)*sin(c)^6 + (5*a^4*b*cos(c
```

)^2 + a^4*b*sin(c)^2)*cos(d*x^2 + 2*c)^4 + (a^5*cos(d*x^2 + 2*c)*cos(c) + a
^4*b*cos(c)^2 + 5*a^4*b*sin(c)^2)*sin(d*x^2 + 2*c)^4 + 2*(5*a^3*b^2*cos(c)^
3 + 3*a^3*b^2*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 + 2*(a^5*cos(d*x^2 + 2*c)
^2*sin(c) + 4*a^4*b*cos(d*x^2 + 2*c)*cos(c)*sin(c) + 3*a^3*b^2*cos(c)^2*sin
(c) + 5*a^3*b^2*sin(c)^3)*sin(d*x^2 + 2*c)^3 + 2*(5*a^2*b^3*cos(c)^4 + 6*a^
2*b^3*cos(c)^2*sin(c)^2 + a^2*b^3*sin(c)^4)*cos(d*x^2 + 2*c)^2 + 2*(a^5*cos
(d*x^2 + 2*c)^3*cos(c) + a^2*b^3*cos(c)^4 + 6*a^2*b^3*cos(c)^2*sin(c)^2 + 5
*a^2*b^3*sin(c)^4 + 3*(a^4*b*cos(c)^2 + a^4*b*sin(c)^2)*cos(d*x^2 + 2*c)^2
+ 3*(a^3*b^2*cos(c)^3 + 3*a^3*b^2*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*
x^2 + 2*c)^2 - ((a^5 - 2*a^3*b^2 - 4*a*b^4)*cos(c)^5 + 2*(a^5 - 2*a^3*b^2 -
4*a*b^4)*cos(c)^3*sin(c)^2 + (a^5 - 2*a^3*b^2 - 4*a*b^4)*cos(c)*sin(c)^4)*
cos(d*x^2 + 2*c) + (a^5*cos(d*x^2 + 2*c)^4*sin(c) + 8*a^4*b*cos(d*x^2 + 2*c
)^3*cos(c)*sin(c) - (a^5 - 2*a^3*b^2 - 4*a*b^4)*cos(c)^4*sin(c) - 2*(a^5 -
2*a^3*b^2 - 4*a*b^4)*cos(c)^2*sin(c)^3 - (a^5 - 2*a^3*b^2 - 4*a*b^4)*sin(c)
^5 + 6*(3*a^3*b^2*cos(c)^2*sin(c) + a^3*b^2*sin(c)^3)*cos(d*x^2 + 2*c)^2 +
16*(a^2*b^3*cos(c)^3*sin(c) + a^2*b^3*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*si
n(d*x^2 + 2*c))*sqrt(-a^2 + b^2))/(a^6*cos(d*x^2 + 2*c)^6 + 6*a^5*b*cos(d*x
^2 + 2*c)^5*cos(c) + a^6*sin(d*x^2 + 2*c)^6 + 6*a^5*b*sin(d*x^2 + 2*c)^5*si
n(c) - (a^6 - 18*a^4*b^2 + 48*a^2*b^4 - 32*b^6)*cos(c)^6 - 3*(a^6 - 18*a^4*
b^2 + 48*a^2*b^4 - 32*b^6)*cos(c)^4*sin(c)^2 - 3*(a^6 - 18*a^4*b^2 + 48*a^2
*b^4 - 32*b^6)*cos(c)^2*sin(c)^4 - (a^6 - 18*a^4*b^2 + 48*a^2*b^4 - 32*b^6)
sin(c)^6 - 3(5*(a^6 - 2*a^4*b^2)*cos(c)^2 + (a^6 - 2*a^4*b^2)*sin(c)^2)*c
os(d*x^2 + 2*c)^4 + 3*(a^6*cos(d*x^2 + 2*c)^2 + 2*a^5*b*cos(d*x^2 + 2*c)*co
s(c) - (a^6 - 2*a^4*b^2)*cos(c)^2 - 5*(a^6 - 2*a^4*b^2)*sin(c)^2)*sin(d*x^2
+ 2*c)^4 - 4*(5*(3*a^5*b - 4*a^3*b^3)*cos(c)^3 + 3*(3*a^5*b - 4*a^3*b^3)*c
os(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 + 4*(3*a^5*b*cos(d*x^2 + 2*c)^2*sin(c) -
6*(a^6 - 2*a^4*b^2)*cos(d*x^2 + 2*c)*cos(c)*sin(c) - 3*(3*a^5*b - 4*a^3*b^
3)*cos(c)^2*sin(c) - 5*(3*a^5*b - 4*a^3*b^3)*sin(c)^3)*sin(d*x^2 + 2*c)^3 +
3*(5*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^4 + 6*(a^6 - 8*a^4*b^2 + 8*a^2*b
^4)*cos(c)^2*sin(c)^2 + (a^6 - 8*a^4*b^2 + 8*a^2*b^4)*sin(c)^4)*cos(d*x^2 +
2*c)^2 + 3*(a^6*cos(d*x^2 + 2*c)^4 + 4*a^5*b*cos(d*x^2 + 2*c)^3*cos(c) + (
a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^4 + 6*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos
(c)^2*sin(c)^2 + 5*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*sin(c)^4 - 6*((a^6 - 2*a^4
*b^2)*cos(c)^2 + (a^6 - 2*a^4*b^2)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 4*((3*a^5
*b - 4*a^3*b^3)*cos(c)^3 + 3*(3*a^5*b - 4*a^3*b^3)*cos(c)*sin(c)^2)*cos(d*x
^2 + 2*c))*sin(d*x^2 + 2*c)^2 + 6*((5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*cos(c)
^5 + 2*(5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*cos(c)^3*sin(c)^2 + (5*a^5*b - 20*
a^3*b^3 + 16*a*b^5)*cos(c)*sin(c)^4)*cos(d*x^2 + 2*c) + 6*(a^5*b*cos(d*x^2
+ 2*c)^4*sin(c) - 4*(a^6 - 2*a^4*b^2)*cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + (5
*a^5*b - 20*a^3*b^3 + 16*a*b^5)*cos(c)^4*sin(c) + 2*(5*a^5*b - 20*a^3*b^3 +
16*a*b^5)*cos(c)^2*sin(c)^3 + (5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*sin(c)^5 -
2*(3*(3*a^5*b - 4*a^3*b^3)*cos(c)^2*sin(c) + (3*a^5*b - 4*a^3*b^3)*sin(c)^
3)*cos(d*x^2 + 2*c)^2 + 4*((a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^3*sin(c) +
(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2
+ 2*c) + 2*(3*a^5*cos(d*x^2 + 2*c)^5*cos(c) + 3*a^5*sin(d*x^2 + 2*c)^5*sin(

```
c) + (3*a^4*b - 16*a^2*b^3 + 16*b^5)*cos(c)^6 + 3*(3*a^4*b - 16*a^2*b^3 + 1
6*b^5)*cos(c)^4*sin(c)^2 + 3*(3*a^4*b - 16*a^2*b^3 + 16*b^5)*cos(c)^2*sin(c
)^4 + (3*a^4*b - 16*a^2*b^3 + 16*b^5)*sin(c)^6 + 3*(5*a^4*b*cos(c)^2 + a^4*
b*sin(c)^2)*cos(d*x^2 + 2*c)^4 + 3*(a^5*cos(d*x^2 + 2*c)*cos(c) + a^4*b*cos
(c)^2 + 5*a^4*b*sin(c)^2)*sin(d*x^2 + 2*c)^4 - 2*(5*(a^5 - 4*a^3*b^2)*cos(c
)^3 + 3*(a^5 - 4*a^3*b^2)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 + 2*(3*a^5*co
s(d*x^2 + 2*c)^2*sin(c) + 12*a^4*b*cos(d*x^2 + 2*c)*cos(c)*sin(c) - 3*(a^5
- 4*a^3*b^2)*cos(c)^2*sin(c) - 5*(a^5 - 4*a^3*b^2)*sin(c)^3)*sin(d*x^2 + 2*
c)^3 - 6*(5*(a^4*b - 2*a^2*b^3)*cos(c)^4 + 6*(a^4*b - 2*a^2*b^3)*cos(c)^2*s
in(c)^2 + (a^4*b - 2*a^2*b^3)*sin(c)^4)*cos(d*x^2 + 2*c)^2 + 6*(a^5*cos(d*x
^2 + 2*c)^3*cos(c) - (a^4*b - 2*a^2*b^3)*cos(c)^4 - 6*(a^4*b - 2*a^2*b^3)*c
os(c)^2*sin(c)^2 - 5*(a^4*b - 2*a^2*b^3)*sin(c)^4 + 3*(a^4*b*cos(c)^2 + a^4
*b*sin(c)^2)*cos(d*x^2 + 2*c)^2 - ((a^5 - 4*a^3*b^2)*cos(c)^3 + 3*(a^5 - 4*
a^3*b^2)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 + 3*((a^5 -
12*a^3*b^2 + 16*a*b^4)*cos(c)^5 + 2*(a^5 - 12*a^3*b^2 + 16*a*b^4)*cos(c)^3*
sin(c)^2 + (a^5 - 12*a^3*b^2 + 16*a*b^4)*cos(c)*sin(c)^4)*cos(d*x^2 + 2*c)
+ 3*(a^5*cos(d*x^2 + 2*c)^4*sin(c) + 8*a^4*b*cos(d*x^2 + 2*c)^3*cos(c)*sin(
c) + (a^5 - 12*a^3*b^2 + 16*a*b^4)*cos(c)^4*sin(c) + 2*(a^5 - 12*a^3*b^2 +
16*a*b^4)*cos(c)^2*sin(c)^3 + (a^5 - 12*a^3*b^2 + 16*a*b^4)*sin(c)^5 - 2*(3
*(a^5 - 4*a^3*b^2)*cos(c)^2*sin(c) + (a^5 - 4*a^3*b^2)*sin(c)^3)*cos(d*x^2
+ 2*c)^2 - 16*((a^4*b - 2*a^2*b^3)*cos(c)^3*sin(c) + (a^4*b - 2*a^2*b^3)*co
s(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c))*sqrt(-a^2 + b^2))))/(sqr
t(-a^2 + b^2)*a*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(55) = 110$.

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 4.21

$$\int \frac{x}{a+b\sec(c+dx^2)}\,dx = \frac{\left(\sqrt{-a^2+b^2}(a-2\,b)d|-a+b| - \sqrt{-a^2+b^2}|a||-a+b||d|\right)\left(\pi\left\lfloor\frac{dx^2+c}{2\,\pi}+\frac{1}{2}\right\rfloor + \arctan\left(\frac{\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)}{\sqrt{-\frac{bd+\sqrt{b^2d^2+(ad+bd)(ad-bd)}}{ad-bd}}}\right)\right)}{2\left((a^2-2\,ab+b^2)a^2d^2+(a^2b-2\,ab^2+b^3)d|a||d|\right)} + \frac{(ad-2\,bd+|a||d|)\left(\pi\left\lfloor\frac{dx^2+c}{2\,\pi}+\frac{1}{2}\right\rfloor + \arctan\left(\frac{\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)}{\sqrt{-\frac{bd-\sqrt{b^2d^2+(ad+bd)(ad-bd)}}{ad-bd}}}\right)\right)}{2\left(a^2d^2-bd|a||d|\right)}$$

```
[In] integrate(x/(a+b*sec(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(-a^2 + b^2)*(a - 2*b)*d*abs(-a + b) - sqrt(-a^2 + b^2)*abs(a)*abs
(-a + b)*abs(d))*(pi*floor(1/2*(d*x^2 + c)/pi + 1/2) + arctan(tan(1/2*d*x^2
```

```
 + 1/2*c)/sqrt(-(b*d + sqrt(b^2*d^2 + (a*d + b*d)*(a*d - b*d)))/(a*d - b*d)
)))/((a^2 - 2*a*b + b^2)*a^2*d^2 + (a^2*b - 2*a*b^2 + b^3)*d*abs(a)*abs(d))
 + 1/2*(a*d - 2*b*d + abs(a)*abs(d))*(pi*floor(1/2*(d*x^2 + c)/pi + 1/2) +
arctan(tan(1/2*d*x^2 + 1/2*c)/sqrt(-(b*d - sqrt(b^2*d^2 + (a*d + b*d)*(a*d
- b*d)))/(a*d - b*d))))/(a^2*d^2 - b*d*abs(a)*abs(d))
```

Mupad [B] (verification not implemented)

Time = 15.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.38

$$\int \frac{x}{a+b\sec(c+dx^2)}\,dx = \frac{x^2}{2\,a}+\frac{b\,\ln\left(2\,b\,x\,e^{d\,x^2\,1i}\,e^{c\,1i}-\frac{b\,x\,\left(a+b\,e^{d\,x^2\,1i}\,e^{c\,1i}\right)\,2i}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{2\,a\,d\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{b\,\ln\left(2\,b\,x\,e^{d\,x^2\,1i}\,e^{c\,1i}+\frac{b\,x\,\left(a+b\,e^{d\,x^2\,1i}\,e^{c\,1i}\right)\,2i}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{2\,a\,d\,\sqrt{a+b}\,\sqrt{a-b}}$$

```
[In] int(x/(a + b/cos(c + d*x^2)),x)
```

```
[Out] x^2/(2*a) + (b*log(2*b*x*exp(d*x^2*1i)*exp(c*1i) - (b*x*(a + b*exp(d*x^2*1i
)*exp(c*1i))*2i)/((a + b)^(1/2)*(a - b)^(1/2))))/(2*a*d*(a + b)^(1/2)*(a -
b)^(1/2)) - (b*log(2*b*x*exp(d*x^2*1i)*exp(c*1i) + (b*x*(a + b*exp(d*x^2*1i
)*exp(c*1i))*2i)/((a + b)^(1/2)*(a - b)^(1/2))))/(2*a*d*(a + b)^(1/2)*(a -
b)^(1/2))
```

3.21 $\int \frac{1}{x(a+b\sec(c+dx^2))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)}\,dx = \text{Int}\left(\frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sec(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)}\,dx = \int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)}\,dx$$

```
[In] Int[1/(x*(a + b*Sec[c + d*x^2])),x]
[Out] Defer[Int][1/(x*(a + b*Sec[c + d*x^2])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*Sec[c + d*x^2])),x]
[Out] Integrate[1/(x*(a + b*Sec[c + d*x^2])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sec\left(d\,x^2+c\right)\right)}dx$$

```
[In] int(1/x/(a+b*sec(d*x^2+c)),x)
[Out] int(1/x/(a+b*sec(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{\left(b\sec\left(dx^2+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sec(d*x^2+c)),x, algorithm="fricas")
[Out] integral(1/(b*x*sec(d*x^2 + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*sec(d*x**2+c)),x)
[Out] Integral(1/(x*(a + b*sec(c + d*x**2))), x)
```

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 249, normalized size of antiderivative = 13.83

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{(b\sec\left(dx^2+c\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*sec(d*x^2+c)),x, algorithm="maxima")
[Out] -(2*a*b*integrate((a*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + 2*b*cos(d*x^2 + c)
^2 + a*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*sin(d*x^2 + c)^2 + a*cos(d*x
^2 + c))/(a^3*x*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*x*cos(d*x^2 + c)^2 + a^3*x*s
in(2*d*x^2 + 2*c)^2 + 4*a^2*b*x*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*a*b^2
*x*sin(d*x^2 + c)^2 + 4*a^2*b*x*cos(d*x^2 + c) + a^3*x + 2*(2*a^2*b*x*cos(d
*x^2 + c) + a^3*x)*cos(2*d*x^2 + 2*c)), x) - log(x))/a
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{(b\sec\left(dx^2+c\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*sec(d*x^2+c)),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x^2 + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 13.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+\frac{b}{\cos(d\,x^2+c)}\right)}\,dx$$

```
[In] int(1/(x*(a + b/cos(c + d*x^2))),x)
[Out] int(1/(x*(a + b/cos(c + d*x^2))), x)
```

3.22 $\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx = -\frac{a}{x} + b\text{Int}\left(\frac{\sec(c+dx^2)}{x^2}, x\right)$$

```
[Out] -a/x+b*Unintegrable(sec(d*x^2+c)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx = \int \frac{a+b\sec(c+dx^2)}{x^2}\,dx$$

```
[In] Int[(a + b*Sec[c + d*x^2])/x^2,x]
[Out] -(a/x) + b*Defer[Int][Sec[c + d*x^2]/x^2, x]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a}{x^2} + \frac{b\sec(c+dx^2)}{x^2}\right) dx$$
$$= -\frac{a}{x} + b\int \frac{\sec(c+dx^2)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\sec(c + dx^2)}{x^2}\, dx = \int \frac{a + b\sec(c + dx^2)}{x^2}\, dx$$

```
[In] Integrate[(a + b*Sec[c + d*x^2])/x^2,x]
[Out] Integrate[(a + b*Sec[c + d*x^2])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a + b\sec(d\,x^2 + c)}{x^2} dx$$

```
[In] int((a+b*sec(d*x^2+c))/x^2,x)
[Out] int((a+b*sec(d*x^2+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\sec(c + dx^2)}{x^2}\, dx = \int \frac{b\sec(dx^2 + c) + a}{x^2}\, dx$$

```
[In] integrate((a+b*sec(d*x^2+c))/x^2,x, algorithm="fricas")
[Out] integral((b*sec(d*x^2 + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a + b\sec(c + dx^2)}{x^2}\, dx = \int \frac{a + b\sec(c + dx^2)}{x^2}\, dx$$

```
[In] integrate((a+b*sec(d*x**2+c))/x**2,x)
[Out] Integral((a + b*sec(c + d*x**2))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 7.38

$$\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx=\int \frac{b\sec(dx^2+c)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sec(d*x^2+c))/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*integrate((cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + sin(2*d*x^2 + 2*c)*sin(d
*x^2 + c) + cos(d*x^2 + c))/(x^2*cos(2*d*x^2 + 2*c)^2 + x^2*sin(2*d*x^2 + 2
*c)^2 + 2*x^2*cos(2*d*x^2 + 2*c) + x^2), x) - a/x
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx=\int \frac{b\sec(dx^2+c)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sec(d*x^2+c))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^2 + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a+b\sec(c+dx^2)}{x^2}\,dx=\int \frac{a+\frac{b}{\cos(d\,x^2+c)}}{x^2}\,dx$$

```
[In] int((a + b/cos(c + d*x^2))/x^2,x)
```

```
[Out] int((a + b/cos(c + d*x^2))/x^2, x)
```

3.23 $\int \frac{x^5}{(a+b\sec(c+dx^2))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1092

$$\int \frac{x^5}{(a+b\sec(c+dx^2))^2}\,dx = -\frac{ib^2x^4}{2a^2(a^2-b^2)d}+\frac{x^6}{6a^2}+\frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+\frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}-\frac{ib^3x^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$+\frac{ibx^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}+\frac{ib^3x^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$-\frac{ibx^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}-\frac{ib^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$-\frac{ib^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$-\frac{b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$+\frac{2bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+\frac{b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$-\frac{2bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$-\frac{ib^3\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$+\frac{2ib\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$+\frac{ib^3\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$-\frac{2ib\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$+\frac{b^2x^4\sin(c+dx^2)}{2a(a^2-b^2)d(b+a\cos(c+dx^2))}$$

```
[Out] 2*I*b*polylog(3,-a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2
)^(1/2)+1/6*x^6/a^2+b^2*x^2*ln(1+a*exp(I*(d*x^2+c))/(b-I*(a^2-b^2)^(1/2)))/
a^2/(a^2-b^2)/d^2+b^2*x^2*ln(1+a*exp(I*(d*x^2+c))/(b+I*(a^2-b^2)^(1/2)))/a^
2/(a^2-b^2)/d^2-I*b*x^4*ln(1+a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/d
/(-a^2+b^2)^(1/2)+I*b^3*polylog(3,-a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))
/a^2/(-a^2+b^2)^(3/2)/d^3-I*b^2*polylog(2,-a*exp(I*(d*x^2+c))/(b+I*(a^2-b^2
)^(1/2)))/a^2/(a^2-b^2)/d^3-2*I*b*polylog(3,-a*exp(I*(d*x^2+c))/(b+(-a^2+b^
2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-b^3*x^2*polylog(2,-a*exp(I*(d*x^2+c))/(
b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+b^3*x^2*polylog(2,-a*exp(I*(d
*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-I*b^2*polylog(2,-a*
exp(I*(d*x^2+c))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3+1/2*I*b^3*x^4*ln(
1+a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+1/2*b^2*x
^4*sin(d*x^2+c)/a/(a^2-b^2)/d/(b+a*cos(d*x^2+c))-I*b^3*polylog(3,-a*exp(I*(
d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-1/2*I*b^2*x^4/a^2/
(a^2-b^2)/d+2*b*x^2*polylog(2,-a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2
/d^2/(-a^2+b^2)^(1/2)-2*b*x^2*polylog(2,-a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(
1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)-1/2*I*b^3*x^4*ln(1+a*exp(I*(d*x^2+c))/(b-(-
a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+I*b*x^4*ln(1+a*exp(I*(d*x^2+c))/(b-
(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 2.67 (sec) , antiderivative size = 1092, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules

used = {4289, 4276, 3405, 3402, 2296, 2221, 2611, 2320, 6724, 4618, 2317, 2438}

$$\int \frac{x^5}{(a+b\sec(c+dx^2))^2}\,dx = \frac{x^6}{6a^2} + \frac{ib\log\left(\frac{e^{i(dx^2+c)}a}{b-\sqrt{b^2-a^2}}+1\right)x^4}{a^2\sqrt{b^2-a^2}d} - \frac{ib^3\log\left(\frac{e^{i(dx^2+c)}a}{b-\sqrt{b^2-a^2}}+1\right)x^4}{2a^2(b^2-a^2)^{3/2}d} - \frac{ib\log\left(\frac{e^{i(dx^2+c)}a}{b+\sqrt{b^2-a^2}}+1\right)x^4}{a^2\sqrt{b^2-a^2}d} + \frac{ib^3\log\left(\frac{e^{i(dx^2+c)}a}{b+\sqrt{b^2-a^2}}+1\right)x^4}{2a^2(b^2-a^2)^{3/2}d} + \frac{b^2\sin(dx^2+c)x^4}{2a(a^2-b^2)d(b+a\cos(dx^2+c))} - \frac{ib^2x^4}{2a^2(a^2-b^2)d} + \frac{b^2\log\left(\frac{e^{i(dx^2+c)}a}{b-i\sqrt{a^2-b^2}}+1\right)x^2}{a^2(a^2-b^2)d^2} + \frac{b^2\log\left(\frac{e^{i(dx^2+c)}a}{b+i\sqrt{a^2-b^2}}+1\right)x^2}{a^2(a^2-b^2)d^2} + \frac{2b\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)x^2}{a^2\sqrt{b^2-a^2}d^2} - \frac{b^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)x^2}{a^2(b^2-a^2)^{3/2}d^2} - \frac{2b\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)x^2}{a^2\sqrt{b^2-a^2}d^2} + \frac{b^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)x^2}{a^2(b^2-a^2)^{3/2}d^2} - \frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} + \frac{2ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2\sqrt{b^2-a^2}d^3} - \frac{ib^3\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2(b^2-a^2)^{3/2}d^3} - \frac{2ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2\sqrt{b^2-a^2}d^3} + \frac{ib^3\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2(b^2-a^2)^{3/2}d^3}$$

```
[In] Int[x^5/(a + b*Sec[c + d*x^2])^2,x]
[Out] ((-1/2*I)*b^2*x^4)/(a^2*(a^2 - b^2)*d) + x^6/(6*a^2) + (b^2*x^2*Log[1 + (a*
E^(I*(c + d*x^2)))/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) + (b^2*x
```

```
^2*Log[1 + (a*E^(I*(c + d*x^2)))/(b + I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)
*d^2) - ((I/2)*b^3*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])
])/(a^2*(-a^2 + b^2)^(3/2)*d) + (I*b*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(b -
 Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) + ((I/2)*b^3*x^4*Log[1 + (a*E
^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - (I*
b*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2
 + b^2]*d) - (I*b^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b - I*Sqrt[a^2 - b^
2]))])/(a^2*(a^2 - b^2)*d^3) - (I*b^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b
 + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (b^3*x^2*PolyLog[2, -((a*E
^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) +
(2*b*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2]))])/(a^2*
Sqrt[-a^2 + b^2]*d^2) + (b^3*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b + Sq
rt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (2*b*x^2*PolyLog[2, -((a*
E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2) - (
I*b^3*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a
^2 + b^2)^(3/2)*d^3) + ((2*I)*b*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(b - Sqr
t[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (I*b^3*PolyLog[3, -((a*E^(I*
(c + d*x^2)))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((2*
I)*b*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt
[-a^2 + b^2]*d^3) + (b^2*x^4*Sin[c + d*x^2])/(2*a*(a^2 - b^2)*d*(b + a*Cos[
c + d*x^2]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
```

```
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4618

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2] + I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a + Rt[-a^2 + b^2,
 2] + I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+b\sec(c+dx))^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x^2}{a^2}+\frac{b^2x^2}{a^2(b+a\cos(c+dx))^2}-\frac{2bx^2}{a^2(b+a\cos(c+dx))}\right)\,dx, x, x^2\right)\\
&= \frac{x^6}{6a^2}-\frac{b\text{Subst}\left(\int \frac{x^2}{b+a\cos(c+dx)}\,dx, x, x^2\right)}{a^2}+\frac{b^2\text{Subst}\left(\int \frac{x^2}{(b+a\cos(c+dx))^2}\,dx, x, x^2\right)}{2a^2}\\
&= \frac{x^6}{6a^2}+\frac{b^2x^4\sin\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\cos\left(c+dx^2\right)\right)}\\
&\quad-\frac{(2b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, x^2\right)}{a^2}\\
&\quad-\frac{b^3\text{Subst}\left(\int \frac{x^2}{b+a\cos(c+dx)}\,dx, x, x^2\right)}{2a^2\left(a^2-b^2\right)}-\frac{b^2\text{Subst}\left(\int \frac{x\sin(c+dx)}{b+a\cos(c+dx)}\,dx, x, x^2\right)}{a\left(a^2-b^2\right)d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{ib^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} + \frac{b^2x^4\sin(c+dx^2)}{2a(a^2-b^2)d(b+a\cos(c+dx^2))} \\
&- \frac{b^3\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, x^2\right)}{a^2(a^2-b^2)} \\
&- \frac{(2b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}} \\
&+ \frac{(2b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}} \\
&- \frac{b^2\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib-\sqrt{a^2-b^2}+iae^{i(c+dx)}}\,dx, x, x^2\right)}{a(a^2-b^2)d} \\
&- \frac{b^2\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+\sqrt{a^2-b^2}+iae^{i(c+dx)}}\,dx, x, x^2\right)}{a(a^2-b^2)d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{ib^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} \\
&+ \frac{ibx^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{ibx^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&+ \frac{b^2x^4\sin(c+dx^2)}{2a(a^2-b^2)d(b+a\cos(c+dx^2))} + \frac{b^3\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^2\right)}{a(-a^2+b^2)^{3/2}} \\
&- \frac{b^3\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^2\right)}{a(-a^2+b^2)^{3/2}} \\
&- \frac{b^2\text{Subst}\left(\int \log\left(1+\frac{iae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)\,dx, x, x^2\right)}{a^2(a^2-b^2)d^2} \\
&- \frac{b^2\text{Subst}\left(\int \log\left(1+\frac{iae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)\,dx, x, x^2\right)}{a^2(a^2-b^2)d^2} \\
&- \frac{(2ib)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{a^2\sqrt{-a^2+b^2}d} \\
&+ \frac{(2ib)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{a^2\sqrt{-a^2+b^2}d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{ib^2x^4}{2a^2\left(a^2-b^2\right)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2} \\
&+ \frac{b^2x^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2} - \frac{ib^3x^4\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} \\
&+ \frac{ibx^4\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3x^4\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} \\
&- \frac{ibx^4\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&- \frac{2bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^2x^4\sin\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\cos\left(c+dx^2\right)\right)} \\
&+ \frac{\left(ib^2\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{iax}{ib-\sqrt{a^2-b^2}}\right)}{x}\,dx,x,e^{i\left(c+dx^2\right)}\right)}{a^2\left(a^2-b^2\right)d^3} \\
&+ \frac{\left(ib^2\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{iax}{ib+\sqrt{a^2-b^2}}\right)}{x}\,dx,x,e^{i\left(c+dx^2\right)}\right)}{a^2\left(a^2-b^2\right)d^3} \\
&- \frac{(2b)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&+ \frac{(2b)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&+ \frac{\left(ib^3\right)\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\left(-a^2+b^2\right)^{3/2}d} \\
&- \frac{\left(ib^3\right)\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}
\end{aligned}
$$

$$= -\frac{ib^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$

$$+ \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{ib^3x^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$

$$+ \frac{ibx^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3x^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$

$$- \frac{ibx^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$

$$- \frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{b^3x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$

$$+ \frac{2bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^3x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$

$$- \frac{2bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^2x^4\sin(c+dx^2)}{2a(a^2-b^2)d(b+a\cos(c+dx^2))}$$

$$+ \frac{(2ib)\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$

$$- \frac{(2ib)\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$

$$+ \frac{b^3\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, x^2\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$

$$- \frac{b^3\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, x^2\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$

$$= -\frac{ib^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{ib^3x^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{ibx^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3x^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{ibx^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{b^3x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$+ \frac{2bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^3x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$- \frac{2bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$- \frac{2ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{b^2x^4\sin(c+dx^2)}{2a(a^2-b^2)d(b+a\cos(c+dx^2))}$$
$$- \frac{(ib^3)\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^2)}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$+ \frac{(ib^3)\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^2)}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$

$$= -\frac{ib^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{ib^3x^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{ibx^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3x^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{ibx^4\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{b^3x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$+ \frac{2bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^3x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$- \frac{2bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{ib^3\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$+ \frac{2ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{ib^3\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$- \frac{2ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{b^2x^4\sin(c+dx^2)}{2a(a^2-b^2)d(b+a\cos(c+dx^2))}$$

Mathematica [A] (verified)

Time = 6.73 (sec) , antiderivative size = 895, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{(a+b\sec(c+dx^2))^2}\,dx$$

$$= \frac{(b+a\cos(c+dx^2))\sec^2(c+dx^2)\left(x^6(b+a\cos(c+dx^2)) - \frac{3b(b+a\cos(c+dx^2))\left(2(1+e^{2ic})\left(ib\sqrt{(-a^2+b^2)e^{2ic}}-2a^2c\right.\right.}{}\right.}{}$$

```
[In] Integrate[x^5/(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x^2])*Sec[c + d*x^2]^2*(x^6*(b + a*Cos[c + d*x^2]) - (3*b
*(b + a*Cos[c + d*x^2])*(2*(1 + E^((2*I)*c))*(I*b*Sqrt[(-a^2 + b^2)*E^((2*I
)*c)] - 2*a^2*d*E^(I*c)*x^2 + b^2*d*E^(I*c)*x^2)*PolyLog[2, -((a*E^(I*(2*c
+ d*x^2)))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 2*(1 + E^((2*I)
*c))*(I*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] + 2*a^2*d*E^(I*c)*x^2 - b^2*d*E^(I
*c)*x^2)*PolyLog[2, -((a*E^(I*(2*c + d*x^2)))/(b*E^(I*c) + Sqrt[(-a^2 + b^2
)*E^((2*I)*c)]))] + I*(d*x^2*(2*b*d*E^((2*I)*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*
c)]*x^2 + (1 + E^((2*I)*c))*((2*I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - 2*a^2
*d*E^(I*c)*x^2 + b^2*d*E^(I*c)*x^2)*Log[1 + (a*E^(I*(2*c + d*x^2)))/(b*E^(I
*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (1 + E^((2*I)*c))*((2*I)*b*Sqrt[(-
a^2 + b^2)*E^((2*I)*c)] + 2*a^2*d*E^(I*c)*x^2 - b^2*d*E^(I*c)*x^2)*Log[1 +
(a*E^(I*(2*c + d*x^2)))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]) - 2*
(2*a^2 - b^2)*E^(I*c)*(1 + E^((2*I)*c))*PolyLog[3, -((a*E^(I*(2*c + d*x^2))
)/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 2*(2*a^2 - b^2)*E^(I*c)*
(1 + E^((2*I)*c))*PolyLog[3, -((a*E^(I*(2*c + d*x^2)))/(b*E^(I*c) + Sqrt[(-
a^2 + b^2)*E^((2*I)*c)]))])))/((a^2 - b^2)*d^3*Sqrt[(-a^2 + b^2)*E^((2*I)*c
)]*(1 + E^((2*I)*c))) + (3*b^2*x^4*(-(b*Sin[c]) + a*Sin[d*x^2]))/((a - b)*(
a + b)*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]))))/(6*a^2*(a + b*Sec[c
 + d*x^2])^2)
```

Maple [F]

$$\int \frac{x^5}{\left(a+b\sec \left(d\,x^2+c\right)\right)^2}dx$$

```
[In] int(x^5/(a+b*sec(d*x^2+c))^2,x)
```

```
[Out] int(x^5/(a+b*sec(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3050 vs. 2(958) = 1916.

Time = 0.51 (sec) , antiderivative size = 3050, normalized size of antiderivative = 2.79

$$\int \frac{x^5}{(a+b\sec(c+dx^2))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d^3*x^6*cos(d*x^2 + c) + 2*(a^4*b - 2*a^2
*b^3 + b^5)*d^3*x^6 + 6*(a^3*b^2 - a*b^4)*d^2*x^4*sin(d*x^2 + c) - 6*(2*I*a
^3*b^2 - I*a*b^4 + (2*I*a^4*b - I*a^2*b^3)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2
)/a^2)*polylog(3, -(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 +
c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) - 6*(-2*I*a^3*b^2 + I*a
```

```
*b^4 + (-2*I*a^4*b + I*a^2*b^3)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*poly
log(3, -(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*si
n(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) - 6*(-2*I*a^3*b^2 + I*a*b^4 + (-2*
I*a^4*b + I*a^2*b^3)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*polylog(3, -(b*
cos(d*x^2 + c) - I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c
))*sqrt(-(a^2 - b^2)/a^2))/a) - 6*(2*I*a^3*b^2 - I*a*b^4 + (2*I*a^4*b - I*a
^2*b^3)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*polylog(3, -(b*cos(d*x^2 + c
) - I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^2
 - b^2)/a^2))/a) - 6*(I*a^2*b^3 - I*b^5 + (I*a^3*b^2 - I*a*b^4)*cos(d*x^2 +
 c) + ((2*a^4*b - a^2*b^3)*d*x^2*cos(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*d*x^2
)*sqrt(-(a^2 - b^2)/a^2))*dilog(-(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (
a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) -
 6*(I*a^2*b^3 - I*b^5 + (I*a^3*b^2 - I*a*b^4)*cos(d*x^2 + c) - ((2*a^4*b -
a^2*b^3)*d*x^2*cos(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*d*x^2)*sqrt(-(a^2 - b^2
)/a^2))*dilog(-(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) +
 I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - 6*(-I*a^2*b^3 + I
*b^5 + (-I*a^3*b^2 + I*a*b^4)*cos(d*x^2 + c) + ((2*a^4*b - a^2*b^3)*d*x^2*c
os(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*d*x^2)*sqrt(-(a^2 - b^2)/a^2))*dilog(-(
b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 +
 c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - 6*(-I*a^2*b^3 + I*b^5 + (-I*a^3*b
^2 + I*a*b^4)*cos(d*x^2 + c) - ((2*a^4*b - a^2*b^3)*d*x^2*cos(d*x^2 + c) +
(2*a^3*b^2 - a*b^4)*d*x^2)*sqrt(-(a^2 - b^2)/a^2))*dilog(-(b*cos(d*x^2 + c)
 - I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^2
- b^2)/a^2) + a)/a + 1) - 3*(2*(a^3*b^2 - a*b^4)*c*cos(d*x^2 + c) + 2*(a^2*
b^3 - b^5)*c + (-I*(2*a^4*b - a^2*b^3)*c^2*cos(d*x^2 + c) - I*(2*a^3*b^2 -
a*b^4)*c^2)*sqrt(-(a^2 - b^2)/a^2))*log(2*a*cos(d*x^2 + c) + 2*I*a*sin(d*x^
2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - 3*(2*(a^3*b^2 - a*b^4)*c*cos(d
*x^2 + c) + 2*(a^2*b^3 - b^5)*c + (I*(2*a^4*b - a^2*b^3)*c^2*cos(d*x^2 + c)
 + I*(2*a^3*b^2 - a*b^4)*c^2)*sqrt(-(a^2 - b^2)/a^2))*log(2*a*cos(d*x^2 + c
) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - 3*(2*(a^3*b^
2 - a*b^4)*c*cos(d*x^2 + c) + 2*(a^2*b^3 - b^5)*c + (-I*(2*a^4*b - a^2*b^3)
*c^2*cos(d*x^2 + c) - I*(2*a^3*b^2 - a*b^4)*c^2)*sqrt(-(a^2 - b^2)/a^2))*lo
g(-2*a*cos(d*x^2 + c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) -
 2*b) - 3*(2*(a^3*b^2 - a*b^4)*c*cos(d*x^2 + c) + 2*(a^2*b^3 - b^5)*c + (I*
(2*a^4*b - a^2*b^3)*c^2*cos(d*x^2 + c) + I*(2*a^3*b^2 - a*b^4)*c^2)*sqrt(-(
a^2 - b^2)/a^2))*log(-2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(
-(a^2 - b^2)/a^2) - 2*b) + 3*(2*(a^2*b^3 - b^5)*d*x^2 + 2*(a^2*b^3 - b^5)*c
 + 2*((a^3*b^2 - a*b^4)*d*x^2 + (a^3*b^2 - a*b^4)*c)*cos(d*x^2 + c) - (I*(2
*a^3*b^2 - a*b^4)*d^2*x^4 - I*(2*a^3*b^2 - a*b^4)*c^2 + (I*(2*a^4*b - a^2*b
^3)*d^2*x^4 - I*(2*a^4*b - a^2*b^3)*c^2)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/
a^2))*log((b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*
sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) + 3*(2*(a^2*b^3 - b^5)*d*x^2
 + 2*(a^2*b^3 - b^5)*c + 2*((a^3*b^2 - a*b^4)*d*x^2 + (a^3*b^2 - a*b^4)*c)*
cos(d*x^2 + c) - (-I*(2*a^3*b^2 - a*b^4)*d^2*x^4 + I*(2*a^3*b^2 - a*b^4)*c^
2 + (-I*(2*a^4*b - a^2*b^3)*d^2*x^4 + I*(2*a^4*b - a^2*b^3)*c^2)*cos(d*x^2
```

```
+ c))*sqrt(-(a^2 - b^2)/a^2))*log((b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) -
(a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) + 3*
(2*(a^2*b^3 - b^5)*d*x^2 + 2*(a^2*b^3 - b^5)*c + 2*((a^3*b^2 - a*b^4)*d*x^2
 + (a^3*b^2 - a*b^4)*c)*cos(d*x^2 + c) - (-I*(2*a^3*b^2 - a*b^4)*d^2*x^4 +
I*(2*a^3*b^2 - a*b^4)*c^2 + (-I*(2*a^4*b - a^2*b^3)*d^2*x^4 + I*(2*a^4*b -
a^2*b^3)*c^2)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))*log((b*cos(d*x^2 + c)
 - I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^2
- b^2)/a^2) + a)/a) + 3*(2*(a^2*b^3 - b^5)*d*x^2 + 2*(a^2*b^3 - b^5)*c + 2*
((a^3*b^2 - a*b^4)*d*x^2 + (a^3*b^2 - a*b^4)*c)*cos(d*x^2 + c) - (I*(2*a^3*
b^2 - a*b^4)*d^2*x^4 - I*(2*a^3*b^2 - a*b^4)*c^2 + (I*(2*a^4*b - a^2*b^3)*d
^2*x^4 - I*(2*a^4*b - a^2*b^3)*c^2)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))
*log((b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d
*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d^3
*cos(d*x^2 + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^3)
```

Sympy [F]

$$\int \frac{x^5}{(a + b\sec(c + dx^2))^2}\,dx = \int \frac{x^5}{(a + b\sec(c + dx^2))^2}\,dx$$

```
[In] integrate(x**5/(a+b*sec(d*x**2+c))**2,x)
```

```
[Out] Integral(x**5/(a + b*sec(c + d*x**2))**2, x)
```

Maxima [F]

$$\int \frac{x^5}{(a + b\sec(c + dx^2))^2}\,dx = \int \frac{x^5}{(b\sec(dx^2 + c) + a)^2}\,dx$$

```
[In] integrate(x^5/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*((a^4 - a^2*b^2)*d*x^6*cos(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^6*c
os(d*x^2 + c)^2 + (a^4 - a^2*b^2)*d*x^6*sin(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 -
 b^4)*d*x^6*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(d*x^2 + c) + 6*a
*b^3*x^4*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^6 + 2*(2*(a^3*b - a*b^3)*d*x^
6*cos(d*x^2 + c) - 3*a*b^3*x^4*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^6)*cos(
2*d*x^2 + 2*c) - 6*((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a
^2*b^4)*d*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^
5*b - a^3*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a^2*b^4)*
d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)
*d + 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x
^2 + 2*c))*integrate(2*(2*(2*a^2*b^2 - b^4)*d*x^5*cos(d*x^2 + c)^2 + 2*(2*a
```

```
^2*b^2 - b^4)*d*x^5*sin(d*x^2 + c)^2 + (2*a^3*b - a*b^3)*d*x^5*cos(d*x^2 +
c) + 2*a*b^3*x^3*sin(d*x^2 + c) + ((2*a^3*b - a*b^3)*d*x^5*cos(d*x^2 + c) -
 2*a*b^3*x^3*sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (2*a*b^3*x^3*cos(d*x^2 +
c) + (2*a^3*b - a*b^3)*d*x^5*sin(d*x^2 + c) + 2*a^2*b^2*x^3)*sin(2*d*x^2 +
2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*cos
(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^5*b - a^3*b^3
)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a^2*b^4)*d*sin(d*x^2 +
 c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d + 2*(2*(a^
5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 + 2*c)), x
) + 2*(3*a*b^3*x^4*cos(d*x^2 + c) + 2*(a^3*b - a*b^3)*d*x^6*sin(d*x^2 + c)
+ 3*a^2*b^2*x^4)*sin(2*d*x^2 + 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^
2 + 4*(a^4*b^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^
2 + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a
^4*b^2 - a^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c)
 + (a^6 - a^4*b^2)*d + 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4
*b^2)*d)*cos(2*d*x^2 + 2*c))
```

Giac [F]

$$\int \frac{x^5}{(a+b\sec(c+dx^2))^2}\,dx = \int \frac{x^5}{(b\sec(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^5/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^5/(b*sec(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+b\sec(c+dx^2))^2}\,dx = \int \frac{x^5}{\left(a+\frac{b}{\cos(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(x^5/(a + b/cos(c + d*x^2))^2,x)
[Out] int(x^5/(a + b/cos(c + d*x^2))^2, x)
```

3.24 $\int \frac{x^4}{\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{(a+b\sec(c+dx^2))^2}\,dx = \text{Int}\left(\frac{x^4}{(a+b\sec(c+dx^2))^2},x\right)$$

```
[Out] Unintegrable(x^4/(a+b*sec(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4}{(a+b\sec(c+dx^2))^2}\,dx = \int \frac{x^4}{(a+b\sec(c+dx^2))^2}\,dx$$

```
[In] Int[x^4/(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] Defer[Int][x^4/(a + b*Sec[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4}{(a+b\sec(c+dx^2))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a+b\sec(c+dx^2))^2}\,dx = \int \frac{x^4}{(a+b\sec(c+dx^2))^2}\,dx$$

```
[In] Integrate[x^4/(a + b*Sec[c + d*x^2])^2,x]
[Out] Integrate[x^4/(a + b*Sec[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+b\sec(d\,x^2+c))^2}dx$$

```
[In] int(x^4/(a+b*sec(d*x^2+c))^2,x)
[Out] int(x^4/(a+b*sec(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^4}{(a+b\sec(c+dx^2))^2}\,dx = \int \frac{x^4}{(b\sec(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^4/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(x^4/(b^2*sec(d*x^2 + c)^2 + 2*a*b*sec(d*x^2 + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{(a+b\sec(c+dx^2))^2}\,dx = \int \frac{x^4}{(a+b\sec(c+dx^2))^2}\,dx$$

```
[In] integrate(x**4/(a+b*sec(d*x**2+c))**2,x)
[Out] Integral(x**4/(a + b*sec(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 1284, normalized size of antiderivative = 71.33

$$\int \frac{x^4}{(a+b\sec(c+dx^2))^2}\,dx = \int \frac{x^4}{(b\sec(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^4/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*((a^4 - a^2*b^2)*d*x^5*cos(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^5*c
os(d*x^2 + c)^2 + (a^4 - a^2*b^2)*d*x^5*sin(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 -
 b^4)*d*x^5*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*x^5*cos(d*x^2 + c) + 5*a
*b^3*x^3*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^5 + (4*(a^3*b - a*b^3)*d*x^5*
cos(d*x^2 + c) - 5*a*b^3*x^3*sin(d*x^2 + c) + 2*(a^4 - a^2*b^2)*d*x^5)*cos(
2*d*x^2 + 2*c) - 5*((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a
^2*b^4)*d*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^
5*b - a^3*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a^2*b^4)*
d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)
*d + 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x
^2 + 2*c))*integrate((4*(2*a^2*b^2 - b^4)*d*x^4*cos(d*x^2 + c)^2 + 4*(2*a^2
*b^2 - b^4)*d*x^4*sin(d*x^2 + c)^2 + 2*(2*a^3*b - a*b^3)*d*x^4*cos(d*x^2 +
c) + 3*a*b^3*x^2*sin(d*x^2 + c) + (2*(2*a^3*b - a*b^3)*d*x^4*cos(d*x^2 + c)
 - 3*a*b^3*x^2*sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (3*a*b^3*x^2*cos(d*x^2
+ c) + 2*(2*a^3*b - a*b^3)*d*x^4*sin(d*x^2 + c) + 3*a^2*b^2*x^2)*sin(2*d*x^
2 + 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d
*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^5*b - a^3
*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a^2*b^4)*d*sin(d*x
^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d + 2*(2
*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 + 2*c)
), x) + (5*a*b^3*x^3*cos(d*x^2 + c) + 4*(a^3*b - a*b^3)*d*x^5*sin(d*x^2 + c
) + 5*a^2*b^2*x^3)*sin(2*d*x^2 + 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c
)^2 + 4*(a^4*b^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*
x^2 + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*
(a^4*b^2 - a^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 +
c) + (a^6 - a^4*b^2)*d + 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a
^4*b^2)*d)*cos(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a+b\sec(c+dx^2))^2}\,dx = \int \frac{x^4}{(b\sec(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^4/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^4/(b*sec(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 12.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(a+b\sec(c+dx^2))^2}\,dx = \int \frac{x^4}{\left(a+\frac{b}{\cos(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(x^4/(a + b/cos(c + d*x^2))^2,x)
[Out] int(x^4/(a + b/cos(c + d*x^2))^2, x)
```

3.25 $\int \frac{x^3}{(a+b\sec(c+dx^2))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 596

$$
\begin{aligned}
\int \frac{x^3}{(a+b\sec(c+dx^2))^2}\,dx &= \frac{x^4}{4a^2} - \frac{ib^3x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} \\
&+ \frac{ibx^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} \\
&- \frac{ibx^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{b^2\log(b+a\cos(c+dx^2))}{2a^2(a^2-b^2)d^2} \\
&- \frac{b^3\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d^2} + \frac{b\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&+ \frac{b^3\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d^2} - \frac{b\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&+ \frac{b^2x^2\sin(c+dx^2)}{2a(a^2-b^2)d(b+a\cos(c+dx^2))}
\end{aligned}
$$

```
[Out] 1/4*x^4/a^2+1/2*b^2*ln(b+a*cos(d*x^2+c))/a^2/(a^2-b^2)/d^2-1/2*I*b^3*x^2*ln
(1+a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+1/2*I*b^
3*x^2*ln(1+a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-
1/2*b^3*polylog(2,-a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^
(3/2)/d^2+1/2*b^3*polylog(2,-a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(
-a^2+b^2)^(3/2)/d^2+1/2*b^2*x^2*sin(d*x^2+c)/a/(a^2-b^2)/d/(b+a*cos(d*x^2+c
```

```
))+I*b*x^2*ln(1+a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(
1/2)-I*b*x^2*ln(1+a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)
^(1/2)+b*polylog(2,-a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+
b^2)^(1/2)-b*polylog(2,-a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-
a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4289, 4276, 3405, 3402, 2296, 2221, 2317, 2438, 2747, 31}

$$\int \frac{x^3}{(a+b\sec(c+dx^2))^2}\,dx = \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}} - \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}} + \frac{b^2\log(a\cos(c+dx^2)+b)}{2a^2d^2(a^2-b^2)} + \frac{ibx^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{a^2d\sqrt{b^2-a^2}} - \frac{ibx^2\log\left(1+\frac{ae^{i(c+dx^2)}}{\sqrt{b^2-a^2}+b}\right)}{a^2d\sqrt{b^2-a^2}} + \frac{b^2x^2\sin(c+dx^2)}{2ad(a^2-b^2)(a\cos(c+dx^2)+b)} - \frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{2a^2d^2(b^2-a^2)^{3/2}} + \frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{2a^2d^2(b^2-a^2)^{3/2}} - \frac{ib^3x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{2a^2d(b^2-a^2)^{3/2}} + \frac{ib^3x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{\sqrt{b^2-a^2}+b}\right)}{2a^2d(b^2-a^2)^{3/2}} + \frac{x^4}{4a^2}$$

```
[In] Int[x^3/(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] x^4/(4*a^2) - ((I/2)*b^3*x^2*Log[1 + (a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 +
 b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + (I*b*x^2*Log[1 + (a*E^(I*(c + d*x^2))
)/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) + ((I/2)*b^3*x^2*Log[1
+ (a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d)
 - (I*b*x^2*Log[1 + (a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqr
t[-a^2 + b^2]*d) + (b^2*Log[b + a*Cos[c + d*x^2]])/(2*a^2*(a^2 - b^2)*d^2)
- (b^3*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2]))])/(2*a^2*
(-a^2 + b^2)^(3/2)*d^2) + (b*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b - Sqrt[-
a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (b^3*PolyLog[2, -((a*E^(I*(c +
d*x^2)))/(b + Sqrt[-a^2 + b^2]))])/(2*a^2*(-a^2 + b^2)^(3/2)*d^2) - (b*Poly
Log[2, -((a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b
^2]*d^2) + (b^2*x^2*Sin[c + d*x^2])/(2*a*(a^2 - b^2)*d*(b + a*Cos[c + d*x^2
]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+b\sec(c+dx))^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x}{a^2}+\frac{b^2x}{a^2(b+a\cos(c+dx))^2}-\frac{2bx}{a^2(b+a\cos(c+dx))}\right)\,dx,x,x^2\right)\\
&= \frac{x^4}{4a^2}-\frac{b\text{Subst}\left(\int \frac{x}{b+a\cos(c+dx)}\,dx,x,x^2\right)}{a^2}+\frac{b^2\text{Subst}\left(\int \frac{x}{(b+a\cos(c+dx))^2}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{x^4}{4a^2}+\frac{b^2x^2\sin\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\cos\left(c+dx^2\right)\right)}\\
&\quad -\frac{(2b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx,x,x^2\right)}{a^2}\\
&\quad -\frac{b^3\text{Subst}\left(\int \frac{x}{b+a\cos(c+dx)}\,dx,x,x^2\right)}{2a^2\left(a^2-b^2\right)}-\frac{b^2\text{Subst}\left(\int \frac{\sin(c+dx)}{b+a\cos(c+dx)}\,dx,x,x^2\right)}{2a\left(a^2-b^2\right)d}
\end{aligned}
$$

$$= \frac{x^4}{4a^2} + \frac{b^2x^2\sin(c+dx^2)}{2a(a^2-b^2)d(b+a\cos(c+dx^2))} - \frac{b^3\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, x^2\right)}{a^2(a^2-b^2)}$$
$$- \frac{(2b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}}$$
$$+ \frac{(2b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}}$$
$$+ \frac{b^2\text{Subst}\left(\int \frac{1}{b+x}\,dx, x, a\cos(c+dx^2)\right)}{2a^2(a^2-b^2)d^2}$$

$$= \frac{x^4}{4a^2} + \frac{ibx^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{ibx^2\log\left(1+\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+ \frac{b^2\log(b+a\cos(c+dx^2))}{2a^2(a^2-b^2)d^2} + \frac{b^2x^2\sin(c+dx^2)}{2a(a^2-b^2)d(b+a\cos(c+dx^2))}$$
$$+ \frac{b^3\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^2\right)}{a(-a^2+b^2)^{3/2}}$$
$$- \frac{b^3\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^2\right)}{a(-a^2+b^2)^{3/2}}$$
$$- \frac{(ib)\text{Subst}\left(\int \log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+ \frac{(ib)\text{Subst}\left(\int \log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$
\begin{aligned}
&= \frac{x^4}{4a^2} - \frac{ib^3x^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} + \frac{ibx^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&+ \frac{ib^3x^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} - \frac{ibx^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&+ \frac{b^2\log\left(b+a\cos\left(c+dx^2\right)\right)}{2a^2\left(a^2-b^2\right)d^2} + \frac{b^2x^2\sin\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\cos\left(c+dx^2\right)\right)} \\
&- \frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i\left(c+dx^2\right)}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&+ \frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i\left(c+dx^2\right)}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&+ \frac{\left(ib^3\right)\text{Subst}\left(\int\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} \\
&- \frac{\left(ib^3\right)\text{Subst}\left(\int\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x^4}{4a^2} - \frac{ib^3x^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} + \frac{ibx^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&+ \frac{ib^3x^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} - \frac{ibx^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&+ \frac{b^2\log\left(b+a\cos\left(c+dx^2\right)\right)}{2a^2\left(a^2-b^2\right)d^2} + \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&- \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^2x^2\sin\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\cos\left(c+dx^2\right)\right)} \\
&+ \frac{b^3\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i\left(c+dx^2\right)}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d^2} \\
&- \frac{b^3\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i\left(c+dx^2\right)}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d^2}
\end{aligned}
$$

$$= \frac{x^4}{4a^2} - \frac{ib^3x^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2} d} + \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3x^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2} d} - \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{b^2 \log(b + a\cos(c+dx^2))}{2a^2(a^2-b^2) d^2} - \frac{b^3 \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2} d^2} + \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^3 \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2} d^2} - \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^2x^2 \sin(c+dx^2)}{2a(a^2-b^2) d(b + a\cos(c+dx^2))}$$

Mathematica [A] (warning: unable to verify)

Time = 11.66 (sec) , antiderivative size = 1118, normalized size of antiderivative = 1.88

$$\int \frac{x^3}{(a + b\sec(c+dx^2))^2}\, dx = \frac{(-c+dx^2)(c+dx^2)(b + a\cos(c+dx^2))^2 \sec^2(c+dx^2)}{4a^2d^2(a + b\sec(c+dx^2))^2} + \frac{(b + a\cos(c+dx^2))\sec^2(c+dx^2)(b^2c\sin(c+dx^2) - b^2(c+dx^2)\sin(c+dx^2))}{2a(-a+b)(a+b)d^2(a + b\sec(c+dx^2))^2} + \frac{b\cos^2\left(\frac{1}{2}(c+dx^2)\right)(b + a\cos(c+dx^2))\left(2(2a^2-b^2)\, c\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right) - \sqrt{a-b}b\sqrt{a+b}\log\right.}{}$$

```
[In] Integrate[x^3/(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] ((-c + d*x^2)*(c + d*x^2)*(b + a*Cos[c + d*x^2])^2*Sec[c + d*x^2]^2)/(4*a^2
*d^2*(a + b*Sec[c + d*x^2])^2) + ((b + a*Cos[c + d*x^2])*Sec[c + d*x^2]^2*(
b^2*c*Sin[c + d*x^2] - b^2*(c + d*x^2)*Sin[c + d*x^2]))/(2*a*(-a + b)*(a +
b)*d^2*(a + b*Sec[c + d*x^2])^2) + (b*Cos[(c + d*x^2)/2]^2*(b + a*Cos[c + d
*x^2])*(2*(2*a^2 - b^2)*c*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x^2)/2])/Sqrt[a +
 b]] - Sqrt[a - b]*b*Sqrt[a + b]*Log[Sec[(c + d*x^2)/2]^2] + Sqrt[a - b]*b*
Sqrt[a + b]*Log[(b + a*Cos[c + d*x^2])*Sec[(c + d*x^2)/2]^2] + I*(2*a^2 - b
^2)*(Log[1 - I*Tan[(c + d*x^2)/2]]*Log[(Sqrt[a + b] - Sqrt[a - b]*Tan[(c +
d*x^2)/2])/(I*Sqrt[a - b] + Sqrt[a + b])] + PolyLog[2, (Sqrt[a - b]*(1 - I*
Tan[(c + d*x^2)/2]))/(Sqrt[a - b] - I*Sqrt[a + b])]) - I*(2*a^2 - b^2)*(Log
[1 - I*Tan[(c + d*x^2)/2]]*Log[(I*(Sqrt[a + b] + Sqrt[a - b]*Tan[(c + d*x^2
)/2]))/(Sqrt[a - b] + I*Sqrt[a + b])] + PolyLog[2, (Sqrt[a - b]*(1 - I*Tan[
(c + d*x^2)/2]))/(Sqrt[a - b] + I*Sqrt[a + b])]) + I*(2*a^2 - b^2)*(Log[1 +
```

```
 I*Tan[(c + d*x^2)/2]]*Log[(Sqrt[a + b] + Sqrt[a - b]*Tan[(c + d*x^2)/2])/(
I*Sqrt[a - b] + Sqrt[a + b])] + PolyLog[2, (Sqrt[a - b]*(1 + I*Tan[(c + d*x
^2)/2]))/(Sqrt[a - b] - I*Sqrt[a + b])]) - I*(2*a^2 - b^2)*(Log[1 + I*Tan[(
c + d*x^2)/2]]*Log[(I*(Sqrt[a + b] - Sqrt[a - b]*Tan[(c + d*x^2)/2]))/(Sqrt
[a - b] + I*Sqrt[a + b])] + PolyLog[2, (Sqrt[a - b]*(1 + I*Tan[(c + d*x^2)/
2]))/(Sqrt[a - b] + I*Sqrt[a + b])]))*Sec[c + d*x^2]^2*((2*a^2 - b^2)*d*x^2
 + a*b*Sin[c + d*x^2])*(Sqrt[a + b] - Sqrt[a - b]*Tan[(c + d*x^2)/2])*(Sqrt
[a + b] + Sqrt[a - b]*Tan[(c + d*x^2)/2]))/(2*a^2*Sqrt[a - b]*Sqrt[a + b]*(
a^2 - b^2)*d^2*(a + b*Sec[c + d*x^2])^2*(-((2*a^2 - b^2)*(c - I*Log[1 - I*T
an[(c + d*x^2)/2]] + I*Log[1 + I*Tan[(c + d*x^2)/2]])) + a*b*Sin[c + d*x^2]
))
```

Maple [F]

$$\int \frac{x^{3}}{\left(a +b \sec \left(d \,x^{2}+c \right)\right)^{2}}d x$$

```
[In] int(x^3/(a+b*sec(d*x^2+c))^2,x)
```

```
[Out] int(x^3/(a+b*sec(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1928 vs. 2(522) = 1044.

Time = 0.44 (sec) , antiderivative size = 1928, normalized size of antiderivative = 3.23

$$\int \frac{x^3}{(a+b\sec(c+dx^2))^2} dx = \text{Too large to display}$$

```
[In] integrate(x^3/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4*cos(d*x^2 + c) + (a^4*b - 2*a^2*b^3
+ b^5)*d^2*x^4 + 2*(a^3*b^2 - a*b^4)*d*x^2*sin(d*x^2 + c) - (2*a^3*b^2 - a*
b^4 + (2*a^4*b - a^2*b^3)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*
cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c
))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + (2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2
*b^3)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*cos(d*x^2 + c) + I*b
*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)
/a^2) + a)/a + 1) - (2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*cos(d*x^2 + c)
)*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) + (a
*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) +
(2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/
a^2)*dilog(-(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*
a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + (-I*(2*a^3*b^2 - a*b
```

```
^4)*d*x^2 - I*(2*a^3*b^2 - a*b^4)*c + (-I*(2*a^4*b - a^2*b^3)*d*x^2 - I*(2*
a^4*b - a^2*b^3)*c)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2
 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-
(a^2 - b^2)/a^2) + a)/a) + (I*(2*a^3*b^2 - a*b^4)*d*x^2 + I*(2*a^3*b^2 - a*
b^4)*c + (I*(2*a^4*b - a^2*b^3)*d*x^2 + I*(2*a^4*b - a^2*b^3)*c)*cos(d*x^2
+ c))*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) - (
a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) + (I*
(2*a^3*b^2 - a*b^4)*d*x^2 + I*(2*a^3*b^2 - a*b^4)*c + (I*(2*a^4*b - a^2*b^3
)*d*x^2 + I*(2*a^4*b - a^2*b^3)*c)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*l
og((b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x
^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) + (-I*(2*a^3*b^2 - a*b^4)*d*x^2 - I
*(2*a^3*b^2 - a*b^4)*c + (-I*(2*a^4*b - a^2*b^3)*d*x^2 - I*(2*a^4*b - a^2*b
^3)*c)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c) - I*b*s
in(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a
^2) + a)/a) + (a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*cos(d*x^2 + c) + (-I*(2*a^
4*b - a^2*b^3)*c*cos(d*x^2 + c) - I*(2*a^3*b^2 - a*b^4)*c)*sqrt(-(a^2 - b^2
)/a^2))*log(2*a*cos(d*x^2 + c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^
2)/a^2) + 2*b) + (a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*cos(d*x^2 + c) + (I*(2*
a^4*b - a^2*b^3)*c*cos(d*x^2 + c) + I*(2*a^3*b^2 - a*b^4)*c)*sqrt(-(a^2 - b
^2)/a^2))*log(2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^2 -
b^2)/a^2) + 2*b) + (a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*cos(d*x^2 + c) + (-I*
(2*a^4*b - a^2*b^3)*c*cos(d*x^2 + c) - I*(2*a^3*b^2 - a*b^4)*c)*sqrt(-(a^2
- b^2)/a^2))*log(-2*a*cos(d*x^2 + c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^
2 - b^2)/a^2) - 2*b) + (a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*cos(d*x^2 + c) +
(I*(2*a^4*b - a^2*b^3)*c*cos(d*x^2 + c) + I*(2*a^3*b^2 - a*b^4)*c)*sqrt(-(a
^2 - b^2)/a^2))*log(-2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-
(a^2 - b^2)/a^2) - 2*b))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d^2*cos(d*x^2 + c) +
(a^6*b - 2*a^4*b^3 + a^2*b^5)*d^2)
```

Sympy [F]

$$\int \frac{x^3}{\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx=\int \frac{x^3}{\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] integrate(x**3/(a+b*sec(d*x**2+c))**2,x)
```

```
[Out] Integral(x**3/(a + b*sec(c + d*x**2))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^3}{\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx = \int \frac{x^3}{\left(b\sec\left(dx^2+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^3/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*sec(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx = \int \frac{x^3}{\left(a+\frac{b}{\cos(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(x^3/(a + b/cos(c + d*x^2))^2,x)
```

```
[Out] int(x^3/(a + b/cos(c + d*x^2))^2, x)
```

3.26 $\int \frac{x^2}{\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{(a+b\sec(c+dx^2))^2}\,dx = \text{Int}\left(\frac{x^2}{(a+b\sec(c+dx^2))^2},x\right)$$

```
[Out] Unintegrable(x^2/(a+b*sec(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{(a+b\sec(c+dx^2))^2}\,dx = \int \frac{x^2}{(a+b\sec(c+dx^2))^2}\,dx$$

```
[In] Int[x^2/(a + b*Sec[c + d*x^2])^2,x]
[Out] Defer[Int][x^2/(a + b*Sec[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{(a+b\sec(c+dx^2))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\sec(c+dx^2))^2}\,dx=\int \frac{x^2}{(a+b\sec(c+dx^2))^2}\,dx$$

```
[In] Integrate[x^2/(a + b*Sec[c + d*x^2])^2,x]
[Out] Integrate[x^2/(a + b*Sec[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+b\sec(d\,x^2+c))^2}dx$$

```
[In] int(x^2/(a+b*sec(d*x^2+c))^2,x)
[Out] int(x^2/(a+b*sec(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^2}{(a+b\sec(c+dx^2))^2}\,dx=\int \frac{x^2}{(b\sec(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(x^2/(b^2*sec(d*x^2 + c)^2 + 2*a*b*sec(d*x^2 + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+b\sec(c+dx^2))^2}\,dx=\int \frac{x^2}{(a+b\sec(c+dx^2))^2}\,dx$$

```
[In] integrate(x**2/(a+b*sec(d*x**2+c))**2,x)
[Out] Integral(x**2/(a + b*sec(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 1261, normalized size of antiderivative = 70.06

$$\int \frac{x^2}{(a+b\sec(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\sec(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((a^4 - a^2*b^2)*d*x^3*cos(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^3*c
os(d*x^2 + c)^2 + (a^4 - a^2*b^2)*d*x^3*sin(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 -
 b^4)*d*x^3*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(d*x^2 + c) + 3*a
*b^3*x*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^3 + (4*(a^3*b - a*b^3)*d*x^3*co
s(d*x^2 + c) - 3*a*b^3*x*sin(d*x^2 + c) + 2*(a^4 - a^2*b^2)*d*x^3)*cos(2*d*
x^2 + 2*c) - 3*((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b
^4)*d*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^5*b
- a^3*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a^2*b^4)*d*si
n(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d +
 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 +
 2*c))*integrate((4*(2*a^2*b^2 - b^4)*d*x^2*cos(d*x^2 + c)^2 + 4*(2*a^2*b^2
 - b^4)*d*x^2*sin(d*x^2 + c)^2 + 2*(2*a^3*b - a*b^3)*d*x^2*cos(d*x^2 + c) +
 a*b^3*sin(d*x^2 + c) + (2*(2*a^3*b - a*b^3)*d*x^2*cos(d*x^2 + c) - a*b^3*s
in(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (a*b^3*cos(d*x^2 + c) + 2*(2*a^3*b - a*
b^3)*d*x^2*sin(d*x^2 + c) + a^2*b^2)*sin(2*d*x^2 + 2*c))/((a^6 - a^4*b^2)*d
*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + (a^6 - a
^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(2*d*x^2 + 2*c)*s
in(d*x^2 + c) + 4*(a^4*b^2 - a^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b
^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d + 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x
^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 + 2*c)), x) + (3*a*b^3*x*cos(d*x^2
 + c) + 4*(a^3*b - a*b^3)*d*x^3*sin(d*x^2 + c) + 3*a^2*b^2*x)*sin(2*d*x^2 +
 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*co
s(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^5*b - a^3*b^
3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a^2*b^4)*d*sin(d*x^2
+ c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d + 2*(2*(a
^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\sec(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\sec(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^2/(b*sec(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 13.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{(a+b\sec(c+dx^2))^2}\,dx = \int \frac{x^2}{\left(a+\frac{b}{\cos(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(x^2/(a + b/cos(c + d*x^2))^2,x)
[Out] int(x^2/(a + b/cos(c + d*x^2))^2, x)
```

3.27 $\int \frac{x}{\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 123

$$\int \frac{x}{(a+b\sec(c+dx^2))^2}\,dx = \frac{x^2}{2a^2} - \frac{b(2a^2-b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2\tan(c+dx^2)}{2a(a^2-b^2)\,d\,(a+b\sec(c+dx^2))}$$

```
[Out] 1/2*x^2/a^2-b*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x^2+1/2*c)/(a+b)^(1
/2))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*b^2*tan(d*x^2+c)/a/(a^2-b^2)/d/(a+b*
sec(d*x^2+c))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4289, 3870, 4004, 3916, 2738, 214}

$$\int \frac{x}{(a+b\sec(c+dx^2))^2}\,dx = -\frac{b(2a^2-b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}} + \frac{b^2\tan(c+dx^2)}{2ad(a^2-b^2)(a+b\sec(c+dx^2))} + \frac{x^2}{2a^2}$$

```
[In] Int[x/(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] x^2/(2*a^2) - (b*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x^2)/2])/Sqr
t[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b^2*Tan[c + d*x^2])/(2*a*
(a^2 - b^2)*d*(a + b*Sec[c + d*x^2]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+b\sec(c+dx))^2}\,dx, x, x^2\right)$$

$$= \frac{b^2 \tan(c+dx^2)}{2a(a^2-b^2)d(a+b\sec(c+dx^2))} - \frac{\text{Subst}\left(\int \frac{-a^2+b^2+ab\sec(c+dx)}{a+b\sec(c+dx)}\,dx, x, x^2\right)}{2a(a^2-b^2)}$$

$$= \frac{x^2}{2a^2} + \frac{b^2 \tan(c+dx^2)}{2a(a^2-b^2)d(a+b\sec(c+dx^2))} - \frac{(b(2a^2-b^2))\,\text{Subst}\left(\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx, x, x^2\right)}{2a^2(a^2-b^2)}$$

$$= \frac{x^2}{2a^2} + \frac{b^2 \tan(c+dx^2)}{2a(a^2-b^2)d(a+b\sec(c+dx^2))} - \frac{(2a^2-b^2)\,\text{Subst}\left(\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx, x, x^2\right)}{2a^2(a^2-b^2)}$$

$$= \frac{x^2}{2a^2} + \frac{b^2 \tan(c+dx^2)}{2a(a^2-b^2)d(a+b\sec(c+dx^2))} - \frac{(2a^2-b^2)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{a^2(a^2-b^2)d}$$

$$= \frac{x^2}{2a^2} - \frac{b(2a^2-b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2 \tan(c+dx^2)}{2a(a^2-b^2)d(a+b\sec(c+dx^2))}$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.24

$$\int \frac{x}{(a+b\sec(c+dx^2))^2}\,dx = \frac{-\frac{2b(-2a^2+b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{a(a^2-b^2)(c+dx^2)\cos(c+dx^2)+b\left((a^2-b^2)(c+dx^2)+ab\sin(c+dx^2)\right)}{b+a\cos(c+dx^2)}}{2a^2(a-b)(a+b)d}$$

```
[In] Integrate[x/(a + b*Sec[c + d*x^2])^2,x]
```

```
[Out] ((-2*b*(-2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]
])/Sqrt[a^2 - b^2] + (a*(a^2 - b^2)*(c + d*x^2)*Cos[c + d*x^2] + b*((a^2 -
b^2)*(c + d*x^2) + a*b*Sin[c + d*x^2]))/(b + a*Cos[c + d*x^2]))/(2*a^2*(a -
 b)*(a + b)*d)
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\frac{2b\left(-\frac{ba\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2 a-\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2 b-a-b\right)}-\frac{\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{a^2}+\frac{2\arctan\left(\tan\left(\frac{dx^2}{2}\right.\right.}{a^2}}{2d}$
default	$\frac{\frac{2b\left(-\frac{ba\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2 a-\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2 b-a-b\right)}-\frac{\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{a^2}+\frac{2\arctan\left(\tan\left(\frac{dx^2}{2}\right.\right.}{a^2}}{2d}$
risch	$\frac{x^2}{2a^2}+\frac{ib^2\left(b\,e^{i\left(dx^2+c\right)}+a\right)}{a^2(a^2-b^2)d\left(a\,e^{2i\left(dx^2+c\right)}+2b\,e^{i\left(dx^2+c\right)}+a\right)}+\frac{b\ln\left(e^{i\left(dx^2+c\right)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{b^3\ln\left(e^{i\left(dx^2+c\right)}\right.}{2\sqrt{a^2-b^2}\,(}$

```
[In] int(x/(a+b*sec(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(2*b/a^2*(-b*a/(a^2-b^2)*tan(1/2*d*x^2+1/2*c)/(tan(1/2*d*x^2+1/2*c)^2
*a-tan(1/2*d*x^2+1/2*c)^2*b-a-b)-(2*a^2-b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2
)*arctanh((a-b)*tan(1/2*d*x^2+1/2*c)/((a-b)*(a+b))^(1/2)))+2/a^2*arctan(tan
(1/2*d*x^2+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(110) = 220$.

Time = 0.30 (sec) , antiderivative size = 525, normalized size of antiderivative = 4.27

$$\int \frac{x}{\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$$
$$= \left[\frac{2\left(a^5-2a^3b^2+ab^4\right)dx^2\cos\left(dx^2+c\right)+2\left(a^4b-2a^2b^3+b^5\right)dx^2+\left(2a^2b^2-b^4+\left(2a^3b-ab^3\right)\cos\left(dx^2\right.\right.}{4\left(\left(a^7-2a^5b^2+a^3b^4\right)d\right.}\right.$$

```
[In] integrate(x/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x^2*cos(d*x^2 + c) + 2*(a^4*b - 2*a^2*b
^3 + b^5)*d*x^2 + (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x^2 + c))*sqrt
(a^2 - b^2)*log((2*a*b*cos(d*x^2 + c) - (a^2 - 2*b^2)*cos(d*x^2 + c)^2 - 2*
sqrt(a^2 - b^2)*(b*cos(d*x^2 + c) + a)*sin(d*x^2 + c) + 2*a^2 - b^2)/(a^2*c
```

```
os(d*x^2 + c)^2 + 2*a*b*cos(d*x^2 + c) + b^2)) + 2*(a^3*b^2 - a*b^4)*sin(d*
x^2 + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x^2 + c) + (a^6*b - 2*a^4*b^
3 + a^2*b^5)*d), 1/2*((a^5 - 2*a^3*b^2 + a*b^4)*d*x^2*cos(d*x^2 + c) + (a^4
*b - 2*a^2*b^3 + b^5)*d*x^2 - (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x^
2 + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x^2 + c) + a)/((
a^2 - b^2)*sin(d*x^2 + c))) + (a^3*b^2 - a*b^4)*sin(d*x^2 + c))/((a^7 - 2*a
^5*b^2 + a^3*b^4)*d*cos(d*x^2 + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]
```

Sympy [F]

$$\int \frac{x}{\left(a + b\sec\left(c + dx^2\right)\right)^2}\,dx = \int \frac{x}{\left(a + b\sec\left(c + dx^2\right)\right)^2}\,dx$$

```
[In] integrate(x/(a+b*sec(d*x**2+c))**2,x)
```

```
[Out] Integral(x/(a + b*sec(c + d*x**2))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8871 vs. 2(110) = 220.

Time = 25.25 (sec) , antiderivative size = 8871, normalized size of antiderivative = 72.12

$$\int \frac{x}{\left(a + b\sec\left(c + dx^2\right)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^6 - 2*a^4*b^2 + a^2*b^4)*d*x^2*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 -
2*a^2*b^4 + b^6)*d*x^2*cos(d*x^2 + c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*d*x^2
*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2*sin(d*x^2 + c)^
2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(d*x^2 + c) + (a^6 - 2*a^4*b^2 +
 a^2*b^4)*d*x^2 + (2*a^4*b - a^2*b^3 + (2*a^4*b - a^2*b^3)*cos(2*d*x^2 + 2*
c)^2 + 4*(2*a^2*b^3 - b^5)*cos(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*sin(2*d*x
^2 + 2*c)^2 + 4*(2*a^3*b^2 - a*b^4)*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(
2*a^2*b^3 - b^5)*sin(d*x^2 + c)^2 + 2*(2*a^4*b - a^2*b^3 + 2*(2*a^3*b^2 - a
*b^4)*cos(d*x^2 + c))*cos(2*d*x^2 + 2*c) + 4*(2*a^3*b^2 - a*b^4)*cos(d*x^2
+ c))*sqrt(-a^2 + b^2)*arctan2(2*(4*(a^6 - a^4*b^2)*cos(d*x^2 + 2*c)^4*cos(
c)*sin(c) - 4*(a^6 - a^4*b^2)*cos(c)*sin(d*x^2 + 2*c)^4*sin(c) + 4*(3*(a^5*
b - a^3*b^3)*cos(c)^2*sin(c) + (a^5*b - a^3*b^3)*sin(c)^3)*cos(d*x^2 + 2*c)
^3 - 4*((a^5*b - a^3*b^3)*cos(c)^3 + 3*(a^5*b - a^3*b^3)*cos(c)*sin(c)^2 +
((a^6 - a^4*b^2)*cos(c)^2 - (a^6 - a^4*b^2)*sin(c)^2)*cos(d*x^2 + 2*c))*sin
(d*x^2 + 2*c)^3 - 4*((a^6 - 5*a^4*b^2 + 4*a^2*b^4)*cos(c)^3*sin(c) + (a^6 -
 5*a^4*b^2 + 4*a^2*b^4)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + 4*((a^6 - 5*a
```

```
^4*b^2 + 4*a^2*b^4)*cos(c)^3*sin(c) + (a^6 - 5*a^4*b^2 + 4*a^2*b^4)*cos(c)*
sin(c)^3 - 3*((a^5*b - a^3*b^3)*cos(c)^2*sin(c) - (a^5*b - a^3*b^3)*sin(c)^
3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - 4*((a^5*b - 3*a^3*b^3 + 2*a*b^5)*
cos(c)^4*sin(c) + 2*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(c)^2*sin(c)^3 + (a^5*
b - 3*a^3*b^3 + 2*a*b^5)*sin(c)^5)*cos(d*x^2 + 2*c) + 4*((a^5*b - 3*a^3*b^3
 + 2*a*b^5)*cos(c)^5 + 2*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(c)^3*sin(c)^2 +
(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(c)*sin(c)^4 - ((a^6 - a^4*b^2)*cos(c)^2 -
 (a^6 - a^4*b^2)*sin(c)^2)*cos(d*x^2 + 2*c)^3 - 3*((a^5*b - a^3*b^3)*cos(c)
^3 - (a^5*b - a^3*b^3)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^2 + ((a^6 - 5*a^4*
b^2 + 4*a^2*b^4)*cos(c)^4 - (a^6 - 5*a^4*b^2 + 4*a^2*b^4)*sin(c)^4)*cos(d*x
^2 + 2*c))*sin(d*x^2 + 2*c) + (a^5*cos(c)*sin(d*x^2 + 2*c)^5 - a^5*cos(d*x^
2 + 2*c)^5*sin(c) - 4*a^4*b*cos(d*x^2 + 2*c)^4*cos(c)*sin(c) - (a^5*cos(d*x
^2 + 2*c)*sin(c) - 4*a^4*b*cos(c)*sin(c))*sin(d*x^2 + 2*c)^4 + 2*(3*(a^5 -
2*a^3*b^2)*cos(c)^2*sin(c) + (a^5 - 2*a^3*b^2)*sin(c)^3)*cos(d*x^2 + 2*c)^3
 + 2*(a^5*cos(d*x^2 + 2*c)^2*cos(c) - (a^5 - 2*a^3*b^2)*cos(c)^3 - 3*(a^5 -
 2*a^3*b^2)*cos(c)*sin(c)^2 + 2*(a^4*b*cos(c)^2 - a^4*b*sin(c)^2)*cos(d*x^2
 + 2*c))*sin(d*x^2 + 2*c)^3 + 4*((3*a^4*b - 4*a^2*b^3)*cos(c)^3*sin(c) + (3
*a^4*b - 4*a^2*b^3)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c)^2 - 2*(a^5*cos(d*x^2
+ 2*c)^3*sin(c) + 2*(3*a^4*b - 4*a^2*b^3)*cos(c)^3*sin(c) + 2*(3*a^4*b - 4*
a^2*b^3)*cos(c)*sin(c)^3 + 3*((a^5 - 2*a^3*b^2)*cos(c)^2*sin(c) - (a^5 - 2*
a^3*b^2)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - ((a^5 - 8*a^3*b^2
 + 8*a*b^4)*cos(c)^4*sin(c) + 2*(a^5 - 8*a^3*b^2 + 8*a*b^4)*cos(c)^2*sin(c)
^3 + (a^5 - 8*a^3*b^2 + 8*a*b^4)*sin(c)^5)*cos(d*x^2 + 2*c) + (a^5*cos(d*x^
2 + 2*c)^4*cos(c) + (a^5 - 8*a^3*b^2 + 8*a*b^4)*cos(c)^5 + 2*(a^5 - 8*a^3*b
^2 + 8*a*b^4)*cos(c)^3*sin(c)^2 + (a^5 - 8*a^3*b^2 + 8*a*b^4)*cos(c)*sin(c)
^4 + 4*(a^4*b*cos(c)^2 - a^4*b*sin(c)^2)*cos(d*x^2 + 2*c)^3 - 6*((a^5 - 2*a
^3*b^2)*cos(c)^3 - (a^5 - 2*a^3*b^2)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^2 -
4*((3*a^4*b - 4*a^2*b^3)*cos(c)^4 - (3*a^4*b - 4*a^2*b^3)*sin(c)^4)*cos(d*x
^2 + 2*c))*sin(d*x^2 + 2*c))*sqrt(-a^2 + b^2))/(a^6*cos(d*x^2 + 2*c)^6 + 6*
a^5*b*cos(d*x^2 + 2*c)^5*cos(c) + a^6*sin(d*x^2 + 2*c)^6 + 6*a^5*b*sin(d*x^
2 + 2*c)^5*sin(c) - (a^6 - 18*a^4*b^2 + 48*a^2*b^4 - 32*b^6)*cos(c)^6 - 3*(
a^6 - 18*a^4*b^2 + 48*a^2*b^4 - 32*b^6)*cos(c)^4*sin(c)^2 - 3*(a^6 - 18*a^4
*b^2 + 48*a^2*b^4 - 32*b^6)*cos(c)^2*sin(c)^4 - (a^6 - 18*a^4*b^2 + 48*a^2*
b^4 - 32*b^6)*sin(c)^6 - 3*(5*(a^6 - 2*a^4*b^2)*cos(c)^2 + (a^6 - 2*a^4*b^2
)*sin(c)^2)*cos(d*x^2 + 2*c)^4 + 3*(a^6*cos(d*x^2 + 2*c)^2 + 2*a^5*b*cos(d*
x^2 + 2*c)*cos(c) - (a^6 - 2*a^4*b^2)*cos(c)^2 - 5*(a^6 - 2*a^4*b^2)*sin(c)
^2)*sin(d*x^2 + 2*c)^4 - 4*(5*(3*a^5*b - 4*a^3*b^3)*cos(c)^3 + 3*(3*a^5*b -
 4*a^3*b^3)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 + 4*(3*a^5*b*cos(d*x^2 + 2*
c)^2*sin(c) - 6*(a^6 - 2*a^4*b^2)*cos(d*x^2 + 2*c)*cos(c)*sin(c) - 3*(3*a^5
*b - 4*a^3*b^3)*cos(c)^2*sin(c) - 5*(3*a^5*b - 4*a^3*b^3)*sin(c)^3)*sin(d*x
^2 + 2*c)^3 + 3*(5*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^4 + 6*(a^6 - 8*a^4*
b^2 + 8*a^2*b^4)*cos(c)^2*sin(c)^2 + (a^6 - 8*a^4*b^2 + 8*a^2*b^4)*sin(c)^4
)*cos(d*x^2 + 2*c)^2 + 3*(a^6*cos(d*x^2 + 2*c)^4 + 4*a^5*b*cos(d*x^2 + 2*c)
^3*cos(c) + (a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^4 + 6*(a^6 - 8*a^4*b^2 + 8
*a^2*b^4)*cos(c)^2*sin(c)^2 + 5*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*sin(c)^4 - 6*
```

((a^6 - 2*a^4*b^2)*cos(c)^2 + (a^6 - 2*a^4*b^2)*sin(c)^2)*cos(d*x^2 + 2*c)^
2 - 4*((3*a^5*b - 4*a^3*b^3)*cos(c)^3 + 3*(3*a^5*b - 4*a^3*b^3)*cos(c)*sin(
c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 + 6*((5*a^5*b - 20*a^3*b^3 + 16*
a*b^5)*cos(c)^5 + 2*(5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*cos(c)^3*sin(c)^2 + (
5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*cos(c)*sin(c)^4)*cos(d*x^2 + 2*c) + 6*(a^5
*b*cos(d*x^2 + 2*c)^4*sin(c) - 4*(a^6 - 2*a^4*b^2)*cos(d*x^2 + 2*c)^3*cos(c
)*sin(c) + (5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*cos(c)^4*sin(c) + 2*(5*a^5*b -
 20*a^3*b^3 + 16*a*b^5)*cos(c)^2*sin(c)^3 + (5*a^5*b - 20*a^3*b^3 + 16*a*b^
5)*sin(c)^5 - 2*(3*(3*a^5*b - 4*a^3*b^3)*cos(c)^2*sin(c) + (3*a^5*b - 4*a^3
*b^3)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + 4*((a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c
)^3*sin(c) + (a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c
))*sin(d*x^2 + 2*c) + 2*(3*a^5*cos(d*x^2 + 2*c)^5*cos(c) + 3*a^5*sin(d*x^2
+ 2*c)^5*sin(c) + (3*a^4*b - 16*a^2*b^3 + 16*b^5)*cos(c)^6 + 3*(3*a^4*b - 1
6*a^2*b^3 + 16*b^5)*cos(c)^4*sin(c)^2 + 3*(3*a^4*b - 16*a^2*b^3 + 16*b^5)*c
os(c)^2*sin(c)^4 + (3*a^4*b - 16*a^2*b^3 + 16*b^5)*sin(c)^6 + 3*(5*a^4*b*co
s(c)^2 + a^4*b*sin(c)^2)*cos(d*x^2 + 2*c)^4 + 3*(a^5*cos(d*x^2 + 2*c)*cos(c
) + a^4*b*cos(c)^2 + 5*a^4*b*sin(c)^2)*sin(d*x^2 + 2*c)^4 - 2*(5*(a^5 - 4*a
^3*b^2)*cos(c)^3 + 3*(a^5 - 4*a^3*b^2)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3
+ 2*(3*a^5*cos(d*x^2 + 2*c)^2*sin(c) + 12*a^4*b*cos(d*x^2 + 2*c)*cos(c)*sin
(c) - 3*(a^5 - 4*a^3*b^2)*cos(c)^2*sin(c) - 5*(a^5 - 4*a^3*b^2)*sin(c)^3)*s
in(d*x^2 + 2*c)^3 - 6*(5*(a^4*b - 2*a^2*b^3)*cos(c)^4 + 6*(a^4*b - 2*a^2*b^
3)*cos(c)^2*sin(c)^2 + (a^4*b - 2*a^2*b^3)*sin(c)^4)*cos(d*x^2 + 2*c)^2 + 6
*(a^5*cos(d*x^2 + 2*c)^3*cos(c) - (a^4*b - 2*a^2*b^3)*cos(c)^4 - 6*(a^4*b -
 2*a^2*b^3)*cos(c)^2*sin(c)^2 - 5*(a^4*b - 2*a^2*b^3)*sin(c)^4 + 3*(a^4*b*c
os(c)^2 + a^4*b*sin(c)^2)*cos(d*x^2 + 2*c)^2 - ((a^5 - 4*a^3*b^2)*cos(c)^3
+ 3*(a^5 - 4*a^3*b^2)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2
 + 3*((a^5 - 12*a^3*b^2 + 16*a*b^4)*cos(c)^5 + 2*(a^5 - 12*a^3*b^2 + 16*a*b
^4)*cos(c)^3*sin(c)^2 + (a^5 - 12*a^3*b^2 + 16*a*b^4)*cos(c)*sin(c)^4)*cos(
d*x^2 + 2*c) + 3*(a^5*cos(d*x^2 + 2*c)^4*sin(c) + 8*a^4*b*cos(d*x^2 + 2*c)^
3*cos(c)*sin(c) + (a^5 - 12*a^3*b^2 + 16*a*b^4)*cos(c)^4*sin(c) + 2*(a^5 -
12*a^3*b^2 + 16*a*b^4)*cos(c)^2*sin(c)^3 + (a^5 - 12*a^3*b^2 + 16*a*b^4)*si
n(c)^5 - 2*(3*(a^5 - 4*a^3*b^2)*cos(c)^2*sin(c) + (a^5 - 4*a^3*b^2)*sin(c)^
3)*cos(d*x^2 + 2*c)^2 - 16*((a^4*b - 2*a^2*b^3)*cos(c)^3*sin(c) + (a^4*b -
2*a^2*b^3)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c))*sqrt(-a^2 +
 b^2)), (a^6*cos(d*x^2 + 2*c)^6 + 6*a^5*b*cos(d*x^2 + 2*c)^5*cos(c) + a^6*s
in(d*x^2 + 2*c)^6 + 6*a^5*b*sin(d*x^2 + 2*c)^5*sin(c) + (a^6 - 8*a^4*b^2 +
8*a^2*b^4)*cos(c)^6 + 3*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^4*sin(c)^2 + 3
*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^2*sin(c)^4 + (a^6 - 8*a^4*b^2 + 8*a^2
*b^4)*sin(c)^6 - (5*(a^6 - 4*a^4*b^2)*cos(c)^2 + (a^6 - 4*a^4*b^2)*sin(c)^2
)*cos(d*x^2 + 2*c)^4 + (3*a^6*cos(d*x^2 + 2*c)^2 + 6*a^5*b*cos(d*x^2 + 2*c)
*cos(c) - (a^6 - 4*a^4*b^2)*cos(c)^2 - 5*(a^6 - 4*a^4*b^2)*sin(c)^2)*sin(d*
x^2 + 2*c)^4 - 4*(5*(a^5*b - 2*a^3*b^3)*cos(c)^3 + 3*(a^5*b - 2*a^3*b^3)*co
s(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 + 4*(3*a^5*b*cos(d*x^2 + 2*c)^2*sin(c) -
2*(a^6 - 4*a^4*b^2)*cos(d*x^2 + 2*c)*cos(c)*sin(c) - 3*(a^5*b - 2*a^3*b^3)*
cos(c)^2*sin(c) - 5*(a^5*b - 2*a^3*b^3)*sin(c)^3)*sin(d*x^2 + 2*c)^3 - (5*(

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a^6 + 4*a^4*b^2 - 8*a^2*b^4)*cos(c)^4 + 6*(a^6 + 4*a^4*b^2 - 8*a^2*b^4)*cos
(c)^2*sin(c)^2 + (a^6 + 4*a^4*b^2 - 8*a^2*b^4)*sin(c)^4)*cos(d*x^2 + 2*c)^2
 + (3*a^6*cos(d*x^2 + 2*c)^4 + 12*a^5*b*cos(d*x^2 + 2*c)^3*cos(c) - (a^6 +
4*a^4*b^2 - 8*a^2*b^4)*cos(c)^4 - 6*(a^6 + 4*a^4*b^2 - 8*a^2*b^4)*cos(c)^2*
sin(c)^2 - 5*(a^6 + 4*a^4*b^2 - 8*a^2*b^4)*sin(c)^4 - 6*((a^6 - 4*a^4*b^2)*
cos(c)^2 + (a^6 - 4*a^4*b^2)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 12*((a^5*b - 2*
a^3*b^3)*cos(c)^3 + 3*(a^5*b - 2*a^3*b^3)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)
)*sin(d*x^2 + 2*c)^2 - 2*((5*a^5*b - 8*a*b^5)*cos(c)^5 + 2*(5*a^5*b - 8*a*b
^5)*cos(c)^3*sin(c)^2 + (5*a^5*b - 8*a*b^5)*cos(c)*sin(c)^4)*cos(d*x^2 + 2*
c) + 2*(3*a^5*b*cos(d*x^2 + 2*c)^4*sin(c) - 4*(a^6 - 4*a^4*b^2)*cos(d*x^2 +
 2*c)^3*cos(c)*sin(c) - (5*a^5*b - 8*a*b^5)*cos(c)^4*sin(c) - 2*(5*a^5*b -
8*a*b^5)*cos(c)^2*sin(c)^3 - (5*a^5*b - 8*a*b^5)*sin(c)^5 - 6*(3*(a^5*b - 2
*a^3*b^3)*cos(c)^2*sin(c) + (a^5*b - 2*a^3*b^3)*sin(c)^3)*cos(d*x^2 + 2*c)^
2 - 4*((a^6 + 4*a^4*b^2 - 8*a^2*b^4)*cos(c)^3*sin(c) + (a^6 + 4*a^4*b^2 - 8
*a^2*b^4)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c) + 4*(a^5*cos(
d*x^2 + 2*c)^5*cos(c) + a^5*sin(d*x^2 + 2*c)^5*sin(c) - (a^4*b - 2*a^2*b^3)
*cos(c)^6 - 3*(a^4*b - 2*a^2*b^3)*cos(c)^4*sin(c)^2 - 3*(a^4*b - 2*a^2*b^3)
*cos(c)^2*sin(c)^4 - (a^4*b - 2*a^2*b^3)*sin(c)^6 + (5*a^4*b*cos(c)^2 + a^4
*b*sin(c)^2)*cos(d*x^2 + 2*c)^4 + (a^5*cos(d*x^2 + 2*c)*cos(c) + a^4*b*cos(
c)^2 + 5*a^4*b*sin(c)^2)*sin(d*x^2 + 2*c)^4 + 2*(5*a^3*b^2*cos(c)^3 + 3*a^3
*b^2*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 + 2*(a^5*cos(d*x^2 + 2*c)^2*sin(c)
 + 4*a^4*b*cos(d*x^2 + 2*c)*cos(c)*sin(c) + 3*a^3*b^2*cos(c)^2*sin(c) + 5*a
^3*b^2*sin(c)^3)*sin(d*x^2 + 2*c)^3 + 2*(5*a^2*b^3*cos(c)^4 + 6*a^2*b^3*cos
(c)^2*sin(c)^2 + a^2*b^3*sin(c)^4)*cos(d*x^2 + 2*c)^2 + 2*(a^5*cos(d*x^2 +
2*c)^3*cos(c) + a^2*b^3*cos(c)^4 + 6*a^2*b^3*cos(c)^2*sin(c)^2 + 5*a^2*b^3*
sin(c)^4 + 3*(a^4*b*cos(c)^2 + a^4*b*sin(c)^2)*cos(d*x^2 + 2*c)^2 + 3*(a^3*
b^2*cos(c)^3 + 3*a^3*b^2*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c
)^2 - ((a^5 - 2*a^3*b^2 - 4*a*b^4)*cos(c)^5 + 2*(a^5 - 2*a^3*b^2 - 4*a*b^4)
*cos(c)^3*sin(c)^2 + (a^5 - 2*a^3*b^2 - 4*a*b^4)*cos(c)*sin(c)^4)*cos(d*x^2
 + 2*c) + (a^5*cos(d*x^2 + 2*c)^4*sin(c) + 8*a^4*b*cos(d*x^2 + 2*c)^3*cos(c
)*sin(c) - (a^5 - 2*a^3*b^2 - 4*a*b^4)*cos(c)^4*sin(c) - 2*(a^5 - 2*a^3*b^2
 - 4*a*b^4)*cos(c)^2*sin(c)^3 - (a^5 - 2*a^3*b^2 - 4*a*b^4)*sin(c)^5 + 6*(3
*a^3*b^2*cos(c)^2*sin(c) + a^3*b^2*sin(c)^3)*cos(d*x^2 + 2*c)^2 + 16*(a^2*b
^3*cos(c)^3*sin(c) + a^2*b^3*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 +
 2*c))*sqrt(-a^2 + b^2))/(a^6*cos(d*x^2 + 2*c)^6 + 6*a^5*b*cos(d*x^2 + 2*c)
^5*cos(c) + a^6*sin(d*x^2 + 2*c)^6 + 6*a^5*b*sin(d*x^2 + 2*c)^5*sin(c) - (a
^6 - 18*a^4*b^2 + 48*a^2*b^4 - 32*b^6)*cos(c)^6 - 3*(a^6 - 18*a^4*b^2 + 48*
a^2*b^4 - 32*b^6)*cos(c)^4*sin(c)^2 - 3*(a^6 - 18*a^4*b^2 + 48*a^2*b^4 - 32
*b^6)*cos(c)^2*sin(c)^4 - (a^6 - 18*a^4*b^2 + 48*a^2*b^4 - 32*b^6)*sin(c)^6
 - 3*(5*(a^6 - 2*a^4*b^2)*cos(c)^2 + (a^6 - 2*a^4*b^2)*sin(c)^2)*cos(d*x^2
+ 2*c)^4 + 3*(a^6*cos(d*x^2 + 2*c)^2 + 2*a^5*b*cos(d*x^2 + 2*c)*cos(c) - (a
^6 - 2*a^4*b^2)*cos(c)^2 - 5*(a^6 - 2*a^4*b^2)*sin(c)^2)*sin(d*x^2 + 2*c)^4
 - 4*(5*(3*a^5*b - 4*a^3*b^3)*cos(c)^3 + 3*(3*a^5*b - 4*a^3*b^3)*cos(c)*sin
(c)^2)*cos(d*x^2 + 2*c)^3 + 4*(3*a^5*b*cos(d*x^2 + 2*c)^2*sin(c) - 6*(a^6 -
 2*a^4*b^2)*cos(d*x^2 + 2*c)*cos(c)*sin(c) - 3*(3*a^5*b - 4*a^3*b^3)*cos(c)
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^2*sin(c) - 5*(3*a^5*b - 4*a^3*b^3)*sin(c)^3)*sin(d*x^2 + 2*c)^3 + 3*(5*(a^
6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^4 + 6*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c
)^2*sin(c)^2 + (a^6 - 8*a^4*b^2 + 8*a^2*b^4)*sin(c)^4)*cos(d*x^2 + 2*c)^2 +
 3*(a^6*cos(d*x^2 + 2*c)^4 + 4*a^5*b*cos(d*x^2 + 2*c)^3*cos(c) + (a^6 - 8*a
^4*b^2 + 8*a^2*b^4)*cos(c)^4 + 6*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^2*sin
(c)^2 + 5*(a^6 - 8*a^4*b^2 + 8*a^2*b^4)*sin(c)^4 - 6*((a^6 - 2*a^4*b^2)*cos
(c)^2 + (a^6 - 2*a^4*b^2)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 4*((3*a^5*b - 4*a^
3*b^3)*cos(c)^3 + 3*(3*a^5*b - 4*a^3*b^3)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)
)*sin(d*x^2 + 2*c)^2 + 6*((5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*cos(c)^5 + 2*(5
*a^5*b - 20*a^3*b^3 + 16*a*b^5)*cos(c)^3*sin(c)^2 + (5*a^5*b - 20*a^3*b^3 +
 16*a*b^5)*cos(c)*sin(c)^4)*cos(d*x^2 + 2*c) + 6*(a^5*b*cos(d*x^2 + 2*c)^4*
sin(c) - 4*(a^6 - 2*a^4*b^2)*cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + (5*a^5*b -
20*a^3*b^3 + 16*a*b^5)*cos(c)^4*sin(c) + 2*(5*a^5*b - 20*a^3*b^3 + 16*a*b^5
)*cos(c)^2*sin(c)^3 + (5*a^5*b - 20*a^3*b^3 + 16*a*b^5)*sin(c)^5 - 2*(3*(3*
a^5*b - 4*a^3*b^3)*cos(c)^2*sin(c) + (3*a^5*b - 4*a^3*b^3)*sin(c)^3)*cos(d*
x^2 + 2*c)^2 + 4*((a^6 - 8*a^4*b^2 + 8*a^2*b^4)*cos(c)^3*sin(c) + (a^6 - 8*
a^4*b^2 + 8*a^2*b^4)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c) +
2*(3*a^5*cos(d*x^2 + 2*c)^5*cos(c) + 3*a^5*sin(d*x^2 + 2*c)^5*sin(c) + (3*a
^4*b - 16*a^2*b^3 + 16*b^5)*cos(c)^6 + 3*(3*a^4*b - 16*a^2*b^3 + 16*b^5)*co
s(c)^4*sin(c)^2 + 3*(3*a^4*b - 16*a^2*b^3 + 16*b^5)*cos(c)^2*sin(c)^4 + (3*
a^4*b - 16*a^2*b^3 + 16*b^5)*sin(c)^6 + 3*(5*a^4*b*cos(c)^2 + a^4*b*sin(c)^
2)*cos(d*x^2 + 2*c)^4 + 3*(a^5*cos(d*x^2 + 2*c)*cos(c) + a^4*b*cos(c)^2 + 5
*a^4*b*sin(c)^2)*sin(d*x^2 + 2*c)^4 - 2*(5*(a^5 - 4*a^3*b^2)*cos(c)^3 + 3*(
a^5 - 4*a^3*b^2)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 + 2*(3*a^5*cos(d*x^2 +
 2*c)^2*sin(c) + 12*a^4*b*cos(d*x^2 + 2*c)*cos(c)*sin(c) - 3*(a^5 - 4*a^3*b
^2)*cos(c)^2*sin(c) - 5*(a^5 - 4*a^3*b^2)*sin(c)^3)*sin(d*x^2 + 2*c)^3 - 6*
(5*(a^4*b - 2*a^2*b^3)*cos(c)^4 + 6*(a^4*b - 2*a^2*b^3)*cos(c)^2*sin(c)^2 +
 (a^4*b - 2*a^2*b^3)*sin(c)^4)*cos(d*x^2 + 2*c)^2 + 6*(a^5*cos(d*x^2 + 2*c)
^3*cos(c) - (a^4*b - 2*a^2*b^3)*cos(c)^4 - 6*(a^4*b - 2*a^2*b^3)*cos(c)^2*s
in(c)^2 - 5*(a^4*b - 2*a^2*b^3)*sin(c)^4 + 3*(a^4*b*cos(c)^2 + a^4*b*sin(c)
^2)*cos(d*x^2 + 2*c)^2 - ((a^5 - 4*a^3*b^2)*cos(c)^3 + 3*(a^5 - 4*a^3*b^2)*
cos(c)*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 + 3*((a^5 - 12*a^3*b^
2 + 16*a*b^4)*cos(c)^5 + 2*(a^5 - 12*a^3*b^2 + 16*a*b^4)*cos(c)^3*sin(c)^2
+ (a^5 - 12*a^3*b^2 + 16*a*b^4)*cos(c)*sin(c)^4)*cos(d*x^2 + 2*c) + 3*(a^5*
cos(d*x^2 + 2*c)^4*sin(c) + 8*a^4*b*cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + (a^5
 - 12*a^3*b^2 + 16*a*b^4)*cos(c)^4*sin(c) + 2*(a^5 - 12*a^3*b^2 + 16*a*b^4)
*cos(c)^2*sin(c)^3 + (a^5 - 12*a^3*b^2 + 16*a*b^4)*sin(c)^5 - 2*(3*(a^5 - 4
*a^3*b^2)*cos(c)^2*sin(c) + (a^5 - 4*a^3*b^2)*sin(c)^3)*cos(d*x^2 + 2*c)^2
- 16*((a^4*b - 2*a^2*b^3)*cos(c)^3*sin(c) + (a^4*b - 2*a^2*b^3)*cos(c)*sin(
c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c))*sqrt(-a^2 + b^2))) + 2*(2*(a^5*b
- 2*a^3*b^3 + a*b^5)*d*x^2*cos(d*x^2 + c) + (a^6 - 2*a^4*b^2 + a^2*b^4)*d*x
^2 - (a^3*b^3 - a*b^5)*sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + 2*(a^4*b^2 - a^
2*b^4 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*sin(d*x^2 + c) + (a^3*b^3 - a*b
^5)*cos(d*x^2 + c))*sin(2*d*x^2 + 2*c) + 2*(a^3*b^3 - a*b^5)*sin(d*x^2 + c)
)/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^6*b^2 - 2*a^4*
```

```
b^4 + a^2*b^6)*d*cos(d*x^2 + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*sin(2*d*x
^2 + 2*c)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*sin(2*d*x^2 + 2*c)*sin(d*x^
2 + c) + 4*(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*d*sin(d*x^2 + c)^2 + 4*(a^7*b -
2*a^5*b^3 + a^3*b^5)*d*cos(d*x^2 + c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d + 2*(
2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x^2 + c) + (a^8 - 2*a^6*b^2 + a^4*b
^4)*d)*cos(2*d*x^2 + 2*c))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.59

$$\int \frac{x}{(a+b\sec(c+dx^2))^2}\,dx = -\frac{b^2\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)}{(a^3d-ab^2d)\left(a\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)^2-a-b\right)} + \frac{(2\,a^2b-b^3)\left(\pi\left\lfloor\frac{dx^2+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4d-a^2b^2d)\sqrt{-a^2+b^2}} + \frac{dx^2+c}{2\,a^2d}$$

```
[In] integrate(x/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] -b^2*tan(1/2*d*x^2 + 1/2*c)/((a^3*d - a*b^2*d)*(a*tan(1/2*d*x^2 + 1/2*c)^2
- b*tan(1/2*d*x^2 + 1/2*c)^2 - a - b)) + (2*a^2*b - b^3)*(pi*floor(1/2*(d*x
^2 + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x^2 + 1/2*c) - b*tan
(1/2*d*x^2 + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*d - a^2*b^2*d)*sqrt(-a^2 + b^
2)) + 1/2*(d*x^2 + c)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.76

$$\int \frac{x}{(a+b\sec(c+dx^2))^2}\,dx = \frac{\frac{b^2}{d\,(a\,b^2\,1\mathrm{i}-a^3\,1\mathrm{i})}+\frac{b^3\,\mathrm{e}^{1\mathrm{i}\,d\,x^2+c\,1\mathrm{i}}}{a\,d\,(a\,b^2\,1\mathrm{i}-a^3\,1\mathrm{i})}}{a+a\,\mathrm{e}^{2\mathrm{i}\,d\,x^2+c\,2\mathrm{i}}+2\,b\,\mathrm{e}^{1\mathrm{i}\,d\,x^2+c\,1\mathrm{i}}} + \frac{x^2}{2\,a^2} + \frac{b\,\ln\left(2\,b\,x\,\mathrm{e}^{1\mathrm{i}\,d\,x^2+c\,1\mathrm{i}}\,(2\,a^2-b^2)-\frac{b\,x\,(a^2-b^2)\,(2\,a^2-b^2)\,\left(a+b\,\mathrm{e}^{1\mathrm{i}\,d\,x^2+c\,1\mathrm{i}}\right)\,2\mathrm{i}}{(a+b)^{3/2}\,(a-b)^{3/2}}\right)\,(2\,a^2-b^2)}{2\,a^2\,d\,(a+b)^{3/2}\,(a-b)^{3/2}} - \frac{b\,\ln\left(2\,b\,x\,\mathrm{e}^{1\mathrm{i}\,d\,x^2+c\,1\mathrm{i}}\,(2\,a^2-b^2)+\frac{b\,x\,(a^2-b^2)\,(2\,a^2-b^2)\,\left(a+b\,\mathrm{e}^{1\mathrm{i}\,d\,x^2+c\,1\mathrm{i}}\right)\,2\mathrm{i}}{(a+b)^{3/2}\,(a-b)^{3/2}}\right)\,(2\,a^2-b^2)}{2\,a^2\,d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

```
[In] int(x/(a + b/cos(c + d*x^2))^2,x)
```

```
[Out] (b^2/(d*(a*b^2*1i - a^3*1i)) + (b^3*exp(c*1i + d*x^2*1i))/(a*d*(a*b^2*1i -
a^3*1i)))/(a + a*exp(c*2i + d*x^2*2i) + 2*b*exp(c*1i + d*x^2*1i)) + x^2/(2*
a^2) + (b*log(2*b*x*exp(c*1i + d*x^2*1i)*(2*a^2 - b^2) - (b*x*(a^2 - b^2)*(
2*a^2 - b^2)*(a + b*exp(c*1i + d*x^2*1i))*2i)/((a + b)^(3/2)*(a - b)^(3/2))
)*(2*a^2 - b^2))/(2*a^2*d*(a + b)^(3/2)*(a - b)^(3/2)) - (b*log(2*b*x*exp(c
*1i + d*x^2*1i)*(2*a^2 - b^2) + (b*x*(a^2 - b^2)*(2*a^2 - b^2)*(a + b*exp(c
*1i + d*x^2*1i))*2i)/((a + b)^(3/2)*(a - b)^(3/2)))*(2*a^2 - b^2))/(2*a^2*d
*(a + b)^(3/2)*(a - b)^(3/2))
```

3.28 $\int \frac{1}{x(a+b\sec(c+dx^2))^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)^2} dx = \operatorname{Int}\left(\frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)^2}, x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sec(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)^2} dx = \int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)^2} dx$$

```
[In] Int[1/(x*(a + b*Sec[c + d*x^2])^2),x]
[Out] Defer[Int][1/(x*(a + b*Sec[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 10.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x*(a + b*Sec[c + d*x^2])^2),x]
[Out] Integrate[1/(x*(a + b*Sec[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\sec\left(d\,x^2+c\right))^2}dx$$

```
[In] int(1/x/(a+b*sec(d*x^2+c))^2,x)
[Out] int(1/x/(a+b*sec(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sec\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x*sec(d*x^2 + c)^2 + 2*a*b*x*sec(d*x^2 + c) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] integrate(1/x/(a+b*sec(d*x**2+c))**2,x)
[Out] Integral(1/(x*(a + b*sec(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 6.02 (sec) , antiderivative size = 4629, normalized size of antiderivative = 257.17

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sec\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] (a^6*d*x^2*cos(2*d*x^2 + 2*c)^2*log(x) + a^6*d*x^2*log(x)*sin(2*d*x^2 + 2*c
)^2 + (a^2*b^4*cos(2*c)^2 + a^2*b^4*sin(2*c)^2)*d*x^2*cos(2*d*x^2)^2*log(x)
 + 4*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(c)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*si
n(c)^2)*d*x^2*cos(d*x^2)^2*log(x) + (a^2*b^4*cos(2*c)^2 + a^2*b^4*sin(2*c)^
2)*d*x^2*log(x)*sin(2*d*x^2)^2 + 4*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(c)^2 +
(a^4*b^2 - 2*a^2*b^4 + b^6)*sin(c)^2)*d*x^2*log(x)*sin(d*x^2)^2 + (a^6 - 2*
a^4*b^2 + a^2*b^4)*d*x^2*log(x) - (a^2*b^4*sin(2*c) + 4*((a^3*b^3 - a*b^5)*
cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*cos(d*x^2)*log(x
) + 2*(a^4*b^2 - a^2*b^4)*d*x^2*cos(2*c)*log(x) + 4*((a^3*b^3 - a*b^5)*cos(
c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^2*log(x)*sin(d*x^2))*c
os(2*d*x^2) - (2*a^4*b^2*d*x^2*cos(2*d*x^2)*cos(2*c)*log(x) - 2*a^4*b^2*d*x
^2*log(x)*sin(2*d*x^2)*sin(2*c) - 4*(a^5*b - a^3*b^3)*d*x^2*cos(d*x^2)*cos(
c)*log(x) + a^3*b^3*sin(d*x^2 + c) + 4*(a^5*b - a^3*b^3)*d*x^2*log(x)*sin(d
*x^2)*sin(c) - 2*(a^6 - a^4*b^2)*d*x^2*log(x))*cos(2*d*x^2 + 2*c) - (a*b^5*
cos(2*c)*sin(2*d*x^2) + a*b^5*cos(2*d*x^2)*sin(2*c) - 2*(a^2*b^4 - b^6)*cos
(c)*sin(d*x^2) - 2*(a^2*b^4 - b^6)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + 2*(2
*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(c)*log(x) + (a^3*b^3 - a*b^5)*sin(c)
)*cos(d*x^2) + (a^8*d*x^2*cos(2*d*x^2 + 2*c)^2 + a^8*d*x^2*sin(2*d*x^2 + 2*
c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^2*cos(2*d*x^2)^2 + 4*(
(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*
sin(c)^2)*d*x^2*cos(d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^2*cos(d*
x^2)*cos(c) + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^2*sin(2*d*x^2)^
2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^
2*b^6)*sin(c)^2)*d*x^2*sin(d*x^2)^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^2
*sin(d*x^2)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*x^2 - 2*(2*((a^5*b^3 - a
^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^2*cos(d*
x^2) + (a^6*b^2 - a^4*b^4)*d*x^2*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*s
in(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^2*sin(d*x^2))*cos(2*d*x^
2) - 2*(a^6*b^2*d*x^2*cos(2*d*x^2)*cos(2*c) - a^6*b^2*d*x^2*sin(2*d*x^2)*si
n(2*c) - 2*(a^7*b - a^5*b^3)*d*x^2*cos(d*x^2)*cos(c) + 2*(a^7*b - a^5*b^3)*
d*x^2*sin(d*x^2)*sin(c) - (a^8 - a^6*b^2)*d*x^2)*cos(2*d*x^2 + 2*c) + 2*(2*
((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))
*d*x^2*cos(d*x^2) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3
*b^5)*sin(2*c)*sin(c))*d*x^2*sin(d*x^2) + (a^6*b^2 - a^4*b^4)*d*x^2*sin(2*c
))*sin(2*d*x^2) - 2*(a^6*b^2*d*x^2*cos(2*c)*sin(2*d*x^2) + a^6*b^2*d*x^2*co
```

```
s(2*d*x^2)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*x^2*cos(c)*sin(d*x^2) - 2*(a^7*
b - a^5*b^3)*d*x^2*cos(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))*integrate(-2*(a^2
*b^4*cos(2*c)*sin(2*d*x^2) + a^2*b^4*cos(2*d*x^2)*sin(2*c) - 2*(a^3*b^3 - a
*b^5)*cos(c)*sin(d*x^2) - 2*(a^3*b^3 - a*b^5)*cos(d*x^2)*sin(c) + (a^3*b^3*
sin(d*x^2 + c) + (2*a^5*b - a^3*b^3)*d*x^2*cos(d*x^2 + c))*cos(2*d*x^2 + 2*
c) + ((2*a^5*b - 3*a^3*b^3 + a*b^5)*d*x^2 + (a*b^5*sin(2*c) - (2*a^3*b^3 -
a*b^5)*d*x^2*cos(2*c))*cos(2*d*x^2) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*x^
2*cos(c) - (a^2*b^4 - b^6)*sin(c))*cos(d*x^2) + (a*b^5*cos(2*c) + (2*a^3*b^
3 - a*b^5)*d*x^2*sin(2*c))*sin(2*d*x^2) - 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*
d*x^2*sin(c) + (a^2*b^4 - b^6)*cos(c))*sin(d*x^2))*cos(d*x^2 + c) - (a^3*b^
3*cos(d*x^2 + c) + a^4*b^2 - (2*a^5*b - a^3*b^3)*d*x^2*sin(d*x^2 + c))*sin(
2*d*x^2 + 2*c) + (a^3*b^3 - a*b^5 - (a*b^5*cos(2*c) + (2*a^3*b^3 - a*b^5)*d
*x^2*sin(2*c))*cos(2*d*x^2) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*x^2*sin(c)
 + (a^2*b^4 - b^6)*cos(c))*cos(d*x^2) + (a*b^5*sin(2*c) - (2*a^3*b^3 - a*b^
5)*d*x^2*cos(2*c))*sin(2*d*x^2) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*x^2*co
s(c) - (a^2*b^4 - b^6)*sin(c))*sin(d*x^2))*sin(d*x^2 + c))/(a^8*d*x^3*cos(2
*d*x^2 + 2*c)^2 + a^8*d*x^3*sin(2*d*x^2 + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^
4*b^4*sin(2*c)^2)*d*x^3*cos(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)
*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^3*cos(d*x^2)^2 +
4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*cos(d*x^2)*cos(c) + (a^4*b^4*cos(2*c)
^2 + a^4*b^4*sin(2*c)^2)*d*x^3*sin(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a
^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^3*sin(d*x^
2)^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*sin(d*x^2)*sin(c) + (a^8 - 2*a
^6*b^2 + a^4*b^4)*d*x^3 - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*
b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^3*cos(d*x^2) + (a^6*b^2 - a^4*b^4)*d*x^
3*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*c
os(2*c)*sin(c))*d*x^3*sin(d*x^2))*cos(2*d*x^2) - 2*(a^6*b^2*d*x^3*cos(2*d*x
^2)*cos(2*c) - a^6*b^2*d*x^3*sin(2*d*x^2)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*
x^3*cos(d*x^2)*cos(c) + 2*(a^7*b - a^5*b^3)*d*x^3*sin(d*x^2)*sin(c) - (a^8
- a^6*b^2)*d*x^3)*cos(2*d*x^2 + 2*c) + 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin
(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^3*cos(d*x^2) - 2*((a^5*b^3
 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^3*si
n(d*x^2) + (a^6*b^2 - a^4*b^4)*d*x^3*sin(2*c))*sin(2*d*x^2) - 2*(a^6*b^2*d*
x^3*cos(2*c)*sin(2*d*x^2) + a^6*b^2*d*x^3*cos(2*d*x^2)*sin(2*c) - 2*(a^7*b
- a^5*b^3)*d*x^3*cos(c)*sin(d*x^2) - 2*(a^7*b - a^5*b^3)*d*x^3*cos(d*x^2)*s
in(c))*sin(2*d*x^2 + 2*c)), x) - (a^2*b^4*cos(2*c) - 4*((a^3*b^3 - a*b^5)*c
os(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^2*cos(d*x^2)*log(x)
 + 4*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c)
)*d*x^2*log(x)*sin(d*x^2) - 2*(a^4*b^2 - a^2*b^4)*d*x^2*log(x)*sin(2*c))*si
n(2*d*x^2) - (2*a^4*b^2*d*x^2*cos(2*c)*log(x)*sin(2*d*x^2) + 2*a^4*b^2*d*x^
2*cos(2*d*x^2)*log(x)*sin(2*c) - a^3*b^3*cos(d*x^2 + c) - 4*(a^5*b - a^3*b^
3)*d*x^2*cos(c)*log(x)*sin(d*x^2) - 4*(a^5*b - a^3*b^3)*d*x^2*cos(d*x^2)*lo
g(x)*sin(c) - a^4*b^2)*sin(2*d*x^2 + 2*c) + (a*b^5*cos(2*d*x^2)*cos(2*c) -
a*b^5*sin(2*d*x^2)*sin(2*c) - a^3*b^3 + a*b^5 - 2*(a^2*b^4 - b^6)*cos(d*x^2
)*cos(c) + 2*(a^2*b^4 - b^6)*sin(d*x^2)*sin(c))*sin(d*x^2 + c) - 2*(2*(a^5*
```

```
b - 2*a^3*b^3 + a*b^5)*d*x^2*log(x)*sin(c) - (a^3*b^3 - a*b^5)*cos(c))*sin(
d*x^2))/(a^8*d*x^2*cos(2*d*x^2 + 2*c)^2 + a^8*d*x^2*sin(2*d*x^2 + 2*c)^2 +
(a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^2*cos(2*d*x^2)^2 + 4*((a^6*b^
2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^
2)*d*x^2*cos(d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^2*cos(d*x^2)*co
s(c) + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^2*sin(2*d*x^2)^2 + 4*(
(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*
sin(c)^2)*d*x^2*sin(d*x^2)^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^2*sin(d*
x^2)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*x^2 - 2*(2*((a^5*b^3 - a^3*b^5)
*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^2*cos(d*x^2) +
(a^6*b^2 - a^4*b^4)*d*x^2*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c)
 - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^2*sin(d*x^2))*cos(2*d*x^2) - 2*
(a^6*b^2*d*x^2*cos(2*d*x^2)*cos(2*c) - a^6*b^2*d*x^2*sin(2*d*x^2)*sin(2*c)
- 2*(a^7*b - a^5*b^3)*d*x^2*cos(d*x^2)*cos(c) + 2*(a^7*b - a^5*b^3)*d*x^2*s
in(d*x^2)*sin(c) - (a^8 - a^6*b^2)*d*x^2)*cos(2*d*x^2 + 2*c) + 2*(2*((a^5*b
^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^2*
cos(d*x^2) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*s
in(2*c)*sin(c))*d*x^2*sin(d*x^2) + (a^6*b^2 - a^4*b^4)*d*x^2*sin(2*c))*sin(
2*d*x^2) - 2*(a^6*b^2*d*x^2*cos(2*c)*sin(2*d*x^2) + a^6*b^2*d*x^2*cos(2*d*x
^2)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*x^2*cos(c)*sin(d*x^2) - 2*(a^7*b - a^5
*b^3)*d*x^2*cos(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sec\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x^2 + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 14.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+\frac{b}{\cos\left(d\,x^2+c\right)}\right)^2}\,dx$$

```
[In] int(1/(x*(a + b/cos(c + d*x^2))^2),x)
[Out] int(1/(x*(a + b/cos(c + d*x^2))^2), x)
```

3.29 $\int \frac{1}{x^2\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx = \operatorname{Int}\left(\frac{1}{x^2\left(a+b\sec\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sec(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{x^2\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x^2*(a + b*Sec[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Sec[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \left(a+b \sec \left(c+d x^2\right)\right)^2} \, dx=\int \frac{1}{x^2 \left(a+b \sec \left(c+d x^2\right)\right)^2} \, dx$$

```
[In] Integrate[1/(x^2*(a + b*Sec[c + d*x^2])^2),x]
[Out] Integrate[1/(x^2*(a + b*Sec[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \left(a +b \sec \left(d \,x^{2}+c \right)\right)^{2}}d x$$

```
[In] int(1/x^2/(a+b*sec(d*x^2+c))^2,x)
[Out] int(1/x^2/(a+b*sec(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^2 \left(a+b \sec \left(c+d x^2\right)\right)^2} \, dx=\int { \frac{1}{{\left(b \sec \left(d x^{2} + c\right) + a\right)}^{2} x^{2}} \,d x }$$

```
[In] integrate(1/x^2/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^2*sec(d*x^2 + c)^2 + 2*a*b*x^2*sec(d*x^2 + c) + a^2*x^2),
 x)
```

Sympy [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 (a+b\sec(c+dx^2))^2}\,dx=\int \frac{1}{x^2 (a+b\sec(c+dx^2))^2}\,dx$$

```
[In] integrate(1/x**2/(a+b*sec(d*x**2+c))**2,x)
[Out] Integral(1/(x**2*(a + b*sec(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 5.77 (sec) , antiderivative size = 4550, normalized size of antiderivative = 252.78

$$\int \frac{1}{x^2 (a+b\sec(c+dx^2))^2}\,dx=\int \frac{1}{(b\sec(dx^2+c)+a)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
[Out] -((a^6 - a^4*b^2)*d*x^2*cos(2*d*x^2 + 2*c)^2 + (a^6 - a^4*b^2)*d*x^2*sin(2*
d*x^2 + 2*c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*d*x^2 + (a^2*b^4*sin(2*c) - (a
^4*b^2 - a^2*b^4)*d*x^2*cos(2*c))*cos(2*d*x^2) + (a^3*b^3*sin(d*x^2 + c) -
(a^4*b^2 - a^2*b^4)*d*x^2*cos(2*d*x^2)*cos(2*c) + 2*(a^5*b - 2*a^3*b^3 + a*
b^5)*d*x^2*cos(d*x^2)*cos(c) + (a^4*b^2 - a^2*b^4)*d*x^2*sin(2*d*x^2)*sin(2
*c) - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*sin(d*x^2)*sin(c) + 2*(a^5*b - a^
3*b^3)*d*x^2*cos(d*x^2 + c) + (2*a^6 - 3*a^4*b^2 + a^2*b^4)*d*x^2)*cos(2*d*
x^2 + 2*c) + (2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2 + (a*b^5*sin(2*c) - 2*(a^
3*b^3 - a*b^5)*d*x^2*cos(2*c))*cos(2*d*x^2) + 2*(2*(a^4*b^2 - 2*a^2*b^4 + b
^6)*d*x^2*cos(c) - (a^2*b^4 - b^6)*sin(c))*cos(d*x^2) + (a*b^5*cos(2*c) + 2
*(a^3*b^3 - a*b^5)*d*x^2*sin(2*c))*sin(2*d*x^2) - 2*(2*(a^4*b^2 - 2*a^2*b^4
 + b^6)*d*x^2*sin(c) + (a^2*b^4 - b^6)*cos(c))*sin(d*x^2))*cos(d*x^2 + c) +
 2*((a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(c) - (a^3*b^3 - a*b^5)*sin(c))*co
s(d*x^2) - (a^8*d*x^3*cos(2*d*x^2 + 2*c)^2 + a^8*d*x^3*sin(2*d*x^2 + 2*c)^2
 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^3*cos(2*d*x^2)^2 + 4*((a^6
*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(
c)^2)*d*x^3*cos(d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*cos(d*x^2)
*cos(c) + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^3*sin(2*d*x^2)^2 +
4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^
6)*sin(c)^2)*d*x^3*sin(d*x^2)^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*sin
(d*x^2)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*x^3 - 2*(2*((a^5*b^3 - a^3*b
^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^3*cos(d*x^2)
```

```
 + (a^6*b^2 - a^4*b^4)*d*x^3*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2
*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^3*sin(d*x^2))*cos(2*d*x^2) -
 2*(a^6*b^2*d*x^3*cos(2*d*x^2)*cos(2*c) - a^6*b^2*d*x^3*sin(2*d*x^2)*sin(2*
c) - 2*(a^7*b - a^5*b^3)*d*x^3*cos(d*x^2)*cos(c) + 2*(a^7*b - a^5*b^3)*d*x^
3*sin(d*x^2)*sin(c) - (a^8 - a^6*b^2)*d*x^3)*cos(2*d*x^2 + 2*c) + 2*(2*((a^
5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x
^3*cos(d*x^2) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5
)*sin(2*c)*sin(c))*d*x^3*sin(d*x^2) + (a^6*b^2 - a^4*b^4)*d*x^3*sin(2*c))*s
in(2*d*x^2) - 2*(a^6*b^2*d*x^3*cos(2*c)*sin(2*d*x^2) + a^6*b^2*d*x^3*cos(2*
d*x^2)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*x^3*cos(c)*sin(d*x^2) - 2*(a^7*b -
a^5*b^3)*d*x^3*cos(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))*integrate(-(3*a^2*b^4
*cos(2*c)*sin(2*d*x^2) + 3*a^2*b^4*cos(2*d*x^2)*sin(2*c) - 6*(a^3*b^3 - a*b
^5)*cos(c)*sin(d*x^2) - 6*(a^3*b^3 - a*b^5)*cos(d*x^2)*sin(c) + (3*a^3*b^3*
sin(d*x^2 + c) + 2*(2*a^5*b - a^3*b^3)*d*x^2*cos(d*x^2 + c))*cos(2*d*x^2 +
2*c) + (2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*d*x^2 + (3*a*b^5*sin(2*c) - 2*(2*a^
3*b^3 - a*b^5)*d*x^2*cos(2*c))*cos(2*d*x^2) + 2*(2*(2*a^4*b^2 - 3*a^2*b^4 +
 b^6)*d*x^2*cos(c) - 3*(a^2*b^4 - b^6)*sin(c))*cos(d*x^2) + (3*a*b^5*cos(2*
c) + 2*(2*a^3*b^3 - a*b^5)*d*x^2*sin(2*c))*sin(2*d*x^2) - 2*(2*(2*a^4*b^2 -
 3*a^2*b^4 + b^6)*d*x^2*sin(c) + 3*(a^2*b^4 - b^6)*cos(c))*sin(d*x^2))*cos(
d*x^2 + c) - (3*a^3*b^3*cos(d*x^2 + c) + 3*a^4*b^2 - 2*(2*a^5*b - a^3*b^3)*
d*x^2*sin(d*x^2 + c))*sin(2*d*x^2 + 2*c) + (3*a^3*b^3 - 3*a*b^5 - (3*a*b^5*
cos(2*c) + 2*(2*a^3*b^3 - a*b^5)*d*x^2*sin(2*c))*cos(2*d*x^2) + 2*(2*(2*a^4
*b^2 - 3*a^2*b^4 + b^6)*d*x^2*sin(c) + 3*(a^2*b^4 - b^6)*cos(c))*cos(d*x^2)
 + (3*a*b^5*sin(2*c) - 2*(2*a^3*b^3 - a*b^5)*d*x^2*cos(2*c))*sin(2*d*x^2) +
 2*(2*(2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*x^2*cos(c) - 3*(a^2*b^4 - b^6)*sin(c)
)*sin(d*x^2))*sin(d*x^2 + c))/(a^8*d*x^4*cos(2*d*x^2 + 2*c)^2 + a^8*d*x^4*s
in(2*d*x^2 + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^4*cos(2
*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*
b^4 + a^2*b^6)*sin(c)^2)*d*x^4*cos(d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^
5)*d*x^4*cos(d*x^2)*cos(c) + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^
4*sin(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 -
 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^4*sin(d*x^2)^2 - 4*(a^7*b - 2*a^5*b^3 +
 a^3*b^5)*d*x^4*sin(d*x^2)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*x^4 - 2*(
2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c
))*d*x^4*cos(d*x^2) + (a^6*b^2 - a^4*b^4)*d*x^4*cos(2*c) + 2*((a^5*b^3 - a^
3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^4*sin(d*x
^2))*cos(2*d*x^2) - 2*(a^6*b^2*d*x^4*cos(2*d*x^2)*cos(2*c) - a^6*b^2*d*x^4*
sin(2*d*x^2)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*x^4*cos(d*x^2)*cos(c) + 2*(a^
7*b - a^5*b^3)*d*x^4*sin(d*x^2)*sin(c) - (a^8 - a^6*b^2)*d*x^4)*cos(2*d*x^2
 + 2*c) + 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*c
os(2*c)*sin(c))*d*x^4*cos(d*x^2) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) +
 (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^4*sin(d*x^2) + (a^6*b^2 - a^4*b^4
)*d*x^4*sin(2*c))*sin(2*d*x^2) - 2*(a^6*b^2*d*x^4*cos(2*c)*sin(2*d*x^2) + a
^6*b^2*d*x^4*cos(2*d*x^2)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*x^4*cos(c)*sin(d
*x^2) - 2*(a^7*b - a^5*b^3)*d*x^4*cos(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c)), x
```

```
) + (a^2*b^4*cos(2*c) + (a^4*b^2 - a^2*b^4)*d*x^2*sin(2*c))*sin(2*d*x^2) -
(a^3*b^3*cos(d*x^2 + c) + a^4*b^2 + (a^4*b^2 - a^2*b^4)*d*x^2*cos(2*c)*sin(
2*d*x^2) - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(c)*sin(d*x^2) + (a^4*b^2
 - a^2*b^4)*d*x^2*cos(2*d*x^2)*sin(2*c) - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x
^2*cos(d*x^2)*sin(c) - 2*(a^5*b - a^3*b^3)*d*x^2*sin(d*x^2 + c))*sin(2*d*x^
2 + 2*c) + (a^3*b^3 - a*b^5 - (a*b^5*cos(2*c) + 2*(a^3*b^3 - a*b^5)*d*x^2*s
in(2*c))*cos(2*d*x^2) + 2*(2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2*sin(c) + (a^
2*b^4 - b^6)*cos(c))*cos(d*x^2) + (a*b^5*sin(2*c) - 2*(a^3*b^3 - a*b^5)*d*x
^2*cos(2*c))*sin(2*d*x^2) + 2*(2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2*cos(c) -
 (a^2*b^4 - b^6)*sin(c))*sin(d*x^2))*sin(d*x^2 + c) - 2*((a^5*b - 2*a^3*b^3
 + a*b^5)*d*x^2*sin(c) + (a^3*b^3 - a*b^5)*cos(c))*sin(d*x^2))/(a^8*d*x^3*c
os(2*d*x^2 + 2*c)^2 + a^8*d*x^3*sin(2*d*x^2 + 2*c)^2 + (a^4*b^4*cos(2*c)^2
+ a^4*b^4*sin(2*c)^2)*d*x^3*cos(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*
b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^3*cos(d*x^2)^
2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*cos(d*x^2)*cos(c) + (a^4*b^4*cos(
2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^3*sin(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4
 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^3*sin(
d*x^2)^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*sin(d*x^2)*sin(c) + (a^8 -
 2*a^6*b^2 + a^4*b^4)*d*x^3 - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (
a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^3*cos(d*x^2) + (a^6*b^2 - a^4*b^4)*
d*x^3*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^
5)*cos(2*c)*sin(c))*d*x^3*sin(d*x^2))*cos(2*d*x^2) - 2*(a^6*b^2*d*x^3*cos(2
*d*x^2)*cos(2*c) - a^6*b^2*d*x^3*sin(2*d*x^2)*sin(2*c) - 2*(a^7*b - a^5*b^3
)*d*x^3*cos(d*x^2)*cos(c) + 2*(a^7*b - a^5*b^3)*d*x^3*sin(d*x^2)*sin(c) - (
a^8 - a^6*b^2)*d*x^3)*cos(2*d*x^2 + 2*c) + 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)
*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^3*cos(d*x^2) - 2*((a^5
*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^
3*sin(d*x^2) + (a^6*b^2 - a^4*b^4)*d*x^3*sin(2*c))*sin(2*d*x^2) - 2*(a^6*b^
2*d*x^3*cos(2*c)*sin(2*d*x^2) + a^6*b^2*d*x^3*cos(2*d*x^2)*sin(2*c) - 2*(a^
7*b - a^5*b^3)*d*x^3*cos(c)*sin(d*x^2) - 2*(a^7*b - a^5*b^3)*d*x^3*cos(d*x^
2)*sin(c))*sin(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \left(a+b\sec \left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sec \left(dx^2+c\right)+a\right)^2 x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x^2 + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 14.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^2 \left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{x^2 \left(a+\frac{b}{\cos(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(1/(x^2*(a + b/cos(c + d*x^2))^2),x)
[Out] int(1/(x^2*(a + b/cos(c + d*x^2))^2), x)
```

3.30 $\int \frac{1}{x^3\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^3\left(a+b\sec\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^3/(a+b*sec(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x^3*(a + b*Sec[c + d*x^2])^2),x]
[Out] Defer[Int][1/(x^3*(a + b*Sec[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^3\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 9.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x^3*(a + b*Sec[c + d*x^2])^2),x]
[Out] Integrate[1/(x^3*(a + b*Sec[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(a+b\sec\left(d\,x^2+c\right)\right)^2}dx$$

```
[In] int(1/x^3/(a+b*sec(d*x^2+c))^2,x)
[Out] int(1/x^3/(a+b*sec(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3\left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\sec\left(dx^2+c\right)+a\right)^2x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^3*sec(d*x^2 + c)^2 + 2*a*b*x^3*sec(d*x^2 + c) + a^2*x^3),
 x)
```

Sympy [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 (a + b\sec(c + dx^2))^2} \, dx = \int \frac{1}{x^3 (a + b\sec(c + dx^2))^2} \, dx$$

```
[In] integrate(1/x**3/(a+b*sec(d*x**2+c))**2,x)
```

```
[Out] Integral(1/(x**3*(a + b*sec(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 5.74 (sec) , antiderivative size = 3521, normalized size of antiderivative = 195.61

$$\int \frac{1}{x^3 (a + b\sec(c + dx^2))^2} \, dx = \int \frac{1}{(b\sec(dx^2 + c) + a)^2 x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a^4 - a^2*b^2)*d*x^2 + ((a^4 - a^2*b^2)*d*x^2*cos(2*c) - 2*a^2*b^2*s
in(2*c))*cos(2*d*x^2) + ((a^4 - a^2*b^2)*d*x^2*cos(2*d*x^2)*cos(2*c) + 2*(a
^3*b - a*b^3)*d*x^2*cos(d*x^2)*cos(c) - (a^4 - a^2*b^2)*d*x^2*sin(2*d*x^2)*
sin(2*c) - 2*(a^3*b - a*b^3)*d*x^2*sin(d*x^2)*sin(c) + (a^4 - a^2*b^2)*d*x^
2)*cos(2*d*x^2 + 2*c) + 2*((a^3*b - a*b^3)*d*x^2 + ((a^3*b - a*b^3)*d*x^2*c
os(2*c) - a*b^3*sin(2*c))*cos(2*d*x^2) + 2*((a^2*b^2 - b^4)*d*x^2*cos(c) -
b^4*sin(c))*cos(d*x^2) - (a*b^3*cos(2*c) + (a^3*b - a*b^3)*d*x^2*sin(2*c))*
sin(2*d*x^2) - 2*(b^4*cos(c) + (a^2*b^2 - b^4)*d*x^2*sin(c))*sin(d*x^2))*co
s(d*x^2 + c) + 2*((a^3*b - a*b^3)*d*x^2*cos(c) - 2*a*b^3*sin(c))*cos(d*x^2)
 - 2*(((a^6 - a^4*b^2)*cos(2*c)^2 + (a^6 - a^4*b^2)*sin(2*c)^2)*d*x^4*cos(2
*d*x^2)^2 + 4*((a^4*b^2 - a^2*b^4)*cos(c)^2 + (a^4*b^2 - a^2*b^4)*sin(c)^2)
*d*x^4*cos(d*x^2)^2 + 4*(a^5*b - a^3*b^3)*d*x^4*cos(d*x^2)*cos(c) + ((a^6 -
 a^4*b^2)*cos(2*c)^2 + (a^6 - a^4*b^2)*sin(2*c)^2)*d*x^4*sin(2*d*x^2)^2 + 4
*((a^4*b^2 - a^2*b^4)*cos(c)^2 + (a^4*b^2 - a^2*b^4)*sin(c)^2)*d*x^4*sin(d*
x^2)^2 - 4*(a^5*b - a^3*b^3)*d*x^4*sin(d*x^2)*sin(c) + (a^6 - a^4*b^2)*d*x^
4 + 2*(2*((a^5*b - a^3*b^3)*cos(2*c)*cos(c) + (a^5*b - a^3*b^3)*sin(2*c)*si
n(c))*d*x^4*cos(d*x^2) + (a^6 - a^4*b^2)*d*x^4*cos(2*c) + 2*((a^5*b - a^3*b
^3)*cos(c)*sin(2*c) - (a^5*b - a^3*b^3)*cos(2*c)*sin(c))*d*x^4*sin(d*x^2))*
cos(2*d*x^2) - 2*(2*((a^5*b - a^3*b^3)*cos(c)*sin(2*c) - (a^5*b - a^3*b^3)*
cos(2*c)*sin(c))*d*x^4*cos(d*x^2) - 2*((a^5*b - a^3*b^3)*cos(2*c)*cos(c) +
(a^5*b - a^3*b^3)*sin(2*c)*sin(c))*d*x^4*sin(d*x^2) + (a^6 - a^4*b^2)*d*x^4
*sin(2*c))*sin(2*d*x^2))*integrate(-2*(2*a^2*b^4*cos(2*c)*sin(2*d*x^2) + 2*
```

```
a^2*b^4*cos(2*d*x^2)*sin(2*c) - 4*(a^3*b^3 - a*b^5)*cos(c)*sin(d*x^2) - 4*(
a^3*b^3 - a*b^5)*cos(d*x^2)*sin(c) + (2*a^3*b^3*sin(d*x^2 + c) + (2*a^5*b -
 a^3*b^3)*d*x^2*cos(d*x^2 + c))*cos(2*d*x^2 + 2*c) + ((2*a^5*b - 3*a^3*b^3
+ a*b^5)*d*x^2 + (2*a*b^5*sin(2*c) - (2*a^3*b^3 - a*b^5)*d*x^2*cos(2*c))*co
s(2*d*x^2) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*x^2*cos(c) - 2*(a^2*b^4 - b
^6)*sin(c))*cos(d*x^2) + (2*a*b^5*cos(2*c) + (2*a^3*b^3 - a*b^5)*d*x^2*sin(
2*c))*sin(2*d*x^2) - 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*x^2*sin(c) + 2*(a^2
*b^4 - b^6)*cos(c))*sin(d*x^2))*cos(d*x^2 + c) - (2*a^3*b^3*cos(d*x^2 + c)
+ 2*a^4*b^2 - (2*a^5*b - a^3*b^3)*d*x^2*sin(d*x^2 + c))*sin(2*d*x^2 + 2*c)
+ (2*a^3*b^3 - 2*a*b^5 - (2*a*b^5*cos(2*c) + (2*a^3*b^3 - a*b^5)*d*x^2*sin(
2*c))*cos(2*d*x^2) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*x^2*sin(c) + 2*(a^2
*b^4 - b^6)*cos(c))*cos(d*x^2) + (2*a*b^5*sin(2*c) - (2*a^3*b^3 - a*b^5)*d*
x^2*cos(2*c))*sin(2*d*x^2) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*x^2*cos(c)
- 2*(a^2*b^4 - b^6)*sin(c))*sin(d*x^2))*sin(d*x^2 + c))/(a^8*d*x^5*cos(2*d*
x^2 + 2*c)^2 + a^8*d*x^5*sin(2*d*x^2 + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b
^4*sin(2*c)^2)*d*x^5*cos(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*co
s(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^5*cos(d*x^2)^2 + 4*(
a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^5*cos(d*x^2)*cos(c) + (a^4*b^4*cos(2*c)^2
+ a^4*b^4*sin(2*c)^2)*d*x^5*sin(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*
b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^5*sin(d*x^2)^
2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^5*sin(d*x^2)*sin(c) + (a^8 - 2*a^6*
b^2 + a^4*b^4)*d*x^5 - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3
 - a^3*b^5)*sin(2*c)*sin(c))*d*x^5*cos(d*x^2) + (a^6*b^2 - a^4*b^4)*d*x^5*c
os(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(
2*c)*sin(c))*d*x^5*sin(d*x^2))*cos(2*d*x^2) - 2*(a^6*b^2*d*x^5*cos(2*d*x^2)
*cos(2*c) - a^6*b^2*d*x^5*sin(2*d*x^2)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*x^5
*cos(d*x^2)*cos(c) + 2*(a^7*b - a^5*b^3)*d*x^5*sin(d*x^2)*sin(c) - (a^8 - a
^6*b^2)*d*x^5)*cos(2*d*x^2 + 2*c) + 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*
c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^5*cos(d*x^2) - 2*((a^5*b^3 -
a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^5*sin(d
*x^2) + (a^6*b^2 - a^4*b^4)*d*x^5*sin(2*c))*sin(2*d*x^2) - 2*(a^6*b^2*d*x^5
*cos(2*c)*sin(2*d*x^2) + a^6*b^2*d*x^5*cos(2*d*x^2)*sin(2*c) - 2*(a^7*b - a
^5*b^3)*d*x^5*cos(c)*sin(d*x^2) - 2*(a^7*b - a^5*b^3)*d*x^5*cos(d*x^2)*sin(
c))*sin(2*d*x^2 + 2*c)), x) - (2*a^2*b^2*cos(2*c) + (a^4 - a^2*b^2)*d*x^2*s
in(2*c))*sin(2*d*x^2) + ((a^4 - a^2*b^2)*d*x^2*cos(2*c)*sin(2*d*x^2) + 2*(a
^3*b - a*b^3)*d*x^2*cos(c)*sin(d*x^2) + (a^4 - a^2*b^2)*d*x^2*cos(2*d*x^2)*
sin(2*c) + 2*(a^3*b - a*b^3)*d*x^2*cos(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c) +
2*(a*b^3 + (a*b^3*cos(2*c) + (a^3*b - a*b^3)*d*x^2*sin(2*c))*cos(2*d*x^2) +
 2*(b^4*cos(c) + (a^2*b^2 - b^4)*d*x^2*sin(c))*cos(d*x^2) + ((a^3*b - a*b^3
)*d*x^2*cos(2*c) - a*b^3*sin(2*c))*sin(2*d*x^2) + 2*((a^2*b^2 - b^4)*d*x^2*
cos(c) - b^4*sin(c))*sin(d*x^2))*sin(d*x^2 + c) - 2*(2*a*b^3*cos(c) + (a^3*
b - a*b^3)*d*x^2*sin(c))*sin(d*x^2))/(((a^6 - a^4*b^2)*cos(2*c)^2 + (a^6 -
a^4*b^2)*sin(2*c)^2)*d*x^4*cos(2*d*x^2)^2 + 4*((a^4*b^2 - a^2*b^4)*cos(c)^2
 + (a^4*b^2 - a^2*b^4)*sin(c)^2)*d*x^4*cos(d*x^2)^2 + 4*(a^5*b - a^3*b^3)*d
*x^4*cos(d*x^2)*cos(c) + ((a^6 - a^4*b^2)*cos(2*c)^2 + (a^6 - a^4*b^2)*sin(
```

```
2*c)^2)*d*x^4*sin(2*d*x^2)^2 + 4*((a^4*b^2 - a^2*b^4)*cos(c)^2 + (a^4*b^2 -
 a^2*b^4)*sin(c)^2)*d*x^4*sin(d*x^2)^2 - 4*(a^5*b - a^3*b^3)*d*x^4*sin(d*x^
2)*sin(c) + (a^6 - a^4*b^2)*d*x^4 + 2*(2*((a^5*b - a^3*b^3)*cos(2*c)*cos(c)
 + (a^5*b - a^3*b^3)*sin(2*c)*sin(c))*d*x^4*cos(d*x^2) + (a^6 - a^4*b^2)*d*
x^4*cos(2*c) + 2*((a^5*b - a^3*b^3)*cos(c)*sin(2*c) - (a^5*b - a^3*b^3)*cos
(2*c)*sin(c))*d*x^4*sin(d*x^2))*cos(2*d*x^2) - 2*(2*((a^5*b - a^3*b^3)*cos(
c)*sin(2*c) - (a^5*b - a^3*b^3)*cos(2*c)*sin(c))*d*x^4*cos(d*x^2) - 2*((a^5
*b - a^3*b^3)*cos(2*c)*cos(c) + (a^5*b - a^3*b^3)*sin(2*c)*sin(c))*d*x^4*si
n(d*x^2) + (a^6 - a^4*b^2)*d*x^4*sin(2*c))*sin(2*d*x^2))
```

Giac [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{\left(b\sec\left(dx^2+c\right)+a\right)^2 x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x^2 + c) + a)^2*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 13.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3 \left(a+b\sec\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{x^3 \left(a+\frac{b}{\cos\left(d\,x^2+c\right)}\right)^2}\,dx$$

```
[In] int(1/(x^3*(a + b/cos(c + d*x^2))^2),x)
[Out] int(1/(x^3*(a + b/cos(c + d*x^2))^2), x)
```

3.31 $\int x^3 \left(a + b \sec\left(c + d\sqrt{x}\right)\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 476

$$\int x^3\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^4}{4}-\frac{4ibx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{14ibx^3\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{14ibx^3\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{84bx^{5/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}-\frac{420ibx^2\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{420ibx^2\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}+\frac{5040ibx\operatorname{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{5040ibx\operatorname{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,-ie^{i(c+d\sqrt{x})}\right)}{d^7}+\frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,ie^{i(c+d\sqrt{x})}\right)}{d^7}-\frac{10080ib\operatorname{PolyLog}\left(8,-ie^{i(c+d\sqrt{x})}\right)}{d^8}+\frac{10080ib\operatorname{PolyLog}\left(8,ie^{i(c+d\sqrt{x})}\right)}{d^8}$$

```
[Out] 1/4*a*x^4+5040*I*b*x*polylog(6,-I*exp(I*(c+d*x^(1/2))))/d^6+10080*I*b*polyl
og(8,I*exp(I*(c+d*x^(1/2))))/d^8-4*I*b*x^(7/2)*arctan(exp(I*(c+d*x^(1/2))))
```

```
/d-84*b*x^(5/2)*polylog(3,-I*exp(I*(c+d*x^(1/2))))/d^3+84*b*x^(5/2)*polylog
(3,I*exp(I*(c+d*x^(1/2))))/d^3-10080*I*b*polylog(8,-I*exp(I*(c+d*x^(1/2))))
/d^8+420*I*b*x^2*polylog(4,I*exp(I*(c+d*x^(1/2))))/d^4+1680*b*x^(3/2)*polyl
og(5,-I*exp(I*(c+d*x^(1/2))))/d^5-1680*b*x^(3/2)*polylog(5,I*exp(I*(c+d*x^(
1/2))))/d^5+14*I*b*x^3*polylog(2,-I*exp(I*(c+d*x^(1/2))))/d^2-5040*I*b*x*po
lylog(6,I*exp(I*(c+d*x^(1/2))))/d^6-14*I*b*x^3*polylog(2,I*exp(I*(c+d*x^(1/
2))))/d^2-420*I*b*x^2*polylog(4,-I*exp(I*(c+d*x^(1/2))))/d^4-10080*b*polylo
g(7,-I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^7+10080*b*polylog(7,I*exp(I*(c+d*x^(
1/2))))*x^(1/2)/d^7
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used

$= \{14, 4289, 4266, 2611, 6744, 2320, 6724\}$

$$\int x^3 \left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^4}{4}-\frac{4ibx^{7/2}\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}-\frac{10080ib\,\text{PolyLog}\left(8,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^8}+\frac{10080ib\,\text{PolyLog}\left(8,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^8}-\frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^7}+\frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^7}+\frac{5040ibx\,\text{PolyLog}\left(6,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}-\frac{5040ibx\,\text{PolyLog}\left(6,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}+\frac{1680bx^{3/2}\,\text{PolyLog}\left(5,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{1680bx^{3/2}\,\text{PolyLog}\left(5,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{420ibx^2\,\text{PolyLog}\left(4,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{420ibx^2\,\text{PolyLog}\left(4,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{84bx^{5/2}\,\text{PolyLog}\left(3,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{84bx^{5/2}\,\text{PolyLog}\left(3,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{14ibx^3\,\text{PolyLog}\left(2,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{14ibx^3\,\text{PolyLog}\left(2,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}$$

```
[In] Int[x^3*(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^4)/4 - ((4*I)*b*x^(7/2)*ArcTan[E^(I*(c + d*Sqrt[x]))])/d + ((14*I)*b*x
^3*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((14*I)*b*x^3*PolyLog[2, I
```

```
*E^(I*(c + d*Sqrt[x]))])/d^2 - (84*b*x^(5/2)*PolyLog[3, (-I)*E^(I*(c + d*Sq
rt[x]))])/d^3 + (84*b*x^(5/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 - ((
420*I)*b*x^2*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((420*I)*b*x^2*P
olyLog[4, I*E^(I*(c + d*Sqrt[x]))])/d^4 + (1680*b*x^(3/2)*PolyLog[5, (-I)*E
^(I*(c + d*Sqrt[x]))])/d^5 - (1680*b*x^(3/2)*PolyLog[5, I*E^(I*(c + d*Sqrt[
x]))])/d^5 + ((5040*I)*b*x*PolyLog[6, (-I)*E^(I*(c + d*Sqrt[x]))])/d^6 - ((
5040*I)*b*x*PolyLog[6, I*E^(I*(c + d*Sqrt[x]))])/d^6 - (10080*b*Sqrt[x]*Pol
yLog[7, (-I)*E^(I*(c + d*Sqrt[x]))])/d^7 + (10080*b*Sqrt[x]*PolyLog[7, I*E^
(I*(c + d*Sqrt[x]))])/d^7 - ((10080*I)*b*PolyLog[8, (-I)*E^(I*(c + d*Sqrt[x
]))])/d^8 + ((10080*I)*b*PolyLog[8, I*E^(I*(c + d*Sqrt[x]))])/d^8
```

<u>Rule 14</u>

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

<u>Rule 2320</u>

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

<u>Rule 2611</u>

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

<u>Rule 4266</u>

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

<u>Rule 4289</u>

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
```

```
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax^3 + bx^3 \sec\left(c + d\sqrt{x}\right)\right)\, dx \\
&= \frac{ax^4}{4} + b\int x^3 \sec\left(c + d\sqrt{x}\right)\, dx \\
&= \frac{ax^4}{4} + (2b)\text{Subst}\left(\int x^7 \sec(c+dx)\, dx, x, \sqrt{x}\right) \\
&= \frac{ax^4}{4} - \frac{4ibx^{7/2}\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d} - \frac{(14b)\text{Subst}\left(\int x^6 \log\left(1 - ie^{i(c+dx)}\right)\, dx, x, \sqrt{x}\right)}{d} \\
&\quad + \frac{(14b)\text{Subst}\left(\int x^6 \log\left(1 + ie^{i(c+dx)}\right)\, dx, x, \sqrt{x}\right)}{d} \\
&= \frac{ax^4}{4} - \frac{4ibx^{7/2}\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d} + \frac{14ibx^3\,\text{PolyLog}\left(2, -ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{14ibx^3\,\text{PolyLog}\left(2, ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{(84ib)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2, -ie^{i(c+dx)}\right)\, dx, x, \sqrt{x}\right)}{d^2} \\
&\quad + \frac{(84ib)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2, ie^{i(c+dx)}\right)\, dx, x, \sqrt{x}\right)}{d^2}
\end{aligned}
$$

$$= \frac{ax^4}{4} - \frac{4ibx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{(420b)\operatorname{Subst}\left(\int x^4\operatorname{PolyLog}\left(3,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3} - \frac{(420b)\operatorname{Subst}\left(\int x^4\operatorname{PolyLog}\left(3,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= \frac{ax^4}{4} - \frac{4ibx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{420ibx^2\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{420ibx^2\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(1680ib)\operatorname{Subst}\left(\int x^3\operatorname{PolyLog}\left(4,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(1680ib)\operatorname{Subst}\left(\int x^3\operatorname{PolyLog}\left(4,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$= \frac{ax^4}{4} - \frac{4ibx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{420ibx^2\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{420ibx^2\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{(5040b)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(5,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5} + \frac{(5040b)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(5,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5}$$

$$= \frac{ax^4}{4} - \frac{4ibx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{420ibx^2\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{420ibx^2\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{5040ibx\operatorname{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{5040ibx\operatorname{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{(10080ib)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(6,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^6} + \frac{(10080ib)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(6,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^6}$$

$$= \frac{ax^4}{4} - \frac{4ibx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{420ibx^2\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{420ibx^2\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{5040ibx\operatorname{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{5040ibx\operatorname{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,-ie^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,ie^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{(10080b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(7,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^7} - \frac{(10080b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(7,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^7}$$

$$= \frac{ax^4}{4} - \frac{4ibx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{420ibx^2\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{420ibx^2\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{1680bx^{3/2}\,\text{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{1680bx^{3/2}\,\text{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{5040ibx\,\text{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{5040ibx\,\text{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,-ie^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,ie^{i(c+d\sqrt{x})}\right)}{d^7} - \frac{(10080ib)\text{Subst}\left(\int \frac{\text{PolyLog}(7,-ix)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^8} + \frac{(10080ib)\text{Subst}\left(\int \frac{\text{PolyLog}(7,ix)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^8}$$

$$= \frac{ax^4}{4} - \frac{4ibx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{420ibx^2\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{420ibx^2\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{1680bx^{3/2}\,\text{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{1680bx^{3/2}\,\text{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{5040ibx\,\text{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{5040ibx\,\text{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,-ie^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,ie^{i(c+d\sqrt{x})}\right)}{d^7} - \frac{10080ib\,\text{PolyLog}\left(8,-ie^{i(c+d\sqrt{x})}\right)}{d^8} + \frac{10080ib\,\text{PolyLog}\left(8,ie^{i(c+d\sqrt{x})}\right)}{d^8}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.01

$$\int x^3\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^4}{4}-\frac{4ibx^{7/2}\arctan\left(e^{ic+id\sqrt{x}}\right)}{d}+\frac{14ibx^3\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{14ibx^3\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{84bx^{5/2}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{84bx^{5/2}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}-\frac{420ibx^2\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{420ibx^2\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{1680bx^{3/2}\,\text{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{1680bx^{3/2}\,\text{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}+\frac{5040ibx\,\text{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{5040ibx\,\text{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,-ie^{i(c+d\sqrt{x})}\right)}{d^7}+\frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,ie^{i(c+d\sqrt{x})}\right)}{d^7}-\frac{10080ib\,\text{PolyLog}\left(8,-ie^{i(c+d\sqrt{x})}\right)}{d^8}+\frac{10080ib\,\text{PolyLog}\left(8,ie^{i(c+d\sqrt{x})}\right)}{d^8}$$

```
[In] Integrate[x^3*(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^4)/4 - ((4*I)*b*x^(7/2)*ArcTan[E^(I*c + I*d*Sqrt[x])])/d + ((14*I)*b*x
^3*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((14*I)*b*x^3*PolyLog[2, I
*E^(I*(c + d*Sqrt[x]))])/d^2 - (84*b*x^(5/2)*PolyLog[3, (-I)*E^(I*(c + d*Sq
rt[x]))])/d^3 + (84*b*x^(5/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 - ((
420*I)*b*x^2*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((420*I)*b*x^2*P
olyLog[4, I*E^(I*(c + d*Sqrt[x]))])/d^4 + (1680*b*x^(3/2)*PolyLog[5, (-I)*E
^(I*(c + d*Sqrt[x]))])/d^5 - (1680*b*x^(3/2)*PolyLog[5, I*E^(I*(c + d*Sqrt[
x]))])/d^5 + ((5040*I)*b*x*PolyLog[6, (-I)*E^(I*(c + d*Sqrt[x]))])/d^6 - ((
5040*I)*b*x*PolyLog[6, I*E^(I*(c + d*Sqrt[x]))])/d^6 - (10080*b*Sqrt[x]*Pol
yLog[7, (-I)*E^(I*(c + d*Sqrt[x]))])/d^7 + (10080*b*Sqrt[x]*PolyLog[7, I*E^
(I*(c + d*Sqrt[x]))])/d^7 - ((10080*I)*b*PolyLog[8, (-I)*E^(I*(c + d*Sqrt[x
]))])/d^8 + ((10080*I)*b*PolyLog[8, I*E^(I*(c + d*Sqrt[x]))])/d^8
```

Maple [F]

$$\int x^3\left(a+b\sec\left(c+d\sqrt{x}\right)\right)dx$$

```
[In] int(x^3*(a+b*sec(c+d*x^(1/2))),x)
[Out] int(x^3*(a+b*sec(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^3\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int \left(b\sec\left(d\sqrt{x}+c\right)+a\right)x^3\,dx$$

```
[In] integrate(x^3*(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(b*x^3*sec(d*sqrt(x) + c) + a*x^3, x)
```

Sympy [F]

$$\int x^3\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int x^3\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate(x**3*(a+b*sec(c+d*x**(1/2))),x)
[Out] Integral(x**3*(a + b*sec(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1512 vs. 2(352) = 704.

Time = 0.51 (sec) , antiderivative size = 1512, normalized size of antiderivative = 3.18

$$\int x^3\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x^3*(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/4*((d*sqrt(x) + c)^8*a - 8*(d*sqrt(x) + c)^7*a*c + 28*(d*sqrt(x) + c)^6*a
*c^2 - 56*(d*sqrt(x) + c)^5*a*c^3 + 70*(d*sqrt(x) + c)^4*a*c^4 - 56*(d*sqrt
(x) + c)^3*a*c^5 + 28*(d*sqrt(x) + c)^2*a*c^6 - 8*(d*sqrt(x) + c)*a*c^7 - 8
*b*c^7*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 8*(I*(d*sqrt(x) + c)^
7*b - 7*I*(d*sqrt(x) + c)^6*b*c + 21*I*(d*sqrt(x) + c)^5*b*c^2 - 35*I*(d*sq
rt(x) + c)^4*b*c^3 + 35*I*(d*sqrt(x) + c)^3*b*c^4 - 21*I*(d*sqrt(x) + c)^2*
b*c^5 + 7*I*(d*sqrt(x) + c)*b*c^6)*arctan2(cos(d*sqrt(x) + c), sin(d*sqrt(x
) + c) + 1) - 8*(I*(d*sqrt(x) + c)^7*b - 7*I*(d*sqrt(x) + c)^6*b*c + 21*I*(
d*sqrt(x) + c)^5*b*c^2 - 35*I*(d*sqrt(x) + c)^4*b*c^3 + 35*I*(d*sqrt(x) + c
)^3*b*c^4 - 21*I*(d*sqrt(x) + c)^2*b*c^5 + 7*I*(d*sqrt(x) + c)*b*c^6)*arcta
n2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) - 56*(I*(d*sqrt(x) + c)^6*b
 - 6*I*(d*sqrt(x) + c)^5*b*c + 15*I*(d*sqrt(x) + c)^4*b*c^2 - 20*I*(d*sqrt(
x) + c)^3*b*c^3 + 15*I*(d*sqrt(x) + c)^2*b*c^4 - 6*I*(d*sqrt(x) + c)*b*c^5
+ I*b*c^6)*dilog(I*e^(I*d*sqrt(x) + I*c)) - 56*(-I*(d*sqrt(x) + c)^6*b + 6*
I*(d*sqrt(x) + c)^5*b*c - 15*I*(d*sqrt(x) + c)^4*b*c^2 + 20*I*(d*sqrt(x) +
c)^3*b*c^3 - 15*I*(d*sqrt(x) + c)^2*b*c^4 + 6*I*(d*sqrt(x) + c)*b*c^5 - I*b
*c^6)*dilog(-I*e^(I*d*sqrt(x) + I*c)) + 4*((d*sqrt(x) + c)^7*b - 7*(d*sqrt(
x) + c)^6*b*c + 21*(d*sqrt(x) + c)^5*b*c^2 - 35*(d*sqrt(x) + c)^4*b*c^3 + 3
5*(d*sqrt(x) + c)^3*b*c^4 - 21*(d*sqrt(x) + c)^2*b*c^5 + 7*(d*sqrt(x) + c)*
b*c^6)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*sqrt(x) +
c) + 1) - 4*((d*sqrt(x) + c)^7*b - 7*(d*sqrt(x) + c)^6*b*c + 21*(d*sqrt(x)
+ c)^5*b*c^2 - 35*(d*sqrt(x) + c)^4*b*c^3 + 35*(d*sqrt(x) + c)^3*b*c^4 - 21
*(d*sqrt(x) + c)^2*b*c^5 + 7*(d*sqrt(x) + c)*b*c^6)*log(cos(d*sqrt(x) + c)^
2 + sin(d*sqrt(x) + c)^2 - 2*sin(d*sqrt(x) + c) + 1) + 40320*I*b*polylog(8,
 I*e^(I*d*sqrt(x) + I*c)) - 40320*I*b*polylog(8, -I*e^(I*d*sqrt(x) + I*c))
+ 40320*((d*sqrt(x) + c)*b - b*c)*polylog(7, I*e^(I*d*sqrt(x) + I*c)) - 403
20*((d*sqrt(x) + c)*b - b*c)*polylog(7, -I*e^(I*d*sqrt(x) + I*c)) - 20160*(
I*(d*sqrt(x) + c)^2*b - 2*I*(d*sqrt(x) + c)*b*c + I*b*c^2)*polylog(6, I*e^(
I*d*sqrt(x) + I*c)) - 20160*(-I*(d*sqrt(x) + c)^2*b + 2*I*(d*sqrt(x) + c)*b
*c - I*b*c^2)*polylog(6, -I*e^(I*d*sqrt(x) + I*c)) - 6720*((d*sqrt(x) + c)^
3*b - 3*(d*sqrt(x) + c)^2*b*c + 3*(d*sqrt(x) + c)*b*c^2 - b*c^3)*polylog(5,
 I*e^(I*d*sqrt(x) + I*c)) + 6720*((d*sqrt(x) + c)^3*b - 3*(d*sqrt(x) + c)^2
*b*c + 3*(d*sqrt(x) + c)*b*c^2 - b*c^3)*polylog(5, -I*e^(I*d*sqrt(x) + I*c)
) - 1680*(-I*(d*sqrt(x) + c)^4*b + 4*I*(d*sqrt(x) + c)^3*b*c - 6*I*(d*sqrt(
```

```
x) + c)^2*b*c^2 + 4*I*(d*sqrt(x) + c)*b*c^3 - I*b*c^4)*polylog(4, I*e^(I*d*
sqrt(x) + I*c)) - 1680*(I*(d*sqrt(x) + c)^4*b - 4*I*(d*sqrt(x) + c)^3*b*c +
 6*I*(d*sqrt(x) + c)^2*b*c^2 - 4*I*(d*sqrt(x) + c)*b*c^3 + I*b*c^4)*polylog
(4, -I*e^(I*d*sqrt(x) + I*c)) + 336*((d*sqrt(x) + c)^5*b - 5*(d*sqrt(x) + c
)^4*b*c + 10*(d*sqrt(x) + c)^3*b*c^2 - 10*(d*sqrt(x) + c)^2*b*c^3 + 5*(d*sq
rt(x) + c)*b*c^4 - b*c^5)*polylog(3, I*e^(I*d*sqrt(x) + I*c)) - 336*((d*sqr
t(x) + c)^5*b - 5*(d*sqrt(x) + c)^4*b*c + 10*(d*sqrt(x) + c)^3*b*c^2 - 10*(
d*sqrt(x) + c)^2*b*c^3 + 5*(d*sqrt(x) + c)*b*c^4 - b*c^5)*polylog(3, -I*e^(
I*d*sqrt(x) + I*c)))/d^8
```

Giac [F]

$$\int x^3 \left(a + b \sec \left(c + d\sqrt{x}\right)\right)\, dx = \int \left(b \sec \left(d\sqrt{x} + c\right) + a\right) x^3\, dx$$

```
[In] integrate(x^3*(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*sec(d*sqrt(x) + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \sec \left(c + d\sqrt{x}\right)\right)\, dx = \int x^3 \left(a + \frac{b}{\cos\left(c + d\sqrt{x}\right)}\right)\, dx$$

```
[In] int(x^3*(a + b/cos(c + d*x^(1/2))),x)
[Out] int(x^3*(a + b/cos(c + d*x^(1/2))), x)
```

3.32 $\int x^2 \left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 348

$$\int x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^3}{3}-\frac{4ibx^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{10ibx^2\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{10ibx^2\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{40bx^{3/2}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{40bx^{3/2}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}-\frac{120ibx\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{120ibx\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{240b\sqrt{x}\,\text{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{240b\sqrt{x}\,\text{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}+\frac{240ib\,\text{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{240ib\,\text{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6}$$

```
[Out] 1/3*a*x^3-4*I*b*x^(5/2)*arctan(exp(I*(c+d*x^(1/2))))/d+10*I*b*x^2*polylog(2
,-I*exp(I*(c+d*x^(1/2))))/d^2-10*I*b*x^2*polylog(2,I*exp(I*(c+d*x^(1/2))))/
d^2-40*b*x^(3/2)*polylog(3,-I*exp(I*(c+d*x^(1/2))))/d^3+40*b*x^(3/2)*polylo
g(3,I*exp(I*(c+d*x^(1/2))))/d^3-120*I*b*x*polylog(4,-I*exp(I*(c+d*x^(1/2)))
)/d^4+120*I*b*x*polylog(4,I*exp(I*(c+d*x^(1/2))))/d^4+240*I*b*polylog(6,-I*
exp(I*(c+d*x^(1/2))))/d^6-240*I*b*polylog(6,I*exp(I*(c+d*x^(1/2))))/d^6+240
*b*polylog(5,-I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^5-240*b*polylog(5,I*exp(I*(
c+d*x^(1/2))))*x^(1/2)/d^5
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {14, 4289, 4266, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=&\frac{ax^3}{3}-\frac{4ibx^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}\\&+\frac{240ib\,\text{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6}\\&-\frac{240ib\,\text{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6}\\&+\frac{240b\sqrt{x}\,\text{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}\\&-\frac{240b\sqrt{x}\,\text{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}\\&-\frac{120ibx\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}\\&+\frac{120ibx\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}\\&-\frac{40bx^{3/2}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}\\&+\frac{40bx^{3/2}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}\\&+\frac{10ibx^2\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}\\&-\frac{10ibx^2\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}\end{aligned}$$

```
[In] Int[x^2*(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^3)/3 - ((4*I)*b*x^(5/2)*ArcTan[E^(I*(c + d*Sqrt[x]))])/d + ((10*I)*b*x
^2*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((10*I)*b*x^2*PolyLog[2, I
*E^(I*(c + d*Sqrt[x]))])/d^2 - (40*b*x^(3/2)*PolyLog[3, (-I)*E^(I*(c + d*Sq
rt[x]))])/d^3 + (40*b*x^(3/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 - ((
120*I)*b*x*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((120*I)*b*x*PolyL
og[4, I*E^(I*(c + d*Sqrt[x]))])/d^4 + (240*b*Sqrt[x]*PolyLog[5, (-I)*E^(I*(
c + d*Sqrt[x]))])/d^5 - (240*b*Sqrt[x]*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))])
```

```
/d^5 + ((240*I)*b*PolyLog[6, (-I)*E^(I*(c + d*Sqrt[x]))])/d^6 - ((240*I)*b*
PolyLog[6, I*E^(I*(c + d*Sqrt[x]))])/d^6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^2 + bx^2 \sec\left(c + d\sqrt{x}\right)\right) dx \\
&= \frac{ax^3}{3} + b\int x^2 \sec\left(c + d\sqrt{x}\right) dx \\
&= \frac{ax^3}{3} + (2b)\text{Subst}\left(\int x^5 \sec(c + dx)\, dx, x, \sqrt{x}\right) \\
&= \frac{ax^3}{3} - \frac{4ibx^{5/2}\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d} - \frac{(10b)\text{Subst}\left(\int x^4 \log\left(1 - ie^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d} \\
&\quad + \frac{(10b)\text{Subst}\left(\int x^4 \log\left(1 + ie^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d} \\
&= \frac{ax^3}{3} - \frac{4ibx^{5/2}\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d} + \frac{10ibx^2\,\text{PolyLog}\left(2, -ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{10ibx^2\,\text{PolyLog}\left(2, ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{(40ib)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2, -ie^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^2} \\
&\quad + \frac{(40ib)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2, ie^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^2} \\
&= \frac{ax^3}{3} - \frac{4ibx^{5/2}\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d} + \frac{10ibx^2\,\text{PolyLog}\left(2, -ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{10ibx^2\,\text{PolyLog}\left(2, ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{40bx^{3/2}\,\text{PolyLog}\left(3, -ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3} \\
&\quad + \frac{40bx^{3/2}\,\text{PolyLog}\left(3, ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3} \\
&\quad + \frac{(120b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3, -ie^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^3} \\
&\quad - \frac{(120b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3, ie^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^3}
\end{aligned}$$

$$= \frac{ax^3}{3} - \frac{4ibx^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{10ibx^2\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{10ibx^2\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{120ibx\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{120ibx\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(240ib)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(4,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(240ib)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(4,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$= \frac{ax^3}{3} - \frac{4ibx^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{10ibx^2\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{10ibx^2\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{120ibx\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{120ibx\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{(240b)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(5,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5} + \frac{(240b)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(5,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5}$$

$$
\begin{aligned}
&= \frac{ax^3}{3} - \frac{4ibx^{5/2}\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d} + \frac{10ibx^2\,\mathrm{PolyLog}\left(2,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{10ibx^2\,\mathrm{PolyLog}\left(2,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{40bx^{3/2}\,\mathrm{PolyLog}\left(3,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3} \\
&\quad + \frac{40bx^{3/2}\,\mathrm{PolyLog}\left(3,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3} - \frac{120ibx\,\mathrm{PolyLog}\left(4,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4} \\
&\quad + \frac{120ibx\,\mathrm{PolyLog}\left(4,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4} + \frac{240b\sqrt{x}\,\mathrm{PolyLog}\left(5,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^5} \\
&\quad - \frac{240b\sqrt{x}\,\mathrm{PolyLog}\left(5,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^5} + \frac{(240ib)\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(5,-ix)}{x}\,dx, x, e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6} \\
&\quad - \frac{(240ib)\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(5,ix)}{x}\,dx, x, e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ax^3}{3} - \frac{4ibx^{5/2}\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d} + \frac{10ibx^2\,\mathrm{PolyLog}\left(2,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{10ibx^2\,\mathrm{PolyLog}\left(2,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{40bx^{3/2}\,\mathrm{PolyLog}\left(3,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3} \\
&\quad + \frac{40bx^{3/2}\,\mathrm{PolyLog}\left(3,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3} - \frac{120ibx\,\mathrm{PolyLog}\left(4,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4} \\
&\quad + \frac{120ibx\,\mathrm{PolyLog}\left(4,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4} + \frac{240b\sqrt{x}\,\mathrm{PolyLog}\left(5,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^5} \\
&\quad - \frac{240b\sqrt{x}\,\mathrm{PolyLog}\left(5,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^5} \\
&\quad + \frac{240ib\,\mathrm{PolyLog}\left(6,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^6} - \frac{240ib\,\mathrm{PolyLog}\left(6,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.01

$$\int x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^3}{3}-\frac{4ibx^{5/2}\arctan\left(e^{ic+id\sqrt{x}}\right)}{d}+\frac{10ibx^2\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{10ibx^2\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{40bx^{3/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}-\frac{120ibx\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{120ibx\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}+\frac{240ib\operatorname{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{240ib\operatorname{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6}$$

```
[In] Integrate[x^2*(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^3)/3 - ((4*I)*b*x^(5/2)*ArcTan[E^(I*c + I*d*Sqrt[x])])/d + ((10*I)*b*x
^2*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((10*I)*b*x^2*PolyLog[2, I
*E^(I*(c + d*Sqrt[x]))])/d^2 - (40*b*x^(3/2)*PolyLog[3, (-I)*E^(I*(c + d*Sq
rt[x]))])/d^3 + (40*b*x^(3/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 - ((
120*I)*b*x*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((120*I)*b*x*PolyL
og[4, I*E^(I*(c + d*Sqrt[x]))])/d^4 + (240*b*Sqrt[x]*PolyLog[5, (-I)*E^(I*(
c + d*Sqrt[x]))])/d^5 - (240*b*Sqrt[x]*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))])
/d^5 + ((240*I)*b*PolyLog[6, (-I)*E^(I*(c + d*Sqrt[x]))])/d^6 - ((240*I)*b*
PolyLog[6, I*E^(I*(c + d*Sqrt[x]))])/d^6
```

Maple [F]

$$\int x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)dx$$

```
[In] int(x^2*(a+b*sec(c+d*x^(1/2))),x)
[Out] int(x^2*(a+b*sec(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\sec\left(d\sqrt{x}+c\right)+a\right)x^2\,dx$$

```
[In] integrate(x^2*(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(b*x^2*sec(d*sqrt(x) + c) + a*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate(x**2*(a+b*sec(c+d*x**(1/2))),x)
[Out] Integral(x**2*(a + b*sec(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 966 vs. $2(256) = 512$.

Time = 0.47 (sec) , antiderivative size = 966, normalized size of antiderivative = 2.78

$$\int x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x^2*(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")
[Out] 1/3*((d*sqrt(x) + c)^6*a - 6*(d*sqrt(x) + c)^5*a*c + 15*(d*sqrt(x) + c)^4*a
*c^2 - 20*(d*sqrt(x) + c)^3*a*c^3 + 15*(d*sqrt(x) + c)^2*a*c^4 - 6*(d*sqrt(
x) + c)*a*c^5 - 6*b*c^5*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 6*(I
*(d*sqrt(x) + c)^5*b - 5*I*(d*sqrt(x) + c)^4*b*c + 10*I*(d*sqrt(x) + c)^3*b
*c^2 - 10*I*(d*sqrt(x) + c)^2*b*c^3 + 5*I*(d*sqrt(x) + c)*b*c^4)*arctan2(co
s(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) - 6*(I*(d*sqrt(x) + c)^5*b - 5*I*
```

```
(d*sqrt(x) + c)^4*b*c + 10*I*(d*sqrt(x) + c)^3*b*c^2 - 10*I*(d*sqrt(x) + c)
^2*b*c^3 + 5*I*(d*sqrt(x) + c)*b*c^4)*arctan2(cos(d*sqrt(x) + c), -sin(d*sq
rt(x) + c) + 1) - 30*(I*(d*sqrt(x) + c)^4*b - 4*I*(d*sqrt(x) + c)^3*b*c + 6
*I*(d*sqrt(x) + c)^2*b*c^2 - 4*I*(d*sqrt(x) + c)*b*c^3 + I*b*c^4)*dilog(I*e
^(I*d*sqrt(x) + I*c)) - 30*(-I*(d*sqrt(x) + c)^4*b + 4*I*(d*sqrt(x) + c)^3*
b*c - 6*I*(d*sqrt(x) + c)^2*b*c^2 + 4*I*(d*sqrt(x) + c)*b*c^3 - I*b*c^4)*di
log(-I*e^(I*d*sqrt(x) + I*c)) + 3*((d*sqrt(x) + c)^5*b - 5*(d*sqrt(x) + c)^
4*b*c + 10*(d*sqrt(x) + c)^3*b*c^2 - 10*(d*sqrt(x) + c)^2*b*c^3 + 5*(d*sqrt
(x) + c)*b*c^4)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*s
qrt(x) + c) + 1) - 3*((d*sqrt(x) + c)^5*b - 5*(d*sqrt(x) + c)^4*b*c + 10*(d
*sqrt(x) + c)^3*b*c^2 - 10*(d*sqrt(x) + c)^2*b*c^3 + 5*(d*sqrt(x) + c)*b*c^
4)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*sin(d*sqrt(x) + c) +
 1) - 720*I*b*polylog(6, I*e^(I*d*sqrt(x) + I*c)) + 720*I*b*polylog(6, -I*e
^(I*d*sqrt(x) + I*c)) - 720*((d*sqrt(x) + c)*b - b*c)*polylog(5, I*e^(I*d*s
qrt(x) + I*c)) + 720*((d*sqrt(x) + c)*b - b*c)*polylog(5, -I*e^(I*d*sqrt(x)
 + I*c)) - 360*(-I*(d*sqrt(x) + c)^2*b + 2*I*(d*sqrt(x) + c)*b*c - I*b*c^2)
*polylog(4, I*e^(I*d*sqrt(x) + I*c)) - 360*(I*(d*sqrt(x) + c)^2*b - 2*I*(d*
sqrt(x) + c)*b*c + I*b*c^2)*polylog(4, -I*e^(I*d*sqrt(x) + I*c)) + 120*((d*
sqrt(x) + c)^3*b - 3*(d*sqrt(x) + c)^2*b*c + 3*(d*sqrt(x) + c)*b*c^2 - b*c^
3)*polylog(3, I*e^(I*d*sqrt(x) + I*c)) - 120*((d*sqrt(x) + c)^3*b - 3*(d*sq
rt(x) + c)^2*b*c + 3*(d*sqrt(x) + c)*b*c^2 - b*c^3)*polylog(3, -I*e^(I*d*sq
rt(x) + I*c)))/d^6
```

Giac [F]

$$\int x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\sec\left(d\sqrt{x}+c\right)+a\right)x^2\,dx$$

```
[In] integrate(x^2*(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*sec(d*sqrt(x) + c) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int x^2\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x^2*(a + b/cos(c + d*x^(1/2))),x)
[Out] int(x^2*(a + b/cos(c + d*x^(1/2))), x)
```

3.33 $\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 220

$$\begin{aligned}\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=&\ \frac{ax^2}{2}-\frac{4ibx^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}\\&+\frac{6ibx\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}\\&-\frac{6ibx\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}\\&-\frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}\\&+\frac{12b\sqrt{x}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}\\&-\frac{12ib\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}\\&+\frac{12ib\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}\end{aligned}$$

```
[Out] 1/2*a*x^2-4*I*b*x^(3/2)*arctan(exp(I*(c+d*x^(1/2))))/d+6*I*b*x*polylog(2,-I
*exp(I*(c+d*x^(1/2))))/d^2-6*I*b*x*polylog(2,I*exp(I*(c+d*x^(1/2))))/d^2-12
*I*b*polylog(4,-I*exp(I*(c+d*x^(1/2))))/d^4+12*I*b*polylog(4,I*exp(I*(c+d*x
^(1/2))))/d^4-12*b*polylog(3,-I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^3+12*b*poly
log(3,I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {14, 4289, 4266, 2611, 6744, 2320, 6724}

$$\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^2}{2}-\frac{4ibx^{3/2}\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}-\frac{12ib\,\text{PolyLog}\left(4,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{12ib\,\text{PolyLog}\left(4,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{12b\sqrt{x}\,\text{PolyLog}\left(3,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{6ibx\,\text{PolyLog}\left(2,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{6ibx\,\text{PolyLog}\left(2,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}$$

```
[In] Int[x*(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^2)/2 - ((4*I)*b*x^(3/2)*ArcTan[E^(I*(c + d*Sqrt[x]))])/d + ((6*I)*b*x*
PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((6*I)*b*x*PolyLog[2, I*E^(I*
(c + d*Sqrt[x]))])/d^2 - (12*b*Sqrt[x]*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x])
)])/d^3 + (12*b*Sqrt[x]*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 - ((12*I)*
b*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((12*I)*b*PolyLog[4, I*E^(I
*(c + d*Sqrt[x]))])/d^4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \left(ax + bx\sec\left(c + d\sqrt{x}\right)\right)\,dx$$
$$= \frac{ax^2}{2} + b\int x\sec\left(c + d\sqrt{x}\right)\,dx$$

$$= \frac{ax^2}{2} + (2b)\text{Subst}\left(\int x^3 \sec(c+dx)\,dx, x, \sqrt{x}\right)$$

$$= \frac{ax^2}{2} - \frac{4ibx^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{(6b)\text{Subst}\left(\int x^2 \log\left(1-ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{(6b)\text{Subst}\left(\int x^2 \log\left(1+ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d}$$

$$= \frac{ax^2}{2} - \frac{4ibx^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6ibx\,\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ibx\,\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{(12ib)\text{Subst}\left(\int x\,\text{PolyLog}\left(2, -ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2} + \frac{(12ib)\text{Subst}\left(\int x\,\text{PolyLog}\left(2, ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$

$$= \frac{ax^2}{2} - \frac{4ibx^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6ibx\,\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ibx\,\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{(12b)\text{Subst}\left(\int \text{PolyLog}\left(3, -ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3} - \frac{(12b)\text{Subst}\left(\int \text{PolyLog}\left(3, ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3}$$

$$= \frac{ax^2}{2} - \frac{4ibx^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6ibx\,\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ibx\,\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{(12ib)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-ix)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(12ib)\text{Subst}\left(\int \frac{\text{PolyLog}(3,ix)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^4}$$

$$= \frac{ax^2}{2} - \frac{4ibx^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6ibx\,\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ibx\,\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{12ib\,\text{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{12ib\,\text{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.01

$$\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx = \frac{ax^2}{2} - \frac{4ibx^{3/2}\arctan\left(e^{ic+id\sqrt{x}}\right)}{d} + \frac{6ibx\,\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ibx\,\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{12ib\,\text{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{12ib\,\text{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4}$$

```
[In] Integrate[x*(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^2)/2 - ((4*I)*b*x^(3/2)*ArcTan[E^(I*c + I*d*Sqrt[x])])/d + ((6*I)*b*x*
PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((6*I)*b*x*PolyLog[2, I*E^(I*
(c + d*Sqrt[x]))])/d^2 - (12*b*Sqrt[x]*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x])
)])/d^3 + (12*b*Sqrt[x]*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 - ((12*I)*
b*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((12*I)*b*PolyLog[4, I*E^(I
*(c + d*Sqrt[x]))])/d^4
```

Maple [F]

$$\int x(a+b\sec(c+d\sqrt{x}))\,dx$$

[In] int(x*(a+b*sec(c+d*x^(1/2))),x)

[Out] int(x*(a+b*sec(c+d*x^(1/2))),x)

Fricas [F]

$$\int x(a+b\sec(c+d\sqrt{x}))\,dx = \int (b\sec(d\sqrt{x}+c)+a)x\,dx$$

[In] integrate(x*(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")

[Out] integral(b*x*sec(d*sqrt(x) + c) + a*x, x)

Sympy [F]

$$\int x(a+b\sec(c+d\sqrt{x}))\,dx = \int x(a+b\sec(c+d\sqrt{x}))\,dx$$

[In] integrate(x*(a+b*sec(c+d*x**(1/2))),x)

[Out] Integral(x*(a + b*sec(c + d*sqrt(x))), x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(160) = 320$.

Time = 0.41 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.45

$$\int x(a+b\sec(c+d\sqrt{x}))\,dx = \frac{(d\sqrt{x}+c)^4a-4(d\sqrt{x}+c)^3ac+6(d\sqrt{x}+c)^2ac^2-4(d\sqrt{x}+c)ac^3-4bc^3\log(\sec(d\sqrt{x}+c)+\tan(d}{}$$

[In] integrate(x*(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")

[Out] 1/2*((d*sqrt(x) + c)^4*a - 4*(d*sqrt(x) + c)^3*a*c + 6*(d*sqrt(x) + c)^2*a*c^2 - 4*(d*sqrt(x) + c)*a*c^3 - 4*b*c^3*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 4*(I*(d*sqrt(x) + c)^3*b - 3*I*(d*sqrt(x) + c)^2*b*c + 3*I*(d*s

```
qrt(x) + c)*b*c^2)*arctan2(cos(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) - 4*
(I*(d*sqrt(x) + c)^3*b - 3*I*(d*sqrt(x) + c)^2*b*c + 3*I*(d*sqrt(x) + c)*b*
c^2)*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) - 12*(I*(d*sqrt(x
) + c)^2*b - 2*I*(d*sqrt(x) + c)*b*c + I*b*c^2)*dilog(I*e^(I*d*sqrt(x) + I*
c)) - 12*(-I*(d*sqrt(x) + c)^2*b + 2*I*(d*sqrt(x) + c)*b*c - I*b*c^2)*dilog
(-I*e^(I*d*sqrt(x) + I*c)) + 2*((d*sqrt(x) + c)^3*b - 3*(d*sqrt(x) + c)^2*b
*c + 3*(d*sqrt(x) + c)*b*c^2)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)
^2 + 2*sin(d*sqrt(x) + c) + 1) - 2*((d*sqrt(x) + c)^3*b - 3*(d*sqrt(x) + c)
^2*b*c + 3*(d*sqrt(x) + c)*b*c^2)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x)
+ c)^2 - 2*sin(d*sqrt(x) + c) + 1) + 24*I*b*polylog(4, I*e^(I*d*sqrt(x) + I
*c)) - 24*I*b*polylog(4, -I*e^(I*d*sqrt(x) + I*c)) + 24*((d*sqrt(x) + c)*b
- b*c)*polylog(3, I*e^(I*d*sqrt(x) + I*c)) - 24*((d*sqrt(x) + c)*b - b*c)*p
olylog(3, -I*e^(I*d*sqrt(x) + I*c)))/d^4
```

Giac [F]

$$\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\sec\left(d\sqrt{x}+c\right)+a\right)x\,dx$$

```
[In] integrate(x*(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*sec(d*sqrt(x) + c) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int x\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x*(a + b/cos(c + d*x^(1/2))),x)
[Out] int(x*(a + b/cos(c + d*x^(1/2))), x)
```

3.34 $\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x}\,dx = a\log(x)+b\text{Int}\left(\frac{\sec\left(c+d\sqrt{x}\right)}{x},x\right)$$

```
[Out] a*ln(x)+b*Unintegrable(sec(c+d*x^(1/2))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x}\,dx = \int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x}\,dx$$

```
[In] Int[(a + b*Sec[c + d*Sqrt[x]])/x,x]
[Out] a*Log[x] + b*Defer[Int][Sec[c + d*Sqrt[x]]/x, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a}{x}+\frac{b\sec\left(c+d\sqrt{x}\right)}{x}\right)dx \\ &= a\log(x)+b\int \frac{\sec\left(c+d\sqrt{x}\right)}{x}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*Sqrt[x]])/x,x]
[Out] Integrate[(a + b*Sec[c + d*Sqrt[x]])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x}dx$$

```
[In] int((a+b*sec(c+d*x^(1/2)))/x,x)
[Out] int((a+b*sec(c+d*x^(1/2)))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x,x, algorithm="fricas")
[Out] integral((b*sec(d*sqrt(x) + c) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate((a+b*sec(c+d*x**(1/2)))/x,x)
[Out] Integral((a + b*sec(c + d*sqrt(x)))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 106, normalized size of antiderivative = 5.89

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x,x, algorithm="maxima")
```

```
[Out] 2*b*integrate((cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + sin(2*d*sqrt(x)
+ 2*c)*sin(d*sqrt(x) + c) + cos(d*sqrt(x) + c))/((cos(2*d*sqrt(x) + 2*c)^2
+ sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1)*x), x) + a*log(x
)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*sqrt(x) + c) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 13.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{a+\frac{b}{\cos\left(c+d\,\sqrt{x}\right)}}{x}\,dx$$

```
[In] int((a + b/cos(c + d*x^(1/2)))/x,x)
```

```
[Out] int((a + b/cos(c + d*x^(1/2)))/x, x)
```

3.35 $\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx=-\frac{a}{x}+b\text{Int}\left(\frac{\sec\left(c+d\sqrt{x}\right)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(sec(c+d*x^(1/2))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] Int[(a + b*Sec[c + d*Sqrt[x]])/x^2,x]
[Out] -(a/x) + b*Defer[Int][Sec[c + d*Sqrt[x]]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x^2}+\frac{b\sec\left(c+d\sqrt{x}\right)}{x^2}\right)dx\\&=-\frac{a}{x}+b\int \frac{\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 12.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^2,x]
[Out] Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}dx$$

```
[In] int((a+b*sec(c+d*x^(1/2)))/x^2,x)
[Out] int((a+b*sec(c+d*x^(1/2)))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^2,x, algorithm="fricas")
[Out] integral((b*sec(d*sqrt(x) + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] integrate((a+b*sec(c+d*x**(1/2)))/x**2,x)
[Out] Integral((a + b*sec(c + d*sqrt(x)))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.11

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx = \int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^2,x, algorithm="maxima")
[Out] (2*b*x*integrate((cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + sin(2*d*sqrt(
x) + 2*c)*sin(d*sqrt(x) + c) + cos(d*sqrt(x) + c))/((cos(2*d*sqrt(x) + 2*c)
^2 + sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1)*x^2), x) - a)
/x
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx = \int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^2,x, algorithm="giac")
[Out] integrate((b*sec(d*sqrt(x) + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 13.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx = \int \frac{a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}}{x^2}\,dx$$

```
[In] int((a + b/cos(c + d*x^(1/2)))/x^2,x)
[Out] int((a + b/cos(c + d*x^(1/2)))/x^2, x)
```

3.36 $\int x^3\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 749

$$\int x^3\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=-\frac{2ib^2x^{7/2}}{d}+\frac{a^2x^4}{4}-\frac{8iabx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}$$
$$+\frac{14b^2x^3\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+\frac{28iabx^3\,\mathrm{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$-\frac{28iabx^3\,\mathrm{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$-\frac{42ib^2x^{5/2}\,\mathrm{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3}$$
$$-\frac{168abx^{5/2}\,\mathrm{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+\frac{168abx^{5/2}\,\mathrm{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+\frac{105b^2x^2\,\mathrm{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4}$$
$$-\frac{840iabx^2\,\mathrm{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+\frac{840iabx^2\,\mathrm{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+\frac{210ib^2x^{3/2}\,\mathrm{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)}{d^5}$$
$$+\frac{3360abx^{3/2}\,\mathrm{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}$$
$$-\frac{3360abx^{3/2}\,\mathrm{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}$$
$$-\frac{315b^2x\,\mathrm{PolyLog}\left(5,-e^{2i(c+d\sqrt{x})}\right)}{d^6}$$
$$+\frac{10080iabx\,\mathrm{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6}$$
$$-\frac{10080iabx\,\mathrm{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6}$$
$$-\frac{315ib^2\sqrt{x}\,\mathrm{PolyLog}\left(6,-e^{2i(c+d\sqrt{x})}\right)}{d^7}$$
$$-\frac{20160ab\sqrt{x}\,\mathrm{PolyLog}\left(7,-ie^{i(c+d\sqrt{x})}\right)}{d^7}$$

```
[Out] 315/2*b^2*polylog(7,-exp(2*I*(c+d*x^(1/2))))/d^8+28*I*a*b*x^3*polylog(2,-I*
exp(I*(c+d*x^(1/2))))/d^2+840*I*a*b*x^2*polylog(4,I*exp(I*(c+d*x^(1/2))))/d
^4+10080*I*a*b*x*polylog(6,-I*exp(I*(c+d*x^(1/2))))/d^6+14*b^2*x^3*ln(1+exp
(2*I*(c+d*x^(1/2))))/d^2+105*b^2*x^2*polylog(3,-exp(2*I*(c+d*x^(1/2))))/d^4
-315*b^2*x*polylog(5,-exp(2*I*(c+d*x^(1/2))))/d^6+2*b^2*x^(7/2)*tan(c+d*x^(
1/2))/d-2*I*b^2*x^(7/2)/d-8*I*a*b*x^(7/2)*arctan(exp(I*(c+d*x^(1/2))))/d-28
*I*a*b*x^3*polylog(2,I*exp(I*(c+d*x^(1/2))))/d^2-840*I*a*b*x^2*polylog(4,-I
*exp(I*(c+d*x^(1/2))))/d^4+20160*I*a*b*polylog(8,I*exp(I*(c+d*x^(1/2))))/d^
8+3360*a*b*x^(3/2)*polylog(5,-I*exp(I*(c+d*x^(1/2))))/d^5-3360*a*b*x^(3/2)*
polylog(5,I*exp(I*(c+d*x^(1/2))))/d^5-20160*a*b*polylog(7,-I*exp(I*(c+d*x^(
1/2))))*x^(1/2)/d^7+20160*a*b*polylog(7,I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^7
-168*a*b*x^(5/2)*polylog(3,-I*exp(I*(c+d*x^(1/2))))/d^3+168*a*b*x^(5/2)*pol
ylog(3,I*exp(I*(c+d*x^(1/2))))/d^3-42*I*b^2*x^(5/2)*polylog(2,-exp(2*I*(c+d
*x^(1/2))))/d^3-20160*I*a*b*polylog(8,-I*exp(I*(c+d*x^(1/2))))/d^8-315*I*b^
2*polylog(6,-exp(2*I*(c+d*x^(1/2))))*x^(1/2)/d^7-10080*I*a*b*x*polylog(6,I*
exp(I*(c+d*x^(1/2))))/d^6+210*I*b^2*x^(3/2)*polylog(4,-exp(2*I*(c+d*x^(1/2)
)))/d^5+1/4*a^2*x^4
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 749, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {4289, 4275, 4266, 2611, 6744, 2320, 6724, 4269, 3800, 2221}

$$\int x^3\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{a^2x^4}{4}-\frac{8iabx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{20160iab\,\mathrm{PolyLog}\left(8,-ie^{i(c+d\sqrt{x})}\right)}{d^8}+\frac{20160iab\,\mathrm{PolyLog}\left(8,ie^{i(c+d\sqrt{x})}\right)}{d^8}-\frac{20160ab\sqrt{x}\,\mathrm{PolyLog}\left(7,-ie^{i(c+d\sqrt{x})}\right)}{d^7}+\frac{20160ab\sqrt{x}\,\mathrm{PolyLog}\left(7,ie^{i(c+d\sqrt{x})}\right)}{d^7}+\frac{10080iabx\,\mathrm{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{10080iabx\,\mathrm{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6}+\frac{3360abx^{3/2}\,\mathrm{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{3360abx^{3/2}\,\mathrm{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{840iabx^2\,\mathrm{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{840iabx^2\,\mathrm{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}-\frac{168abx^{5/2}\,\mathrm{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{168abx^{5/2}\,\mathrm{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{28iabx^3\,\mathrm{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{28iabx^3\,\mathrm{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}+\frac{315b^2\,\mathrm{PolyLog}\left(7,-e^{2i(c+d\sqrt{x})}\right)}{2d^8}-\frac{315ib^2\sqrt{x}\,\mathrm{PolyLog}\left(6,-e^{2i(c+d\sqrt{x})}\right)}{d^7}-\frac{315b^2x\,\mathrm{PolyLog}\left(5,-e^{2i(c+d\sqrt{x})}\right)}{d^6}$$

$$210ib^2x^{3/2}\,\mathrm{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)$$

```
[In] Int[x^3*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x^(7/2))/d + (a^2*x^4)/4 - ((8*I)*a*b*x^(7/2)*ArcTan[E^(I*(c +
d*Sqrt[x]))])/d + (14*b^2*x^3*Log[1 + E^((2*I)*(c + d*Sqrt[x]))])/d^2 + ((2
8*I)*a*b*x^3*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((28*I)*a*b*x^3*
PolyLog[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - ((42*I)*b^2*x^(5/2)*PolyLog[2, -
E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (168*a*b*x^(5/2)*PolyLog[3, (-I)*E^(I*(c
+ d*Sqrt[x]))])/d^3 + (168*a*b*x^(5/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])
/d^3 + (105*b^2*x^2*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))])/d^4 - ((840*I)*
a*b*x^2*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((840*I)*a*b*x^2*Poly
Log[4, I*E^(I*(c + d*Sqrt[x]))])/d^4 + ((210*I)*b^2*x^(3/2)*PolyLog[4, -E^(
(2*I)*(c + d*Sqrt[x]))])/d^5 + (3360*a*b*x^(3/2)*PolyLog[5, (-I)*E^(I*(c +
d*Sqrt[x]))])/d^5 - (3360*a*b*x^(3/2)*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))])/
d^5 - (315*b^2*x*PolyLog[5, -E^((2*I)*(c + d*Sqrt[x]))])/d^6 + ((10080*I)*a
*b*x*PolyLog[6, (-I)*E^(I*(c + d*Sqrt[x]))])/d^6 - ((10080*I)*a*b*x*PolyLog
[6, I*E^(I*(c + d*Sqrt[x]))])/d^6 - ((315*I)*b^2*Sqrt[x]*PolyLog[6, -E^((2*
I)*(c + d*Sqrt[x]))])/d^7 - (20160*a*b*Sqrt[x]*PolyLog[7, (-I)*E^(I*(c + d*
Sqrt[x]))])/d^7 + (20160*a*b*Sqrt[x]*PolyLog[7, I*E^(I*(c + d*Sqrt[x]))])/d
^7 + (315*b^2*PolyLog[7, -E^((2*I)*(c + d*Sqrt[x]))])/(2*d^8) - ((20160*I)*
a*b*PolyLog[8, (-I)*E^(I*(c + d*Sqrt[x]))])/d^8 + ((20160*I)*a*b*PolyLog[8,
 I*E^(I*(c + d*Sqrt[x]))])/d^8 + (2*b^2*x^(7/2)*Tan[c + d*Sqrt[x]])/d
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^7(a+b\sec(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(a^2x^7+2abx^7\sec(c+dx)+b^2x^7\sec^2(c+dx)\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{a^2x^4}{4}+(4ab)\text{Subst}\left(\int x^7\sec(c+dx)\,dx, x, \sqrt{x}\right)+\left(2b^2\right)\text{Subst}\left(\int x^7\sec^2(c+dx)\,dx, x, \sqrt{x}\right)\\
&= \frac{a^2x^4}{4}-\frac{8iabx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{2b^2x^{7/2}\tan\left(c+d\sqrt{x}\right)}{d}\\
&\quad-\frac{(28ab)\text{Subst}\left(\int x^6\log\left(1-ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad+\frac{(28ab)\text{Subst}\left(\int x^6\log\left(1+ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad-\frac{\left(14b^2\right)\text{Subst}\left(\int x^6\tan(c+dx)\,dx, x, \sqrt{x}\right)}{d}\\
&= -\frac{2ib^2x^{7/2}}{d}+\frac{a^2x^4}{4}-\frac{8iabx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}\\
&\quad+\frac{28iabx^3\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{28iabx^3\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}\\
&\quad+\frac{2b^2x^{7/2}\tan\left(c+d\sqrt{x}\right)}{d}-\frac{(168iab)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}\\
&\quad+\frac{(168iab)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}\\
&\quad+\frac{\left(28ib^2\right)\text{Subst}\left(\int \frac{e^{2i(c+dx)}x^6}{1+e^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{d}\\
&= -\frac{2ib^2x^{7/2}}{d}+\frac{a^2x^4}{4}-\frac{8iabx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{14b^2x^3\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}\\
&\quad+\frac{28iabx^3\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{28iabx^3\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}\\
&\quad-\frac{168abx^{5/2}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{168abx^{5/2}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}\\
&\quad+\frac{2b^2x^{7/2}\tan\left(c+d\sqrt{x}\right)}{d}+\frac{(840ab)\text{Subst}\left(\int x^4\,\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3}\\
&\quad-\frac{(840ab)\text{Subst}\left(\int x^4\,\text{PolyLog}\left(3,ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3}\\
&\quad-\frac{\left(84b^2\right)\text{Subst}\left(\int x^5\log\left(1+e^{2i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}
\end{aligned}
$$

$$= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8iabx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14b^2x^3\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{28iabx^3\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{28iabx^3\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{42ib^2x^{5/2}\,\text{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{168abx^{5/2}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{168abx^{5/2}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{840iabx^2\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+ \frac{840iabx^2\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{2b^2x^{7/2}\tan\left(c+d\sqrt{x}\right)}{d}$$
$$+ \frac{(3360iab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(4,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$
$$- \frac{(3360iab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(4,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$
$$+ \frac{(210ib^2)\,\text{Subst}\left(\int x^4\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8iabx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}$$
$$+ \frac{14b^2x^3\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{28iabx^3\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{28iabx^3\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{42ib^2x^{5/2}\,\text{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3}$$
$$- \frac{168abx^{5/2}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{168abx^{5/2}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{105b^2x^2\,\text{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{840iabx^2\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+ \frac{840iabx^2\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}$$
$$- \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{2b^2x^{7/2}\tan\left(c+d\sqrt{x}\right)}{d}$$
$$- \frac{(10080ab)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(5,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5}$$
$$+ \frac{(10080ab)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(5,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5}$$
$$- \frac{(420b^2)\,\text{Subst}\left(\int x^3\,\text{PolyLog}\left(3,-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8iabx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14b^2x^3\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{28iabx^3\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{28iabx^3\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{42ib^2x^{5/2}\,\text{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{168abx^{5/2}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{168abx^{5/2}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{105b^2x^2\,\text{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4}$$
$$- \frac{840iabx^2\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{840iabx^2\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+ \frac{210ib^2x^{3/2}\,\text{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)}{d^5} + \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}$$
$$- \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{10080iabx\,\text{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6}$$
$$- \frac{10080iabx\,\text{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6} + \frac{2b^2x^{7/2}\tan\left(c+d\sqrt{x}\right)}{d}$$
$$- \frac{(20160iab)\text{Subst}\left(\int x\,\text{PolyLog}\left(6,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^6}$$
$$+ \frac{(20160iab)\text{Subst}\left(\int x\,\text{PolyLog}\left(6,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^6}$$
$$- \frac{(630ib^2)\,\text{Subst}\left(\int x^2\,\text{PolyLog}\left(4,-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5}$$

$$= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8iabx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14b^2x^3\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{28iabx^3\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{28iabx^3\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{42ib^2x^{5/2}\,\text{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{168abx^{5/2}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{168abx^{5/2}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{105b^2x^2\,\text{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4}$$
$$- \frac{840iabx^2\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{840iabx^2\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+ \frac{210ib^2x^{3/2}\,\text{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)}{d^5} + \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}$$
$$- \frac{3360abx^{3/2}\,\text{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{315b^2x\,\text{PolyLog}\left(5,-e^{2i(c+d\sqrt{x})}\right)}{d^6}$$
$$+ \frac{10080iabx\,\text{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{10080iabx\,\text{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6}$$
$$- \frac{20160ab\sqrt{x}\,\text{PolyLog}\left(7,-ie^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{20160ab\sqrt{x}\,\text{PolyLog}\left(7,ie^{i(c+d\sqrt{x})}\right)}{d^7}$$
$$+ \frac{2b^2x^{7/2}\tan\left(c+d\sqrt{x}\right)}{d} + \frac{(20160ab)\text{Subst}\left(\int \text{PolyLog}\left(7,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^7}$$
$$- \frac{(20160ab)\text{Subst}\left(\int \text{PolyLog}\left(7,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^7}$$
$$+ \frac{(630b^2)\,\text{Subst}\left(\int x\,\text{PolyLog}\left(5,-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^6}$$

$$= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8iabx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14b^2x^3\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{28iabx^3\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{28iabx^3\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{42ib^2x^{5/2}\operatorname{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{105b^2x^2\operatorname{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4}$$
$$- \frac{840iabx^2\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{840iabx^2\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+ \frac{210ib^2x^{3/2}\operatorname{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)}{d^5} + \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}$$
$$- \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{315b^2x\operatorname{PolyLog}\left(5,-e^{2i(c+d\sqrt{x})}\right)}{d^6}$$
$$+ \frac{10080iabx\operatorname{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{10080iabx\operatorname{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6}$$
$$- \frac{315ib^2\sqrt{x}\operatorname{PolyLog}\left(6,-e^{2i(c+d\sqrt{x})}\right)}{d^7} - \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,-ie^{i(c+d\sqrt{x})}\right)}{d^7}$$
$$+ \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,ie^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{2b^2x^{7/2}\tan\left(c+d\sqrt{x}\right)}{d}$$
$$- \frac{(20160iab)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(7,-ix)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^8}$$
$$+ \frac{(20160iab)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(7,ix)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^8}$$
$$+ \frac{(315ib^2)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(6,-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^7}$$

$$
\begin{aligned}
&= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8iabx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} \\
&+ \frac{14b^2x^3\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{28iabx^3\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
&- \frac{28iabx^3\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{42ib^2x^{5/2}\operatorname{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} \\
&- \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
&+ \frac{105b^2x^2\operatorname{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{840iabx^2\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
&+ \frac{840iabx^2\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{210ib^2x^{3/2}\operatorname{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)}{d^5} \\
&+ \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} \\
&- \frac{315b^2x\operatorname{PolyLog}\left(5,-e^{2i(c+d\sqrt{x})}\right)}{d^6} + \frac{10080iabx\operatorname{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6} \\
&- \frac{10080iabx\operatorname{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{315ib^2\sqrt{x}\operatorname{PolyLog}\left(6,-e^{2i(c+d\sqrt{x})}\right)}{d^7} \\
&- \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,-ie^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,ie^{i(c+d\sqrt{x})}\right)}{d^7} \\
&- \frac{20160iab\operatorname{PolyLog}\left(8,-ie^{i(c+d\sqrt{x})}\right)}{d^8} + \frac{20160iab\operatorname{PolyLog}\left(8,ie^{i(c+d\sqrt{x})}\right)}{d^8} \\
&+ \frac{2b^2x^{7/2}\tan\left(c+d\sqrt{x}\right)}{d} + \frac{(315b^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(6,-x)}{x}\,dx,x,e^{2i(c+d\sqrt{x})}\right)}{2d^8}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8iabx^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} \\
&+ \frac{14b^2x^3\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{28iabx^3\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
&- \frac{28iabx^3\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{42ib^2x^{5/2}\operatorname{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} \\
&- \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
&+ \frac{105b^2x^2\operatorname{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{840iabx^2\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
&+ \frac{840iabx^2\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{210ib^2x^{3/2}\operatorname{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)}{d^5} \\
&+ \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} \\
&- \frac{315b^2x\operatorname{PolyLog}\left(5,-e^{2i(c+d\sqrt{x})}\right)}{d^6} + \frac{10080iabx\operatorname{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6} \\
&- \frac{10080iabx\operatorname{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{315ib^2\sqrt{x}\operatorname{PolyLog}\left(6,-e^{2i(c+d\sqrt{x})}\right)}{d^7} \\
&- \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,-ie^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,ie^{i(c+d\sqrt{x})}\right)}{d^7} \\
&+ \frac{315b^2\operatorname{PolyLog}\left(7,-e^{2i(c+d\sqrt{x})}\right)}{2d^8} - \frac{20160iab\operatorname{PolyLog}\left(8,-ie^{i(c+d\sqrt{x})}\right)}{d^8} \\
&+ \frac{20160iab\operatorname{PolyLog}\left(8,ie^{i(c+d\sqrt{x})}\right)}{d^8} + \frac{2b^2x^{7/2}\tan\left(c+d\sqrt{x}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.15 (sec) , antiderivative size = 739, normalized size of antiderivative = 0.99

$$
\int x^3\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx
= \frac{-8ib^2d^7x^{7/2}+a^2d^8x^4-32iabd^7x^{7/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)+56b^2d^6x^3\log\left(1+e^{2i(c+d\sqrt{x})}\right)+112iabd^6x^3\operatorname{Pol}}{}
$$

```
[In] Integrate[x^3*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-8*I)*b^2*d^7*x^(7/2) + a^2*d^8*x^4 - (32*I)*a*b*d^7*x^(7/2)*ArcTan[E^(I*
(c + d*Sqrt[x]))] + 56*b^2*d^6*x^3*Log[1 + E^((2*I)*(c + d*Sqrt[x]))] + (11
```

```
2*I)*a*b*d^6*x^3*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (112*I)*a*b*d^6*x
^3*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))] - (168*I)*b^2*d^5*x^(5/2)*PolyLog[2,
 -E^((2*I)*(c + d*Sqrt[x]))] - 672*a*b*d^5*x^(5/2)*PolyLog[3, (-I)*E^(I*(c
+ d*Sqrt[x]))] + 672*a*b*d^5*x^(5/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))] +
420*b^2*d^4*x^2*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))] - (3360*I)*a*b*d^4*x
^2*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))] + (3360*I)*a*b*d^4*x^2*PolyLog[4,
 I*E^(I*(c + d*Sqrt[x]))] + (840*I)*b^2*d^3*x^(3/2)*PolyLog[4, -E^((2*I)*(c
 + d*Sqrt[x]))] + 13440*a*b*d^3*x^(3/2)*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]
))] - 13440*a*b*d^3*x^(3/2)*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))] - 1260*b^2*
d^2*x*PolyLog[5, -E^((2*I)*(c + d*Sqrt[x]))] + (40320*I)*a*b*d^2*x*PolyLog[
6, (-I)*E^(I*(c + d*Sqrt[x]))] - (40320*I)*a*b*d^2*x*PolyLog[6, I*E^(I*(c +
 d*Sqrt[x]))] - (1260*I)*b^2*d*Sqrt[x]*PolyLog[6, -E^((2*I)*(c + d*Sqrt[x])
)] - 80640*a*b*d*Sqrt[x]*PolyLog[7, (-I)*E^(I*(c + d*Sqrt[x]))] + 80640*a*b
*d*Sqrt[x]*PolyLog[7, I*E^(I*(c + d*Sqrt[x]))] + 630*b^2*PolyLog[7, -E^((2*
I)*(c + d*Sqrt[x]))] - (80640*I)*a*b*PolyLog[8, (-I)*E^(I*(c + d*Sqrt[x]))]
 + (80640*I)*a*b*PolyLog[8, I*E^(I*(c + d*Sqrt[x]))] + 8*b^2*d^7*x^(7/2)*Ta
n[c + d*Sqrt[x]])/(4*d^8)
```

Maple [F]

$$\int x^3 \left(a + b\sec\left(c + d\sqrt{x}\right)\right)^2 dx$$

```
[In] int(x^3*(a+b*sec(c+d*x^(1/2)))^2,x)
[Out] int(x^3*(a+b*sec(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x^3 \left(a + b\sec\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b\sec\left(d\sqrt{x} + c\right) + a\right)^2 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(b^2*x^3*sec(d*sqrt(x) + c)^2 + 2*a*b*x^3*sec(d*sqrt(x) + c) + a^2*
x^3, x)
```

Sympy [F]

$$\int x^3 \left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x^3 \left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx$$

```
[In] integrate(x**3*(a+b*sec(c+d*x**(1/2)))**2,x)
[Out] Integral(x**3*(a + b*sec(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6347 vs. $2(574) = 1148$.

Time = 0.69 (sec) , antiderivative size = 6347, normalized size of antiderivative = 8.47

$$\int x^3 \left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] 1/4*((d*sqrt(x) + c)^8*a^2 - 8*(d*sqrt(x) + c)^7*a^2*c + 28*(d*sqrt(x) + c)
^6*a^2*c^2 - 56*(d*sqrt(x) + c)^5*a^2*c^3 + 70*(d*sqrt(x) + c)^4*a^2*c^4 -
56*(d*sqrt(x) + c)^3*a^2*c^5 + 28*(d*sqrt(x) + c)^2*a^2*c^6 - 8*(d*sqrt(x)
+ c)*a^2*c^7 - 16*a*b*c^7*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 8*
(60*b^2*c^7 + 60*((d*sqrt(x) + c)^7*a*b - 7*(d*sqrt(x) + c)^6*a*b*c + 21*(d
*sqrt(x) + c)^5*a*b*c^2 - 35*(d*sqrt(x) + c)^4*a*b*c^3 + 35*(d*sqrt(x) + c)
^3*a*b*c^4 - 21*(d*sqrt(x) + c)^2*a*b*c^5 + 7*(d*sqrt(x) + c)*a*b*c^6 + ((d
*sqrt(x) + c)^7*a*b - 7*(d*sqrt(x) + c)^6*a*b*c + 21*(d*sqrt(x) + c)^5*a*b*
c^2 - 35*(d*sqrt(x) + c)^4*a*b*c^3 + 35*(d*sqrt(x) + c)^3*a*b*c^4 - 21*(d*s
qrt(x) + c)^2*a*b*c^5 + 7*(d*sqrt(x) + c)*a*b*c^6)*cos(2*d*sqrt(x) + 2*c) +
 (I*(d*sqrt(x) + c)^7*a*b - 7*I*(d*sqrt(x) + c)^6*a*b*c + 21*I*(d*sqrt(x) +
 c)^5*a*b*c^2 - 35*I*(d*sqrt(x) + c)^4*a*b*c^3 + 35*I*(d*sqrt(x) + c)^3*a*b
*c^4 - 21*I*(d*sqrt(x) + c)^2*a*b*c^5 + 7*I*(d*sqrt(x) + c)*a*b*c^6)*sin(2*
d*sqrt(x) + 2*c))*arctan2(cos(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) + 60*
((d*sqrt(x) + c)^7*a*b - 7*(d*sqrt(x) + c)^6*a*b*c + 21*(d*sqrt(x) + c)^5*a
*b*c^2 - 35*(d*sqrt(x) + c)^4*a*b*c^3 + 35*(d*sqrt(x) + c)^3*a*b*c^4 - 21*(
d*sqrt(x) + c)^2*a*b*c^5 + 7*(d*sqrt(x) + c)*a*b*c^6 + ((d*sqrt(x) + c)^7*a
*b - 7*(d*sqrt(x) + c)^6*a*b*c + 21*(d*sqrt(x) + c)^5*a*b*c^2 - 35*(d*sqrt(
x) + c)^4*a*b*c^3 + 35*(d*sqrt(x) + c)^3*a*b*c^4 - 21*(d*sqrt(x) + c)^2*a*b
*c^5 + 7*(d*sqrt(x) + c)*a*b*c^6)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) +
c)^7*a*b - 7*I*(d*sqrt(x) + c)^6*a*b*c + 21*I*(d*sqrt(x) + c)^5*a*b*c^2 - 3
5*I*(d*sqrt(x) + c)^4*a*b*c^3 + 35*I*(d*sqrt(x) + c)^3*a*b*c^4 - 21*I*(d*sq
rt(x) + c)^2*a*b*c^5 + 7*I*(d*sqrt(x) + c)*a*b*c^6)*sin(2*d*sqrt(x) + 2*c))
*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) - 14*(80*(d*sqrt(x) +
```

c)^6*b^2 - 288*(d*sqrt(x) + c)^5*b^2*c + 450*(d*sqrt(x) + c)^4*b^2*c^2 - 4
00*(d*sqrt(x) + c)^3*b^2*c^3 + 225*(d*sqrt(x) + c)^2*b^2*c^4 - 90*(d*sqrt(x
) + c)*b^2*c^5 + 15*b^2*c^6 + (80*(d*sqrt(x) + c)^6*b^2 - 288*(d*sqrt(x) +
c)^5*b^2*c + 450*(d*sqrt(x) + c)^4*b^2*c^2 - 400*(d*sqrt(x) + c)^3*b^2*c^3
+ 225*(d*sqrt(x) + c)^2*b^2*c^4 - 90*(d*sqrt(x) + c)*b^2*c^5 + 15*b^2*c^6)*
cos(2*d*sqrt(x) + 2*c) - (-80*I*(d*sqrt(x) + c)^6*b^2 + 288*I*(d*sqrt(x) +
c)^5*b^2*c - 450*I*(d*sqrt(x) + c)^4*b^2*c^2 + 400*I*(d*sqrt(x) + c)^3*b^2*
c^3 - 225*I*(d*sqrt(x) + c)^2*b^2*c^4 + 90*I*(d*sqrt(x) + c)*b^2*c^5 - 15*I
*b^2*c^6)*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(2*d*sqrt(x) + 2*c), cos(2*d*s
qrt(x) + 2*c) + 1) + 60*((d*sqrt(x) + c)^7*b^2 - 7*(d*sqrt(x) + c)^6*b^2*c
+ 21*(d*sqrt(x) + c)^5*b^2*c^2 - 35*(d*sqrt(x) + c)^4*b^2*c^3 + 35*(d*sqrt(
x) + c)^3*b^2*c^4 - 21*(d*sqrt(x) + c)^2*b^2*c^5 + 7*(d*sqrt(x) + c)*b^2*c^
6)*cos(2*d*sqrt(x) + 2*c) + 210*(16*(d*sqrt(x) + c)^5*b^2 - 48*(d*sqrt(x) +
c)^4*b^2*c + 60*(d*sqrt(x) + c)^3*b^2*c^2 - 40*(d*sqrt(x) + c)^2*b^2*c^3 +
15*(d*sqrt(x) + c)*b^2*c^4 - 3*b^2*c^5 + (16*(d*sqrt(x) + c)^5*b^2 - 48*(d
*sqrt(x) + c)^4*b^2*c + 60*(d*sqrt(x) + c)^3*b^2*c^2 - 40*(d*sqrt(x) + c)^2
*b^2*c^3 + 15*(d*sqrt(x) + c)*b^2*c^4 - 3*b^2*c^5)*cos(2*d*sqrt(x) + 2*c) +
(16*I*(d*sqrt(x) + c)^5*b^2 - 48*I*(d*sqrt(x) + c)^4*b^2*c + 60*I*(d*sqrt(
x) + c)^3*b^2*c^2 - 40*I*(d*sqrt(x) + c)^2*b^2*c^3 + 15*I*(d*sqrt(x) + c)*b
^2*c^4 - 3*I*b^2*c^5)*sin(2*d*sqrt(x) + 2*c))*dilog(-e^(2*I*d*sqrt(x) + 2*I
c)) + 420((d*sqrt(x) + c)^6*a*b - 6*(d*sqrt(x) + c)^5*a*b*c + 15*(d*sqrt(
x) + c)^4*a*b*c^2 - 20*(d*sqrt(x) + c)^3*a*b*c^3 + 15*(d*sqrt(x) + c)^2*a*b
c^4 - 6(d*sqrt(x) + c)*a*b*c^5 + a*b*c^6 + ((d*sqrt(x) + c)^6*a*b - 6*(d*
sqrt(x) + c)^5*a*b*c + 15*(d*sqrt(x) + c)^4*a*b*c^2 - 20*(d*sqrt(x) + c)^3*
a*b*c^3 + 15*(d*sqrt(x) + c)^2*a*b*c^4 - 6*(d*sqrt(x) + c)*a*b*c^5 + a*b*c^
6)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^6*a*b - 6*I*(d*sqrt(x) + c)^
5*a*b*c + 15*I*(d*sqrt(x) + c)^4*a*b*c^2 - 20*I*(d*sqrt(x) + c)^3*a*b*c^3 +
15*I*(d*sqrt(x) + c)^2*a*b*c^4 - 6*I*(d*sqrt(x) + c)*a*b*c^5 + I*a*b*c^6)*
sin(2*d*sqrt(x) + 2*c))*dilog(I*e^(I*d*sqrt(x) + I*c)) - 420*((d*sqrt(x) +
c)^6*a*b - 6*(d*sqrt(x) + c)^5*a*b*c + 15*(d*sqrt(x) + c)^4*a*b*c^2 - 20*(d
*sqrt(x) + c)^3*a*b*c^3 + 15*(d*sqrt(x) + c)^2*a*b*c^4 - 6*(d*sqrt(x) + c)*
a*b*c^5 + a*b*c^6 + ((d*sqrt(x) + c)^6*a*b - 6*(d*sqrt(x) + c)^5*a*b*c + 15
*(d*sqrt(x) + c)^4*a*b*c^2 - 20*(d*sqrt(x) + c)^3*a*b*c^3 + 15*(d*sqrt(x) +
c)^2*a*b*c^4 - 6*(d*sqrt(x) + c)*a*b*c^5 + a*b*c^6)*cos(2*d*sqrt(x) + 2*c)
- (-I*(d*sqrt(x) + c)^6*a*b + 6*I*(d*sqrt(x) + c)^5*a*b*c - 15*I*(d*sqrt(x
) + c)^4*a*b*c^2 + 20*I*(d*sqrt(x) + c)^3*a*b*c^3 - 15*I*(d*sqrt(x) + c)^2*
a*b*c^4 + 6*I*(d*sqrt(x) + c)*a*b*c^5 - I*a*b*c^6)*sin(2*d*sqrt(x) + 2*c))*
dilog(-I*e^(I*d*sqrt(x) + I*c)) + 7*(80*I*(d*sqrt(x) + c)^6*b^2 - 288*I*(d*
sqrt(x) + c)^5*b^2*c + 450*I*(d*sqrt(x) + c)^4*b^2*c^2 - 400*I*(d*sqrt(x) +
c)^3*b^2*c^3 + 225*I*(d*sqrt(x) + c)^2*b^2*c^4 - 90*I*(d*sqrt(x) + c)*b^2*
c^5 + 15*I*b^2*c^6 + (80*I*(d*sqrt(x) + c)^6*b^2 - 288*I*(d*sqrt(x) + c)^5*
b^2*c + 450*I*(d*sqrt(x) + c)^4*b^2*c^2 - 400*I*(d*sqrt(x) + c)^3*b^2*c^3 +
225*I*(d*sqrt(x) + c)^2*b^2*c^4 - 90*I*(d*sqrt(x) + c)*b^2*c^5 + 15*I*b^2*
c^6)*cos(2*d*sqrt(x) + 2*c) - (80*(d*sqrt(x) + c)^6*b^2 - 288*(d*sqrt(x) +
c)^5*b^2*c + 450*(d*sqrt(x) + c)^4*b^2*c^2 - 400*(d*sqrt(x) + c)^3*b^2*c^3

+ 225*(d*sqrt(x) + c)^2*b^2*c^4 - 90*(d*sqrt(x) + c)*b^2*c^5 + 15*b^2*c^6)*
sin(2*d*sqrt(x) + 2*c))*log(cos(2*d*sqrt(x) + 2*c)^2 + sin(2*d*sqrt(x) + 2*
c)^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1) + 30*(I*(d*sqrt(x) + c)^7*a*b - 7*I*(d
*sqrt(x) + c)^6*a*b*c + 21*I*(d*sqrt(x) + c)^5*a*b*c^2 - 35*I*(d*sqrt(x) +
c)^4*a*b*c^3 + 35*I*(d*sqrt(x) + c)^3*a*b*c^4 - 21*I*(d*sqrt(x) + c)^2*a*b*
c^5 + 7*I*(d*sqrt(x) + c)*a*b*c^6 + (I*(d*sqrt(x) + c)^7*a*b - 7*I*(d*sqrt(
x) + c)^6*a*b*c + 21*I*(d*sqrt(x) + c)^5*a*b*c^2 - 35*I*(d*sqrt(x) + c)^4*a
*b*c^3 + 35*I*(d*sqrt(x) + c)^3*a*b*c^4 - 21*I*(d*sqrt(x) + c)^2*a*b*c^5 +
7*I*(d*sqrt(x) + c)*a*b*c^6)*cos(2*d*sqrt(x) + 2*c) - ((d*sqrt(x) + c)^7*a*
b - 7*(d*sqrt(x) + c)^6*a*b*c + 21*(d*sqrt(x) + c)^5*a*b*c^2 - 35*(d*sqrt(x
) + c)^4*a*b*c^3 + 35*(d*sqrt(x) + c)^3*a*b*c^4 - 21*(d*sqrt(x) + c)^2*a*b*
c^5 + 7*(d*sqrt(x) + c)*a*b*c^6)*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x)
+ c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*sqrt(x) + c) + 1) + 30*(-I*(d*sqrt(
x) + c)^7*a*b + 7*I*(d*sqrt(x) + c)^6*a*b*c - 21*I*(d*sqrt(x) + c)^5*a*b*c^
2 + 35*I*(d*sqrt(x) + c)^4*a*b*c^3 - 35*I*(d*sqrt(x) + c)^3*a*b*c^4 + 21*I*
(d*sqrt(x) + c)^2*a*b*c^5 - 7*I*(d*sqrt(x) + c)*a*b*c^6 + (-I*(d*sqrt(x) +
c)^7*a*b + 7*I*(d*sqrt(x) + c)^6*a*b*c - 21*I*(d*sqrt(x) + c)^5*a*b*c^2 + 3
5*I*(d*sqrt(x) + c)^4*a*b*c^3 - 35*I*(d*sqrt(x) + c)^3*a*b*c^4 + 21*I*(d*sq
rt(x) + c)^2*a*b*c^5 - 7*I*(d*sqrt(x) + c)*a*b*c^6)*cos(2*d*sqrt(x) + 2*c)
+ ((d*sqrt(x) + c)^7*a*b - 7*(d*sqrt(x) + c)^6*a*b*c + 21*(d*sqrt(x) + c)^5
*a*b*c^2 - 35*(d*sqrt(x) + c)^4*a*b*c^3 + 35*(d*sqrt(x) + c)^3*a*b*c^4 - 21
*(d*sqrt(x) + c)^2*a*b*c^5 + 7*(d*sqrt(x) + c)*a*b*c^6)*sin(2*d*sqrt(x) + 2
*c))*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*sin(d*sqrt(x) + c)
 + 1) - 302400*(a*b*cos(2*d*sqrt(x) + 2*c) + I*a*b*sin(2*d*sqrt(x) + 2*c) +
 a*b)*polylog(8, I*e^(I*d*sqrt(x) + I*c)) + 302400*(a*b*cos(2*d*sqrt(x) + 2
*c) + I*a*b*sin(2*d*sqrt(x) + 2*c) + a*b)*polylog(8, -I*e^(I*d*sqrt(x) + I*
c)) + 12600*(I*b^2*cos(2*d*sqrt(x) + 2*c) - b^2*sin(2*d*sqrt(x) + 2*c) + I*
b^2)*polylog(7, -e^(2*I*d*sqrt(x) + 2*I*c)) + 302400*(I*(d*sqrt(x) + c)*a*b
 - I*a*b*c + (I*(d*sqrt(x) + c)*a*b - I*a*b*c)*cos(2*d*sqrt(x) + 2*c) - ((d
*sqrt(x) + c)*a*b - a*b*c)*sin(2*d*sqrt(x) + 2*c))*polylog(7, I*e^(I*d*sqrt
(x) + I*c)) + 302400*(-I*(d*sqrt(x) + c)*a*b + I*a*b*c + (-I*(d*sqrt(x) + c
)*a*b + I*a*b*c)*cos(2*d*sqrt(x) + 2*c) + ((d*sqrt(x) + c)*a*b - a*b*c)*sin
(2*d*sqrt(x) + 2*c))*polylog(7, -I*e^(I*d*sqrt(x) + I*c)) + 5040*(5*(d*sqrt
(x) + c)*b^2 - 3*b^2*c + (5*(d*sqrt(x) + c)*b^2 - 3*b^2*c)*cos(2*d*sqrt(x)
+ 2*c) + (5*I*(d*sqrt(x) + c)*b^2 - 3*I*b^2*c)*sin(2*d*sqrt(x) + 2*c))*poly
log(6, -e^(2*I*d*sqrt(x) + 2*I*c)) + 151200*((d*sqrt(x) + c)^2*a*b - 2*(d*s
qrt(x) + c)*a*b*c + a*b*c^2 + ((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a*
b*c + a*b*c^2)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^2*a*b - 2*I*(d*s
qrt(x) + c)*a*b*c + I*a*b*c^2)*sin(2*d*sqrt(x) + 2*c))*polylog(6, I*e^(I*d*
sqrt(x) + I*c)) - 151200*((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c +
 a*b*c^2 + ((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c + a*b*c^2)*cos(
2*d*sqrt(x) + 2*c) - (-I*(d*sqrt(x) + c)^2*a*b + 2*I*(d*sqrt(x) + c)*a*b*c
- I*a*b*c^2)*sin(2*d*sqrt(x) + 2*c))*polylog(6, -I*e^(I*d*sqrt(x) + I*c)) +
 630*(-40*I*(d*sqrt(x) + c)^2*b^2 + 48*I*(d*sqrt(x) + c)*b^2*c - 15*I*b^2*c
^2 + (-40*I*(d*sqrt(x) + c)^2*b^2 + 48*I*(d*sqrt(x) + c)*b^2*c - 15*I*b^2*c

^2)*cos(2*d*sqrt(x) + 2*c) + (40*(d*sqrt(x) + c)^2*b^2 - 48*(d*sqrt(x) + c)
*b^2*c + 15*b^2*c^2)*sin(2*d*sqrt(x) + 2*c))*polylog(5, -e^(2*I*d*sqrt(x) +
 2*I*c)) + 50400*(-I*(d*sqrt(x) + c)^3*a*b + 3*I*(d*sqrt(x) + c)^2*a*b*c -
3*I*(d*sqrt(x) + c)*a*b*c^2 + I*a*b*c^3 + (-I*(d*sqrt(x) + c)^3*a*b + 3*I*(
d*sqrt(x) + c)^2*a*b*c - 3*I*(d*sqrt(x) + c)*a*b*c^2 + I*a*b*c^3)*cos(2*d*s
qrt(x) + 2*c) + ((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3*(d*s
qrt(x) + c)*a*b*c^2 - a*b*c^3)*sin(2*d*sqrt(x) + 2*c))*polylog(5, I*e^(I*d*
sqrt(x) + I*c)) + 50400*(I*(d*sqrt(x) + c)^3*a*b - 3*I*(d*sqrt(x) + c)^2*a*
b*c + 3*I*(d*sqrt(x) + c)*a*b*c^2 - I*a*b*c^3 + (I*(d*sqrt(x) + c)^3*a*b -
3*I*(d*sqrt(x) + c)^2*a*b*c + 3*I*(d*sqrt(x) + c)*a*b*c^2 - I*a*b*c^3)*cos(
2*d*sqrt(x) + 2*c) - ((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3
*(d*sqrt(x) + c)*a*b*c^2 - a*b*c^3)*sin(2*d*sqrt(x) + 2*c))*polylog(5, -I*e
^(I*d*sqrt(x) + I*c)) - 420*(40*(d*sqrt(x) + c)^3*b^2 - 72*(d*sqrt(x) + c)^
2*b^2*c + 45*(d*sqrt(x) + c)*b^2*c^2 - 10*b^2*c^3 + (40*(d*sqrt(x) + c)^3*b
^2 - 72*(d*sqrt(x) + c)^2*b^2*c + 45*(d*sqrt(x) + c)*b^2*c^2 - 10*b^2*c^3)*
cos(2*d*sqrt(x) + 2*c) - (-40*I*(d*sqrt(x) + c)^3*b^2 + 72*I*(d*sqrt(x) + c
)^2*b^2*c - 45*I*(d*sqrt(x) + c)*b^2*c^2 + 10*I*b^2*c^3)*sin(2*d*sqrt(x) +
2*c))*polylog(4, -e^(2*I*d*sqrt(x) + 2*I*c)) - 12600*((d*sqrt(x) + c)^4*a*b
 - 4*(d*sqrt(x) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4*(d*sqrt(x) +
 c)*a*b*c^3 + a*b*c^4 + ((d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b*c
+ 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^3 + a*b*c^4)*cos(2*
d*sqrt(x) + 2*c) - (-I*(d*sqrt(x) + c)^4*a*b + 4*I*(d*sqrt(x) + c)^3*a*b*c
- 6*I*(d*sqrt(x) + c)^2*a*b*c^2 + 4*I*(d*sqrt(x) + c)*a*b*c^3 - I*a*b*c^4)*
sin(2*d*sqrt(x) + 2*c))*polylog(4, I*e^(I*d*sqrt(x) + I*c)) + 12600*((d*sqr
t(x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 -
 4*(d*sqrt(x) + c)*a*b*c^3 + a*b*c^4 + ((d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x
) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^3 +
a*b*c^4)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^4*a*b - 4*I*(d*sqrt(x)
 + c)^3*a*b*c + 6*I*(d*sqrt(x) + c)^2*a*b*c^2 - 4*I*(d*sqrt(x) + c)*a*b*c^3
 + I*a*b*c^4)*sin(2*d*sqrt(x) + 2*c))*polylog(4, -I*e^(I*d*sqrt(x) + I*c))
+ 105*(80*I*(d*sqrt(x) + c)^4*b^2 - 192*I*(d*sqrt(x) + c)^3*b^2*c + 180*I*(
d*sqrt(x) + c)^2*b^2*c^2 - 80*I*(d*sqrt(x) + c)*b^2*c^3 + 15*I*b^2*c^4 + (8
0*I*(d*sqrt(x) + c)^4*b^2 - 192*I*(d*sqrt(x) + c)^3*b^2*c + 180*I*(d*sqrt(x
) + c)^2*b^2*c^2 - 80*I*(d*sqrt(x) + c)*b^2*c^3 + 15*I*b^2*c^4)*cos(2*d*sqr
t(x) + 2*c) - (80*(d*sqrt(x) + c)^4*b^2 - 192*(d*sqrt(x) + c)^3*b^2*c + 180
*(d*sqrt(x) + c)^2*b^2*c^2 - 80*(d*sqrt(x) + c)*b^2*c^3 + 15*b^2*c^4)*sin(2
*d*sqrt(x) + 2*c))*polylog(3, -e^(2*I*d*sqrt(x) + 2*I*c)) + 2520*(I*(d*sqrt
(x) + c)^5*a*b - 5*I*(d*sqrt(x) + c)^4*a*b*c + 10*I*(d*sqrt(x) + c)^3*a*b*c
^2 - 10*I*(d*sqrt(x) + c)^2*a*b*c^3 + 5*I*(d*sqrt(x) + c)*a*b*c^4 - I*a*b*c
^5 + (I*(d*sqrt(x) + c)^5*a*b - 5*I*(d*sqrt(x) + c)^4*a*b*c + 10*I*(d*sqrt(
x) + c)^3*a*b*c^2 - 10*I*(d*sqrt(x) + c)^2*a*b*c^3 + 5*I*(d*sqrt(x) + c)*a*
b*c^4 - I*a*b*c^5)*cos(2*d*sqrt(x) + 2*c) - ((d*sqrt(x) + c)^5*a*b - 5*(d*s
qrt(x) + c)^4*a*b*c + 10*(d*sqrt(x) + c)^3*a*b*c^2 - 10*(d*sqrt(x) + c)^2*a
*b*c^3 + 5*(d*sqrt(x) + c)*a*b*c^4 - a*b*c^5)*sin(2*d*sqrt(x) + 2*c))*polyl
og(3, I*e^(I*d*sqrt(x) + I*c)) + 2520*(-I*(d*sqrt(x) + c)^5*a*b + 5*I*(d*sq

```
rt(x) + c)^4*a*b*c - 10*I*(d*sqrt(x) + c)^3*a*b*c^2 + 10*I*(d*sqrt(x) + c)^
2*a*b*c^3 - 5*I*(d*sqrt(x) + c)*a*b*c^4 + I*a*b*c^5 + (-I*(d*sqrt(x) + c)^5
*a*b + 5*I*(d*sqrt(x) + c)^4*a*b*c - 10*I*(d*sqrt(x) + c)^3*a*b*c^2 + 10*I*
(d*sqrt(x) + c)^2*a*b*c^3 - 5*I*(d*sqrt(x) + c)*a*b*c^4 + I*a*b*c^5)*cos(2*
d*sqrt(x) + 2*c) + ((d*sqrt(x) + c)^5*a*b - 5*(d*sqrt(x) + c)^4*a*b*c + 10*
(d*sqrt(x) + c)^3*a*b*c^2 - 10*(d*sqrt(x) + c)^2*a*b*c^3 + 5*(d*sqrt(x) + c
)*a*b*c^4 - a*b*c^5)*sin(2*d*sqrt(x) + 2*c))*polylog(3, -I*e^(I*d*sqrt(x) +
 I*c)) + 60*(I*(d*sqrt(x) + c)^7*b^2 - 7*I*(d*sqrt(x) + c)^6*b^2*c + 21*I*(
d*sqrt(x) + c)^5*b^2*c^2 - 35*I*(d*sqrt(x) + c)^4*b^2*c^3 + 35*I*(d*sqrt(x)
 + c)^3*b^2*c^4 - 21*I*(d*sqrt(x) + c)^2*b^2*c^5 + 7*I*(d*sqrt(x) + c)*b^2*
c^6)*sin(2*d*sqrt(x) + 2*c))/(-30*I*cos(2*d*sqrt(x) + 2*c) + 30*sin(2*d*sqr
t(x) + 2*c) - 30*I))/d^8
```

Giac [F]

$$\int x^3 \left(a + b \sec \left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b \sec \left(d\sqrt{x} + c\right) + a\right)^2 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*sec(d*sqrt(x) + c) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \sec \left(c + d\sqrt{x}\right)\right)^2 \, dx = \int x^3 \left(a + \frac{b}{\cos \left(c + d\sqrt{x}\right)}\right)^2 dx$$

```
[In] int(x^3*(a + b/cos(c + d*x^(1/2)))^2,x)
[Out] int(x^3*(a + b/cos(c + d*x^(1/2)))^2, x)
```

3.37 $\int x^2 \left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 551

$$\int x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=-\frac{2ib^2x^{5/2}}{d}+\frac{a^2x^3}{3}-\frac{8iabx^{5/2}\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}+\frac{10b^2x^2\log\left(1+e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^2}+\frac{20iabx^2\operatorname{PolyLog}\left(2,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{20iabx^2\operatorname{PolyLog}\left(2,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{20ib^2x^{3/2}\operatorname{PolyLog}\left(2,-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^3}-\frac{80abx^{3/2}\operatorname{PolyLog}\left(3,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{80abx^{3/2}\operatorname{PolyLog}\left(3,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{30b^2x\operatorname{PolyLog}\left(3,-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{240iabx\operatorname{PolyLog}\left(4,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{240iabx\operatorname{PolyLog}\left(4,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{30ib^2\sqrt{x}\operatorname{PolyLog}\left(4,-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^5}+\frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{15b^2\operatorname{PolyLog}\left(5,-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^6}+\frac{480iab\operatorname{PolyLog}\left(6,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}-\frac{480iab\operatorname{PolyLog}\left(6,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}+\frac{2b^2x^{5/2}\tan\left(c+d\sqrt{x}\right)}{d}$$

```
[Out] 20*I*a*b*x^2*polylog(2,-I*exp(I*(c+d*x^(1/2))))/d^2+1/3*a^2*x^3-20*I*b^2*x^
(3/2)*polylog(2,-exp(2*I*(c+d*x^(1/2))))/d^3+10*b^2*x^2*ln(1+exp(2*I*(c+d*x
^(1/2))))/d^2+240*I*a*b*x*polylog(4,I*exp(I*(c+d*x^(1/2))))/d^4-2*I*b^2*x^(
5/2)/d-8*I*a*b*x^(5/2)*arctan(exp(I*(c+d*x^(1/2))))/d-80*a*b*x^(3/2)*polylo
g(3,-I*exp(I*(c+d*x^(1/2))))/d^3+80*a*b*x^(3/2)*polylog(3,I*exp(I*(c+d*x^(1
/2))))/d^3+30*b^2*x*polylog(3,-exp(2*I*(c+d*x^(1/2))))/d^4-20*I*a*b*x^2*pol
ylog(2,I*exp(I*(c+d*x^(1/2))))/d^2+30*I*b^2*polylog(4,-exp(2*I*(c+d*x^(1/2)
)))*x^(1/2)/d^5-15*b^2*polylog(5,-exp(2*I*(c+d*x^(1/2))))/d^6-480*I*a*b*pol
ylog(6,I*exp(I*(c+d*x^(1/2))))/d^6-240*I*a*b*x*polylog(4,-I*exp(I*(c+d*x^(1
/2))))/d^4+480*I*a*b*polylog(6,-I*exp(I*(c+d*x^(1/2))))/d^6+480*a*b*polylog
(5,-I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^5-480*a*b*polylog(5,I*exp(I*(c+d*x^(1
/2))))*x^(1/2)/d^5+2*b^2*x^(5/2)*tan(c+d*x^(1/2))/d
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {4289, 4275, 4266, 2611, 6744, 2320, 6724, 4269, 3800, 2221}

$$\int x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{a^2x^3}{3}-\frac{8iabx^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{480iab\,\text{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{480iab\,\text{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6}+\frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{240iabx\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{240iabx\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}-\frac{80abx^{3/2}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{80abx^{3/2}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{20iabx^2\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{20iabx^2\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{15b^2\,\text{PolyLog}\left(5,-e^{2i(c+d\sqrt{x})}\right)}{d^6}+\frac{30ib^2\sqrt{x}\,\text{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)}{d^5}+\frac{30b^2x\,\text{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4}-\frac{20ib^2x^{3/2}\,\text{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3}+\frac{10b^2x^2\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}+\frac{2b^2x^{5/2}\tan\left(c+d\sqrt{x}\right)}{d}-\frac{2ib^2x^{5/2}}{d}$$

```
[In] Int[x^2*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x^(5/2))/d + (a^2*x^3)/3 - ((8*I)*a*b*x^(5/2)*ArcTan[E^(I*(c +
d*Sqrt[x]))])/d + (10*b^2*x^2*Log[1 + E^((2*I)*(c + d*Sqrt[x]))])/d^2 + ((2
0*I)*a*b*x^2*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((20*I)*a*b*x^2*
PolyLog[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - ((20*I)*b^2*x^(3/2)*PolyLog[2, -
E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (80*a*b*x^(3/2)*PolyLog[3, (-I)*E^(I*(c +
 d*Sqrt[x]))])/d^3 + (80*a*b*x^(3/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d
^3 + (30*b^2*x*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))])/d^4 - ((240*I)*a*b*x
*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((240*I)*a*b*x*PolyLog[4, I*
E^(I*(c + d*Sqrt[x]))])/d^4 + ((30*I)*b^2*Sqrt[x]*PolyLog[4, -E^((2*I)*(c +
 d*Sqrt[x]))])/d^5 + (480*a*b*Sqrt[x]*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))
])/d^5 - (480*a*b*Sqrt[x]*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))])/d^5 - (15*b^
2*PolyLog[5, -E^((2*I)*(c + d*Sqrt[x]))])/d^6 + ((480*I)*a*b*PolyLog[6, (-I
)*E^(I*(c + d*Sqrt[x]))])/d^6 - ((480*I)*a*b*PolyLog[6, I*E^(I*(c + d*Sqrt[
x]))])/d^6 + (2*b^2*x^(5/2)*Tan[c + d*Sqrt[x]])/d
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int x^5(a+b\sec(c+dx))^2\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(a^2x^5 + 2abx^5\sec(c+dx) + b^2x^5\sec^2(c+dx)\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{a^2x^3}{3} + (4ab)\text{Subst}\left(\int x^5\sec(c+dx)\,dx, x, \sqrt{x}\right) + \left(2b^2\right)\text{Subst}\left(\int x^5\sec^2(c+dx)\,dx, x, \sqrt{x}\right)$$

$$= \frac{a^2x^3}{3} - \frac{8iabx^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{2b^2x^{5/2}\tan\left(c+d\sqrt{x}\right)}{d} - \frac{(20ab)\text{Subst}\left(\int x^4\log\left(1-ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{(20ab)\text{Subst}\left(\int x^4\log\left(1+ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d} - \frac{\left(10b^2\right)\text{Subst}\left(\int x^4\tan(c+dx)\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8iabx^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{20iabx^2\,\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\,\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} + \frac{2b^2x^{5/2}\tan\left(c+d\sqrt{x}\right)}{d} - \frac{(80iab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2, -ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2} + \frac{(80iab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2, ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2} + \frac{\left(20ib^2\right)\text{Subst}\left(\int \frac{e^{2i(c+dx)}x^4}{1+e^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8iabx^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{10b^2x^2\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{20iabx^2\,\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\,\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{80abx^{3/2}\,\text{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{80abx^{3/2}\,\text{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{2b^2x^{5/2}\tan\left(c+d\sqrt{x}\right)}{d} + \frac{(240ab)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3, -ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3} - \frac{(240ab)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3, ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3} - \frac{\left(40b^2\right)\text{Subst}\left(\int x^3\log\left(1+e^{2i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$

$$= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8iabx^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{10b^2x^2\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{20iabx^2\,\mathrm{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\,\mathrm{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{20ib^2x^{3/2}\,\mathrm{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{80abx^{3/2}\,\mathrm{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{80abx^{3/2}\,\mathrm{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{240iabx\,\mathrm{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+ \frac{240iabx\,\mathrm{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{2b^2x^{5/2}\tan\left(c+d\sqrt{x}\right)}{d}$$
$$+ \frac{(480iab)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(4,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$
$$- \frac{(480iab)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(4,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$
$$+ \frac{(60ib^2)\,\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(2,-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8iabx^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{10b^2x^2\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{20iabx^2\,\mathrm{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\,\mathrm{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{20ib^2x^{3/2}\,\mathrm{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{80abx^{3/2}\,\mathrm{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{80abx^{3/2}\,\mathrm{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{30b^2x\,\mathrm{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4}$$
$$- \frac{240iabx\,\mathrm{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{240iabx\,\mathrm{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+ \frac{480ab\sqrt{x}\,\mathrm{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{480ab\sqrt{x}\,\mathrm{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}$$
$$+ \frac{2b^2x^{5/2}\tan\left(c+d\sqrt{x}\right)}{d} - \frac{(480ab)\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(5,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5}$$
$$+ \frac{(480ab)\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(5,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5}$$
$$- \frac{(60b^2)\,\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(3,-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8iabx^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{10b^2x^2\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{20iabx^2\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{20ib^2x^{3/2}\operatorname{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{30b^2x\operatorname{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4}$$
$$- \frac{240iabx\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{240iabx\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+ \frac{30ib^2\sqrt{x}\operatorname{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)}{d^5} + \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}$$
$$- \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{2b^2x^{5/2}\tan\left(c+d\sqrt{x}\right)}{d}$$
$$+ \frac{(480iab)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(5,-ix)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^6}$$
$$- \frac{(480iab)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(5,ix)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^6}$$
$$- \frac{(30ib^2)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(4,-e^{2i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^5}$$

$$= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8iabx^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}$$
$$+ \frac{10b^2x^2\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{20iabx^2\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{20iabx^2\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20ib^2x^{3/2}\operatorname{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3}$$
$$- \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{30b^2x\operatorname{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{240iabx\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+ \frac{240iabx\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{30ib^2\sqrt{x}\operatorname{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)}{d^5}$$
$$+ \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}$$
$$+ \frac{480iab\operatorname{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{480iab\operatorname{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6}$$
$$+ \frac{2b^2x^{5/2}\tan\left(c+d\sqrt{x}\right)}{d} - \frac{(15b^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(4,-x)}{x}\,dx,x,e^{2i(c+d\sqrt{x})}\right)}{d^6}$$

$$= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8iabx^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{10b^2x^2\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{20iabx^2\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20ib^2x^{3/2}\,\text{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{80abx^{3/2}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{80abx^{3/2}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{30b^2x\,\text{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{240iabx\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{240iabx\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{30ib^2\sqrt{x}\,\text{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)}{d^5} + \frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{15b^2\,\text{PolyLog}\left(5,-e^{2i(c+d\sqrt{x})}\right)}{d^6} + \frac{480iab\,\text{PolyLog}\left(6,-ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{480iab\,\text{PolyLog}\left(6,ie^{i(c+d\sqrt{x})}\right)}{d^6} + \frac{2b^2x^{5/2}\tan\left(c+d\sqrt{x}\right)}{d}$$

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 543, normalized size of antiderivative = 0.99

$$\int x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx$$

$$= \frac{-6ib^2d^5x^{5/2} + a^2d^6x^3 - 24iabd^5x^{5/2}\arctan\left(e^{i(c+d\sqrt{x})}\right) + 30b^2d^4x^2\log\left(1+e^{2i(c+d\sqrt{x})}\right) + 60iabd^4x^2\,\text{PolyL}}{}$$

```
[In] Integrate[x^2*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-6*I)*b^2*d^5*x^(5/2) + a^2*d^6*x^3 - (24*I)*a*b*d^5*x^(5/2)*ArcTan[E^(I*
(c + d*Sqrt[x]))] + 30*b^2*d^4*x^2*Log[1 + E^((2*I)*(c + d*Sqrt[x]))] + (60
*I)*a*b*d^4*x^2*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (60*I)*a*b*d^4*x^2
*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))] - (60*I)*b^2*d^3*x^(3/2)*PolyLog[2, -E
^((2*I)*(c + d*Sqrt[x]))] - 240*a*b*d^3*x^(3/2)*PolyLog[3, (-I)*E^(I*(c + d
*Sqrt[x]))] + 240*a*b*d^3*x^(3/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))] + 90*
b^2*d^2*x*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))] - (720*I)*a*b*d^2*x*PolyLo
g[4, (-I)*E^(I*(c + d*Sqrt[x]))] + (720*I)*a*b*d^2*x*PolyLog[4, I*E^(I*(c +
 d*Sqrt[x]))] + (90*I)*b^2*d*Sqrt[x]*PolyLog[4, -E^((2*I)*(c + d*Sqrt[x]))]
```

```
 + 1440*a*b*d*Sqrt[x]*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))] - 1440*a*b*d*S
qrt[x]*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))] - 45*b^2*PolyLog[5, -E^((2*I)*(c
 + d*Sqrt[x]))] + (1440*I)*a*b*PolyLog[6, (-I)*E^(I*(c + d*Sqrt[x]))] - (14
40*I)*a*b*PolyLog[6, I*E^(I*(c + d*Sqrt[x]))] + 6*b^2*d^5*x^(5/2)*Tan[c + d
*Sqrt[x]])/(3*d^6)
```

Maple [F]

$$\int x^2 \left(a + b \sec \left(c + d\sqrt{x}\right)\right)^2 dx$$

```
[In] int(x^2*(a+b*sec(c+d*x^(1/2)))^2,x)
[Out] int(x^2*(a+b*sec(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x^2 \left(a + b \sec \left(c + d\sqrt{x}\right)\right)^2 dx = \int \left(b \sec \left(d\sqrt{x} + c\right) + a\right)^2 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*sec(d*sqrt(x) + c)^2 + 2*a*b*x^2*sec(d*sqrt(x) + c) + a^2*
x^2, x)
```

Sympy [F]

$$\int x^2 \left(a + b \sec \left(c + d\sqrt{x}\right)\right)^2 dx = \int x^2 \left(a + b \sec \left(c + d\sqrt{x}\right)\right)^2 dx$$

```
[In] integrate(x**2*(a+b*sec(c+d*x**(1/2)))**2,x)
[Out] Integral(x**2*(a + b*sec(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3879 vs. $2(422) = 844$.

Time = 0.61 (sec) , antiderivative size = 3879, normalized size of antiderivative = 7.04

$$\int x^2 \left(a + b \sec \left(c + d\sqrt{x}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate(x^2*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((d*sqrt(x) + c)^6*a^2 - 6*(d*sqrt(x) + c)^5*a^2*c + 15*(d*sqrt(x) + c)
^4*a^2*c^2 - 20*(d*sqrt(x) + c)^3*a^2*c^3 + 15*(d*sqrt(x) + c)^2*a^2*c^4 -
6*(d*sqrt(x) + c)*a^2*c^5 - 12*a*b*c^5*log(sec(d*sqrt(x) + c) + tan(d*sqrt(
x) + c)) - 6*(12*b^2*c^5 + 12*((d*sqrt(x) + c)^5*a*b - 5*(d*sqrt(x) + c)^4*
a*b*c + 10*(d*sqrt(x) + c)^3*a*b*c^2 - 10*(d*sqrt(x) + c)^2*a*b*c^3 + 5*(d*
sqrt(x) + c)*a*b*c^4 + ((d*sqrt(x) + c)^5*a*b - 5*(d*sqrt(x) + c)^4*a*b*c +
 10*(d*sqrt(x) + c)^3*a*b*c^2 - 10*(d*sqrt(x) + c)^2*a*b*c^3 + 5*(d*sqrt(x)
 + c)*a*b*c^4)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^5*a*b - 5*I*(d*s
qrt(x) + c)^4*a*b*c + 10*I*(d*sqrt(x) + c)^3*a*b*c^2 - 10*I*(d*sqrt(x) + c)
^2*a*b*c^3 + 5*I*(d*sqrt(x) + c)*a*b*c^4)*sin(2*d*sqrt(x) + 2*c))*arctan2(c
os(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) + 12*((d*sqrt(x) + c)^5*a*b - 5*
(d*sqrt(x) + c)^4*a*b*c + 10*(d*sqrt(x) + c)^3*a*b*c^2 - 10*(d*sqrt(x) + c)
^2*a*b*c^3 + 5*(d*sqrt(x) + c)*a*b*c^4 + ((d*sqrt(x) + c)^5*a*b - 5*(d*sqrt
(x) + c)^4*a*b*c + 10*(d*sqrt(x) + c)^3*a*b*c^2 - 10*(d*sqrt(x) + c)^2*a*b*
c^3 + 5*(d*sqrt(x) + c)*a*b*c^4)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c
)^5*a*b - 5*I*(d*sqrt(x) + c)^4*a*b*c + 10*I*(d*sqrt(x) + c)^3*a*b*c^2 - 10
*I*(d*sqrt(x) + c)^2*a*b*c^3 + 5*I*(d*sqrt(x) + c)*a*b*c^4)*sin(2*d*sqrt(x)
 + 2*c))*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) - 10*(6*(d*sq
rt(x) + c)^4*b^2 - 16*(d*sqrt(x) + c)^3*b^2*c + 18*(d*sqrt(x) + c)^2*b^2*c^
2 - 12*(d*sqrt(x) + c)*b^2*c^3 + 3*b^2*c^4 + (6*(d*sqrt(x) + c)^4*b^2 - 16*
(d*sqrt(x) + c)^3*b^2*c + 18*(d*sqrt(x) + c)^2*b^2*c^2 - 12*(d*sqrt(x) + c)
*b^2*c^3 + 3*b^2*c^4)*cos(2*d*sqrt(x) + 2*c) - (-6*I*(d*sqrt(x) + c)^4*b^2
+ 16*I*(d*sqrt(x) + c)^3*b^2*c - 18*I*(d*sqrt(x) + c)^2*b^2*c^2 + 12*I*(d*s
qrt(x) + c)*b^2*c^3 - 3*I*b^2*c^4)*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(2*d*
sqrt(x) + 2*c), cos(2*d*sqrt(x) + 2*c) + 1) + 12*((d*sqrt(x) + c)^5*b^2 - 5
*(d*sqrt(x) + c)^4*b^2*c + 10*(d*sqrt(x) + c)^3*b^2*c^2 - 10*(d*sqrt(x) + c
)^2*b^2*c^3 + 5*(d*sqrt(x) + c)*b^2*c^4)*cos(2*d*sqrt(x) + 2*c) + 60*(2*(d*
sqrt(x) + c)^3*b^2 - 4*(d*sqrt(x) + c)^2*b^2*c + 3*(d*sqrt(x) + c)*b^2*c^2
- b^2*c^3 + (2*(d*sqrt(x) + c)^3*b^2 - 4*(d*sqrt(x) + c)^2*b^2*c + 3*(d*sqr
t(x) + c)*b^2*c^2 - b^2*c^3)*cos(2*d*sqrt(x) + 2*c) + (2*I*(d*sqrt(x) + c)^
3*b^2 - 4*I*(d*sqrt(x) + c)^2*b^2*c + 3*I*(d*sqrt(x) + c)*b^2*c^2 - I*b^2*c
^3)*sin(2*d*sqrt(x) + 2*c))*dilog(-e^(2*I*d*sqrt(x) + 2*I*c)) + 60*((d*sqrt
(x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 -
4*(d*sqrt(x) + c)*a*b*c^3 + a*b*c^4 + ((d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x)
 + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^3 + a
*b*c^4)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^4*a*b - 4*I*(d*sqrt(x)
+ c)^3*a*b*c + 6*I*(d*sqrt(x) + c)^2*a*b*c^2 - 4*I*(d*sqrt(x) + c)*a*b*c^3
+ I*a*b*c^4)*sin(2*d*sqrt(x) + 2*c))*dilog(I*e^(I*d*sqrt(x) + I*c)) - 60*((
d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*
c^2 - 4*(d*sqrt(x) + c)*a*b*c^3 + a*b*c^4 + ((d*sqrt(x) + c)^4*a*b - 4*(d*s
qrt(x) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c
^3 + a*b*c^4)*cos(2*d*sqrt(x) + 2*c) - (-I*(d*sqrt(x) + c)^4*a*b + 4*I*(d*s
qrt(x) + c)^3*a*b*c - 6*I*(d*sqrt(x) + c)^2*a*b*c^2 + 4*I*(d*sqrt(x) + c)*a
*b*c^3 - I*a*b*c^4)*sin(2*d*sqrt(x) + 2*c))*dilog(-I*e^(I*d*sqrt(x) + I*c))
 + 5*(6*I*(d*sqrt(x) + c)^4*b^2 - 16*I*(d*sqrt(x) + c)^3*b^2*c + 18*I*(d*sq
```

rt(x) + c)^2*b^2*c^2 - 12*I*(d*sqrt(x) + c)*b^2*c^3 + 3*I*b^2*c^4 + (6*I*(d
*sqrt(x) + c)^4*b^2 - 16*I*(d*sqrt(x) + c)^3*b^2*c + 18*I*(d*sqrt(x) + c)^2
*b^2*c^2 - 12*I*(d*sqrt(x) + c)*b^2*c^3 + 3*I*b^2*c^4)*cos(2*d*sqrt(x) + 2*
c) - (6*(d*sqrt(x) + c)^4*b^2 - 16*(d*sqrt(x) + c)^3*b^2*c + 18*(d*sqrt(x)
+ c)^2*b^2*c^2 - 12*(d*sqrt(x) + c)*b^2*c^3 + 3*b^2*c^4)*sin(2*d*sqrt(x) +
2*c))*log(cos(2*d*sqrt(x) + 2*c)^2 + sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*s
qrt(x) + 2*c) + 1) + 6*(I*(d*sqrt(x) + c)^5*a*b - 5*I*(d*sqrt(x) + c)^4*a*b
*c + 10*I*(d*sqrt(x) + c)^3*a*b*c^2 - 10*I*(d*sqrt(x) + c)^2*a*b*c^3 + 5*I*
(d*sqrt(x) + c)*a*b*c^4 + (I*(d*sqrt(x) + c)^5*a*b - 5*I*(d*sqrt(x) + c)^4*
a*b*c + 10*I*(d*sqrt(x) + c)^3*a*b*c^2 - 10*I*(d*sqrt(x) + c)^2*a*b*c^3 + 5
I(d*sqrt(x) + c)*a*b*c^4)*cos(2*d*sqrt(x) + 2*c) - ((d*sqrt(x) + c)^5*a*b
 - 5*(d*sqrt(x) + c)^4*a*b*c + 10*(d*sqrt(x) + c)^3*a*b*c^2 - 10*(d*sqrt(x)
 + c)^2*a*b*c^3 + 5*(d*sqrt(x) + c)*a*b*c^4)*sin(2*d*sqrt(x) + 2*c))*log(co
s(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*sqrt(x) + c) + 1) + 6*(
-I*(d*sqrt(x) + c)^5*a*b + 5*I*(d*sqrt(x) + c)^4*a*b*c - 10*I*(d*sqrt(x) +
c)^3*a*b*c^2 + 10*I*(d*sqrt(x) + c)^2*a*b*c^3 - 5*I*(d*sqrt(x) + c)*a*b*c^4
 + (-I*(d*sqrt(x) + c)^5*a*b + 5*I*(d*sqrt(x) + c)^4*a*b*c - 10*I*(d*sqrt(x
) + c)^3*a*b*c^2 + 10*I*(d*sqrt(x) + c)^2*a*b*c^3 - 5*I*(d*sqrt(x) + c)*a*b
*c^4)*cos(2*d*sqrt(x) + 2*c) + ((d*sqrt(x) + c)^5*a*b - 5*(d*sqrt(x) + c)^4
*a*b*c + 10*(d*sqrt(x) + c)^3*a*b*c^2 - 10*(d*sqrt(x) + c)^2*a*b*c^3 + 5*(d
*sqrt(x) + c)*a*b*c^4)*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) + c)^2 + s
in(d*sqrt(x) + c)^2 - 2*sin(d*sqrt(x) + c) + 1) + 1440*(a*b*cos(2*d*sqrt(x)
 + 2*c) + I*a*b*sin(2*d*sqrt(x) + 2*c) + a*b)*polylog(6, I*e^(I*d*sqrt(x) +
 I*c)) - 1440*(a*b*cos(2*d*sqrt(x) + 2*c) + I*a*b*sin(2*d*sqrt(x) + 2*c) +
a*b)*polylog(6, -I*e^(I*d*sqrt(x) + I*c)) + 90*(-I*b^2*cos(2*d*sqrt(x) + 2*
c) + b^2*sin(2*d*sqrt(x) + 2*c) - I*b^2)*polylog(5, -e^(2*I*d*sqrt(x) + 2*I
c)) + 1440(-I*(d*sqrt(x) + c)*a*b + I*a*b*c + (-I*(d*sqrt(x) + c)*a*b + I
*a*b*c)*cos(2*d*sqrt(x) + 2*c) + ((d*sqrt(x) + c)*a*b - a*b*c)*sin(2*d*sqrt
(x) + 2*c))*polylog(5, I*e^(I*d*sqrt(x) + I*c)) + 1440*(I*(d*sqrt(x) + c)*a
*b - I*a*b*c + (I*(d*sqrt(x) + c)*a*b - I*a*b*c)*cos(2*d*sqrt(x) + 2*c) - (
(d*sqrt(x) + c)*a*b - a*b*c)*sin(2*d*sqrt(x) + 2*c))*polylog(5, -I*e^(I*d*s
qrt(x) + I*c)) - 60*(3*(d*sqrt(x) + c)*b^2 - 2*b^2*c + (3*(d*sqrt(x) + c)*b
^2 - 2*b^2*c)*cos(2*d*sqrt(x) + 2*c) - (-3*I*(d*sqrt(x) + c)*b^2 + 2*I*b^2*
c)*sin(2*d*sqrt(x) + 2*c))*polylog(4, -e^(2*I*d*sqrt(x) + 2*I*c)) - 720*((d
*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c + a*b*c^2 + ((d*sqrt(x) + c)^
2*a*b - 2*(d*sqrt(x) + c)*a*b*c + a*b*c^2)*cos(2*d*sqrt(x) + 2*c) - (-I*(d*
sqrt(x) + c)^2*a*b + 2*I*(d*sqrt(x) + c)*a*b*c - I*a*b*c^2)*sin(2*d*sqrt(x)
 + 2*c))*polylog(4, I*e^(I*d*sqrt(x) + I*c)) + 720*((d*sqrt(x) + c)^2*a*b -
 2*(d*sqrt(x) + c)*a*b*c + a*b*c^2 + ((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x)
+ c)*a*b*c + a*b*c^2)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^2*a*b - 2
I(d*sqrt(x) + c)*a*b*c + I*a*b*c^2)*sin(2*d*sqrt(x) + 2*c))*polylog(4, -I
*e^(I*d*sqrt(x) + I*c)) + 30*(6*I*(d*sqrt(x) + c)^2*b^2 - 8*I*(d*sqrt(x) +
c)*b^2*c + 3*I*b^2*c^2 + (6*I*(d*sqrt(x) + c)^2*b^2 - 8*I*(d*sqrt(x) + c)*b
^2*c + 3*I*b^2*c^2)*cos(2*d*sqrt(x) + 2*c) - (6*(d*sqrt(x) + c)^2*b^2 - 8*(
d*sqrt(x) + c)*b^2*c + 3*b^2*c^2)*sin(2*d*sqrt(x) + 2*c))*polylog(3, -e^(2*

```
I*d*sqrt(x) + 2*I*c)) + 240*(I*(d*sqrt(x) + c)^3*a*b - 3*I*(d*sqrt(x) + c)^
2*a*b*c + 3*I*(d*sqrt(x) + c)*a*b*c^2 - I*a*b*c^3 + (I*(d*sqrt(x) + c)^3*a*
b - 3*I*(d*sqrt(x) + c)^2*a*b*c + 3*I*(d*sqrt(x) + c)*a*b*c^2 - I*a*b*c^3)*
cos(2*d*sqrt(x) + 2*c) - ((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c
 + 3*(d*sqrt(x) + c)*a*b*c^2 - a*b*c^3)*sin(2*d*sqrt(x) + 2*c))*polylog(3,
I*e^(I*d*sqrt(x) + I*c)) + 240*(-I*(d*sqrt(x) + c)^3*a*b + 3*I*(d*sqrt(x) +
 c)^2*a*b*c - 3*I*(d*sqrt(x) + c)*a*b*c^2 + I*a*b*c^3 + (-I*(d*sqrt(x) + c)
^3*a*b + 3*I*(d*sqrt(x) + c)^2*a*b*c - 3*I*(d*sqrt(x) + c)*a*b*c^2 + I*a*b*
c^3)*cos(2*d*sqrt(x) + 2*c) + ((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*
a*b*c + 3*(d*sqrt(x) + c)*a*b*c^2 - a*b*c^3)*sin(2*d*sqrt(x) + 2*c))*polylo
g(3, -I*e^(I*d*sqrt(x) + I*c)) + 12*(I*(d*sqrt(x) + c)^5*b^2 - 5*I*(d*sqrt(
x) + c)^4*b^2*c + 10*I*(d*sqrt(x) + c)^3*b^2*c^2 - 10*I*(d*sqrt(x) + c)^2*b
^2*c^3 + 5*I*(d*sqrt(x) + c)*b^2*c^4)*sin(2*d*sqrt(x) + 2*c))/(-6*I*cos(2*d
*sqrt(x) + 2*c) + 6*sin(2*d*sqrt(x) + 2*c) - 6*I))/d^6
```

Giac [F]

$$\int x^2(a+b\sec(c+d\sqrt{x}))^2\,dx=\int(b\sec(d\sqrt{x}+c)+a)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*sec(d*sqrt(x) + c) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\sec(c+d\sqrt{x}))^2\,dx=\int x^2\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2\,dx$$

```
[In] int(x^2*(a + b/cos(c + d*x^(1/2)))^2,x)
[Out] int(x^2*(a + b/cos(c + d*x^(1/2)))^2, x)
```

3.38 $\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 355

$$\begin{aligned}\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx &= -\frac{2ib^2x^{3/2}}{d}+\frac{a^2x^2}{2}-\frac{8iabx^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}\\
&\quad+\frac{6b^2x\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}\\
&\quad+\frac{12iabx\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}\\
&\quad-\frac{12iabx\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}\\
&\quad-\frac{6ib^2\sqrt{x}\operatorname{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3}\\
&\quad-\frac{24ab\sqrt{x}\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}\\
&\quad+\frac{24ab\sqrt{x}\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}\\
&\quad+\frac{3b^2\operatorname{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4}\\
&\quad-\frac{24iab\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}\\
&\quad+\frac{24iab\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{2b^2x^{3/2}\tan\left(c+d\sqrt{x}\right)}{d}\end{aligned}$$

```
[Out] -2*I*b^2*x^(3/2)/d+1/2*a^2*x^2-8*I*a*b*x^(3/2)*arctan(exp(I*(c+d*x^(1/2))))
/d+6*b^2*x*ln(1+exp(2*I*(c+d*x^(1/2))))/d^2+12*I*a*b*x*polylog(2,-I*exp(I*(
c+d*x^(1/2))))/d^2-12*I*a*b*x*polylog(2,I*exp(I*(c+d*x^(1/2))))/d^2+3*b^2*p
olylog(3,-exp(2*I*(c+d*x^(1/2))))/d^4-24*I*a*b*polylog(4,-I*exp(I*(c+d*x^(1
/2))))/d^4+24*I*a*b*polylog(4,I*exp(I*(c+d*x^(1/2))))/d^4-6*I*b^2*polylog(2
,-exp(2*I*(c+d*x^(1/2))))*x^(1/2)/d^3-24*a*b*polylog(3,-I*exp(I*(c+d*x^(1/2
))))*x^(1/2)/d^3+24*a*b*polylog(3,I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^3+2*b^2
*x^(3/2)*tan(c+d*x^(1/2))/d
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4289, 4275, 4266, 2611, 6744, 2320, 6724, 4269, 3800, 2221}

$$\begin{aligned}\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=&\frac{a^2x^2}{2}-\frac{8iabx^{3/2}\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}\\&-\frac{24iab\,\text{PolyLog}\left(4,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}\\&+\frac{24iab\,\text{PolyLog}\left(4,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}\\&-\frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\&+\frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\&+\frac{12iabx\,\text{PolyLog}\left(2,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\&-\frac{12iabx\,\text{PolyLog}\left(2,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\&+\frac{3b^2\,\text{PolyLog}\left(3,-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^4}\\&-\frac{6ib^2\sqrt{x}\,\text{PolyLog}\left(2,-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\&+\frac{6b^2x\log\left(1+e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\&+\frac{2b^2x^{3/2}\tan\left(c+d\sqrt{x}\right)}{d}-\frac{2ib^2x^{3/2}}{d}\end{aligned}$$

```
[In] Int[x*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x^(3/2))/d + (a^2*x^2)/2 - ((8*I)*a*b*x^(3/2)*ArcTan[E^(I*(c +
d*Sqrt[x]))])/d + (6*b^2*x*Log[1 + E^((2*I)*(c + d*Sqrt[x]))])/d^2 + ((12*I
)*a*b*x*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((12*I)*a*b*x*PolyLog
[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - ((6*I)*b^2*Sqrt[x]*PolyLog[2, -E^((2*I)
*(c + d*Sqrt[x]))])/d^3 - (24*a*b*Sqrt[x]*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[
x]))])/d^3 + (24*a*b*Sqrt[x]*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 + (3*
b^2*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))])/d^4 - ((24*I)*a*b*PolyLog[4, (-
I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((24*I)*a*b*PolyLog[4, I*E^(I*(c + d*Sqrt[
x]))])/d^4 + (2*b^2*x^(3/2)*Tan[c + d*Sqrt[x]])/d
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^3(a+b\sec(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(a^2x^3+2abx^3\sec(c+dx)+b^2x^3\sec^2(c+dx)\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{a^2x^2}{2}+(4ab)\text{Subst}\left(\int x^3\sec(c+dx)\,dx, x, \sqrt{x}\right)+\left(2b^2\right)\text{Subst}\left(\int x^3\sec^2(c+dx)\,dx, x, \sqrt{x}\right)
\end{aligned}$$

$$= \frac{a^2x^2}{2} - \frac{8iabx^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{2b^2x^{3/2}\tan\left(c+d\sqrt{x}\right)}{d}$$
$$- \frac{(12ab)\text{Subst}\left(\int x^2\log\left(1-ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d}$$
$$+ \frac{(12ab)\text{Subst}\left(\int x^2\log\left(1+ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d}$$
$$- \frac{(6b^2)\,\text{Subst}\left(\int x^2\tan(c+dx)\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8iabx^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}$$
$$+ \frac{12iabx\,\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12iabx\,\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{2b^2x^{3/2}\tan\left(c+d\sqrt{x}\right)}{d} - \frac{(24iab)\text{Subst}\left(\int x\,\text{PolyLog}\left(2, -ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$
$$+ \frac{(24iab)\text{Subst}\left(\int x\,\text{PolyLog}\left(2, ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$
$$+ \frac{(12ib^2)\,\text{Subst}\left(\int \frac{e^{2i(c+dx)}x^2}{1+e^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8iabx^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6b^2x\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{12iabx\,\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12iabx\,\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{2b^2x^{3/2}\tan\left(c+d\sqrt{x}\right)}{d} + \frac{(24ab)\text{Subst}\left(\int \text{PolyLog}\left(3, -ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3}$$
$$- \frac{(24ab)\text{Subst}\left(\int \text{PolyLog}\left(3, ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3}$$
$$- \frac{(12b^2)\,\text{Subst}\left(\int x\log\left(1+e^{2i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$

$$= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8iabx^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6b^2x\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{12iabx\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12iabx\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ib^2\sqrt{x}\,\text{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{2b^2x^{3/2}\tan\left(c+d\sqrt{x}\right)}{d} - \frac{(24iab)\text{Subst}\left(\int\frac{\text{PolyLog}(3,-ix)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(24iab)\text{Subst}\left(\int\frac{\text{PolyLog}(3,ix)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(6ib^2)\,\text{Subst}\left(\int\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8iabx^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6b^2x\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{12iabx\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12iabx\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ib^2\sqrt{x}\,\text{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{24iab\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{24iab\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{2b^2x^{3/2}\tan\left(c+d\sqrt{x}\right)}{d} + \frac{(3b^2)\,\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{2i(c+d\sqrt{x})}\right)}{d^4}$$

$$= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8iabx^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6b^2x\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{12iabx\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12iabx\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ib^2\sqrt{x}\,\text{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{3b^2\,\text{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{24iab\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{24iab\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{2b^2x^{3/2}\tan\left(c+d\sqrt{x}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.98

$$\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx = \frac{-4ib^2d^3x^{3/2}+a^2d^4x^2-16iabd^3x^{3/2}\arctan\left(e^{i(c+d\sqrt{x})}\right)+12b^2d^2x\log\left(1+e^{2i(c+d\sqrt{x})}\right)+24iabd^2x\,\text{PolyLo}}{}$$

```
[In] Integrate[x*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-4*I)*b^2*d^3*x^(3/2) + a^2*d^4*x^2 - (16*I)*a*b*d^3*x^(3/2)*ArcTan[E^(I*
(c + d*Sqrt[x]))] + 12*b^2*d^2*x*Log[1 + E^((2*I)*(c + d*Sqrt[x]))] + (24*I
)*a*b*d^2*x*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (24*I)*a*b*d^2*x*PolyL
og[2, I*E^(I*(c + d*Sqrt[x]))] - (12*I)*b^2*d*Sqrt[x]*PolyLog[2, -E^((2*I)*
(c + d*Sqrt[x]))] - 48*a*b*d*Sqrt[x]*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))]
 + 48*a*b*d*Sqrt[x]*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))] + 6*b^2*PolyLog[3,
-E^((2*I)*(c + d*Sqrt[x]))] - (48*I)*a*b*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x
]))] + (48*I)*a*b*PolyLog[4, I*E^(I*(c + d*Sqrt[x]))] + 4*b^2*d^3*x^(3/2)*T
an[c + d*Sqrt[x]])/(2*d^4)
```

Maple [F]

$$\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2 dx$$

```
[In] int(x*(a+b*sec(c+d*x^(1/2)))^2,x)
[Out] int(x*(a+b*sec(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(b^2*x*sec(d*sqrt(x) + c)^2 + 2*a*b*x*sec(d*sqrt(x) + c) + a^2*x, x
)
```

Sympy [F]

$$\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx$$

```
[In] integrate(x*(a+b*sec(c+d*x**(1/2)))**2,x)
[Out] Integral(x*(a + b*sec(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1991 vs. $2(270) = 540$.

Time = 0.45 (sec) , antiderivative size = 1991, normalized size of antiderivative = 5.61

$$\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(x*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] 1/2*((d*sqrt(x) + c)^4*a^2 - 4*(d*sqrt(x) + c)^3*a^2*c + 6*(d*sqrt(x) + c)^
2*a^2*c^2 - 4*(d*sqrt(x) + c)*a^2*c^3 - 8*a*b*c^3*log(sec(d*sqrt(x) + c) +
tan(d*sqrt(x) + c)) - 4*(4*b^2*c^3 + 4*((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x
) + c)^2*a*b*c + 3*(d*sqrt(x) + c)*a*b*c^2 + ((d*sqrt(x) + c)^3*a*b - 3*(d*
sqrt(x) + c)^2*a*b*c + 3*(d*sqrt(x) + c)*a*b*c^2)*cos(2*d*sqrt(x) + 2*c) +
```

```
(I*(d*sqrt(x) + c)^3*a*b - 3*I*(d*sqrt(x) + c)^2*a*b*c + 3*I*(d*sqrt(x) + c
)*a*b*c^2)*sin(2*d*sqrt(x) + 2*c))*arctan2(cos(d*sqrt(x) + c), sin(d*sqrt(x
) + c) + 1) + 4*((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3*(d*s
qrt(x) + c)*a*b*c^2 + ((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c +
3*(d*sqrt(x) + c)*a*b*c^2)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^3*a*
b - 3*I*(d*sqrt(x) + c)^2*a*b*c + 3*I*(d*sqrt(x) + c)*a*b*c^2)*sin(2*d*sqrt
(x) + 2*c))*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) - 6*((d*sq
rt(x) + c)^2*b^2 - 2*(d*sqrt(x) + c)*b^2*c + b^2*c^2 + ((d*sqrt(x) + c)^2*b
^2 - 2*(d*sqrt(x) + c)*b^2*c + b^2*c^2)*cos(2*d*sqrt(x) + 2*c) - (-I*(d*sqr
t(x) + c)^2*b^2 + 2*I*(d*sqrt(x) + c)*b^2*c - I*b^2*c^2)*sin(2*d*sqrt(x) +
2*c))*arctan2(sin(2*d*sqrt(x) + 2*c), cos(2*d*sqrt(x) + 2*c) + 1) + 4*((d*s
qrt(x) + c)^3*b^2 - 3*(d*sqrt(x) + c)^2*b^2*c + 3*(d*sqrt(x) + c)*b^2*c^2)*
cos(2*d*sqrt(x) + 2*c) + 6*((d*sqrt(x) + c)*b^2 - b^2*c + ((d*sqrt(x) + c)*
b^2 - b^2*c)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)*b^2 - I*b^2*c)*sin
(2*d*sqrt(x) + 2*c))*dilog(-e^(2*I*d*sqrt(x) + 2*I*c)) + 12*((d*sqrt(x) + c
)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c + a*b*c^2 + ((d*sqrt(x) + c)^2*a*b - 2*(d
*sqrt(x) + c)*a*b*c + a*b*c^2)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^
2*a*b - 2*I*(d*sqrt(x) + c)*a*b*c + I*a*b*c^2)*sin(2*d*sqrt(x) + 2*c))*dilo
g(I*e^(I*d*sqrt(x) + I*c)) - 12*((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*
a*b*c + a*b*c^2 + ((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c + a*b*c^
2)*cos(2*d*sqrt(x) + 2*c) - (-I*(d*sqrt(x) + c)^2*a*b + 2*I*(d*sqrt(x) + c)
*a*b*c - I*a*b*c^2)*sin(2*d*sqrt(x) + 2*c))*dilog(-I*e^(I*d*sqrt(x) + I*c))
 + 3*(I*(d*sqrt(x) + c)^2*b^2 - 2*I*(d*sqrt(x) + c)*b^2*c + I*b^2*c^2 + (I*
(d*sqrt(x) + c)^2*b^2 - 2*I*(d*sqrt(x) + c)*b^2*c + I*b^2*c^2)*cos(2*d*sqrt
(x) + 2*c) - ((d*sqrt(x) + c)^2*b^2 - 2*(d*sqrt(x) + c)*b^2*c + b^2*c^2)*si
n(2*d*sqrt(x) + 2*c))*log(cos(2*d*sqrt(x) + 2*c)^2 + sin(2*d*sqrt(x) + 2*c)
^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1) + 2*(I*(d*sqrt(x) + c)^3*a*b - 3*I*(d*sq
rt(x) + c)^2*a*b*c + 3*I*(d*sqrt(x) + c)*a*b*c^2 + (I*(d*sqrt(x) + c)^3*a*b
 - 3*I*(d*sqrt(x) + c)^2*a*b*c + 3*I*(d*sqrt(x) + c)*a*b*c^2)*cos(2*d*sqrt(
x) + 2*c) - ((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3*(d*sqrt(
x) + c)*a*b*c^2)*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) + c)^2 + sin(d*s
qrt(x) + c)^2 + 2*sin(d*sqrt(x) + c) + 1) + 2*(-I*(d*sqrt(x) + c)^3*a*b + 3
*I*(d*sqrt(x) + c)^2*a*b*c - 3*I*(d*sqrt(x) + c)*a*b*c^2 + (-I*(d*sqrt(x) +
 c)^3*a*b + 3*I*(d*sqrt(x) + c)^2*a*b*c - 3*I*(d*sqrt(x) + c)*a*b*c^2)*cos(
2*d*sqrt(x) + 2*c) + ((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3
*(d*sqrt(x) + c)*a*b*c^2)*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) + c)^2
+ sin(d*sqrt(x) + c)^2 - 2*sin(d*sqrt(x) + c) + 1) - 24*(a*b*cos(2*d*sqrt(x
) + 2*c) + I*a*b*sin(2*d*sqrt(x) + 2*c) + a*b)*polylog(4, I*e^(I*d*sqrt(x)
+ I*c)) + 24*(a*b*cos(2*d*sqrt(x) + 2*c) + I*a*b*sin(2*d*sqrt(x) + 2*c) + a
*b)*polylog(4, -I*e^(I*d*sqrt(x) + I*c)) + 3*(I*b^2*cos(2*d*sqrt(x) + 2*c)
- b^2*sin(2*d*sqrt(x) + 2*c) + I*b^2)*polylog(3, -e^(2*I*d*sqrt(x) + 2*I*c)
) + 24*(I*(d*sqrt(x) + c)*a*b - I*a*b*c + (I*(d*sqrt(x) + c)*a*b - I*a*b*c)
*cos(2*d*sqrt(x) + 2*c) - ((d*sqrt(x) + c)*a*b - a*b*c)*sin(2*d*sqrt(x) + 2
*c))*polylog(3, I*e^(I*d*sqrt(x) + I*c)) + 24*(-I*(d*sqrt(x) + c)*a*b + I*a
*b*c + (-I*(d*sqrt(x) + c)*a*b + I*a*b*c)*cos(2*d*sqrt(x) + 2*c) + ((d*sqrt
```

```
(x) + c)*a*b - a*b*c)*sin(2*d*sqrt(x) + 2*c))*polylog(3, -I*e^(I*d*sqrt(x)
+ I*c)) + 4*(I*(d*sqrt(x) + c)^3*b^2 - 3*I*(d*sqrt(x) + c)^2*b^2*c + 3*I*(d
*sqrt(x) + c)*b^2*c^2)*sin(2*d*sqrt(x) + 2*c))/(-2*I*cos(2*d*sqrt(x) + 2*c)
 + 2*sin(2*d*sqrt(x) + 2*c) - 2*I))/d^4
```

Giac [F]

$$\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*sec(d*sqrt(x) + c) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2\,dx$$

```
[In] int(x*(a + b/cos(c + d*x^(1/2)))^2,x)
[Out] int(x*(a + b/cos(c + d*x^(1/2)))^2, x)
```

3.39 $\int \frac{(a+b\sec(c+d\sqrt{x}))^2}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\text{Int}\left(\frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x},x\right)$$

```
[Out] Unintegrable((a+b*sec(c+d*x^(1/2)))^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

```
[In] Int[(a + b*Sec[c + d*Sqrt[x]])^2/x,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*Sqrt[x]])^2/x, x]
```

Rubi steps

$$\text{integral}=\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 109.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x,x]
[Out] Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.71 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x}dx$$

```
[In] int((a+b*sec(c+d*x^(1/2)))^2/x,x)
[Out] int((a+b*sec(c+d*x^(1/2)))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x,x, algorithm="fricas")
[Out] integral((b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 8.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

```
[In] integrate((a+b*sec(c+d*x**(1/2)))**2/x,x)
[Out] Integral((a + b*sec(c + d*sqrt(x)))**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 319, normalized size of antiderivative = 15.95

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x,x, algorithm="maxima")
```

```
[Out] (4*b^2*sqrt(x)*sin(2*d*sqrt(x) + 2*c) + (d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin
(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x*integrate(2*(b^2*
sqrt(x)*sin(2*d*sqrt(x) + 2*c) + 2*(a*b*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt
(x) + c) + a*b*d*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*cos(d*sq
rt(x) + c))*x)/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 +
2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^2), x) + (a^2*d*cos(2*d*sqrt(x) + 2*c)^2
+ a^2*d*sin(2*d*sqrt(x) + 2*c)^2 + 2*a^2*d*cos(2*d*sqrt(x) + 2*c) + a^2*d)*
x*log(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*c
os(2*d*sqrt(x) + 2*c) + d)*x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*sqrt(x) + c) + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 13.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+\frac{b}{\cos\left(c+d\,\sqrt{x}\right)}\right)^2}{x}\,dx$$

```
[In] int((a + b/cos(c + d*x^(1/2)))^2/x,x)
```

```
[Out] int((a + b/cos(c + d*x^(1/2)))^2/x, x)
```

3.40 $\int \frac{(a+b\sec(c+d\sqrt{x}))^2}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\text{Int}\left(\frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*sec(c+d*x^(1/2)))^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

```
[In] Int[(a + b*Sec[c + d*Sqrt[x]])^2/x^2,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*Sqrt[x]])^2/x^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 74.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x^2,x]
[Out] Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.78 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2}dx$$

```
[In] int((a+b*sec(c+d*x^(1/2)))^2/x^2,x)
[Out] int((a+b*sec(c+d*x^(1/2)))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2)/x^2, x
)
```

Sympy [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sec(c+d*x**(1/2)))**2/x**2,x)
[Out] Integral((a + b*sec(c + d*sqrt(x)))**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 321, normalized size of antiderivative = 16.05

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x^2,x, algorithm="maxima")
[Out] ((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqr
t(x) + 2*c) + d)*x^2*integrate(2*(3*b^2*sqrt(x)*sin(2*d*sqrt(x) + 2*c) + 2*
(a*b*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + a*b*d*sin(2*d*sqrt(x) +
2*c)*sin(d*sqrt(x) + c) + a*b*d*cos(d*sqrt(x) + c))*x)/((d*cos(2*d*sqrt(x)
+ 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^3
), x) + 4*b^2*sqrt(x)*sin(2*d*sqrt(x) + 2*c) - (a^2*d*cos(2*d*sqrt(x) + 2*c
)^2 + a^2*d*sin(2*d*sqrt(x) + 2*c)^2 + 2*a^2*d*cos(2*d*sqrt(x) + 2*c) + a^2
*d)*x)/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(
2*d*sqrt(x) + 2*c) + d)*x^2)
```

Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x^2,x, algorithm="giac")
[Out] integrate((b*sec(d*sqrt(x) + c) + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 13.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+\frac{b}{\cos\left(c+d\,\sqrt{x}\right)}\right)^2}{x^2}\,dx$$

```
[In] int((a + b/cos(c + d*x^(1/2)))^2/x^2,x)
[Out] int((a + b/cos(c + d*x^(1/2)))^2/x^2, x)
```

3.41 $\int \frac{x^3}{a+b\sec(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 1041

$$\int \frac{x^3}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx=\frac{x^4}{4a}+\frac{2ibx^{7/2}\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^{7/2}\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+\frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$-\frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+\frac{84ibx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$-\frac{84ibx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$-\frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$+\frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$-\frac{1680ibx^{3/2}\,\text{PolyLog}\left(5,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$+\frac{1680ibx^{3/2}\,\text{PolyLog}\left(5,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$+\frac{5040bx\,\text{PolyLog}\left(6,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$
$$-\frac{5040bx\,\text{PolyLog}\left(6,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$
$$+\frac{10080ib\sqrt{x}\,\text{PolyLog}\left(7,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7}$$
$$-\frac{10080ib\sqrt{x}\,\text{PolyLog}\left(7,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7}$$
$$-\frac{10080b\,\text{PolyLog}\left(8,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8}$$
$$+\frac{10080b\,\text{PolyLog}\left(8,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8}$$

```
[Out] 1/4*x^4/a-84*I*b*x^(5/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1
/2)))/a/d^3/(-a^2+b^2)^(1/2)+2*I*b*x^(7/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-(
-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)^(1/2)+14*b*x^3*polylog(2,-a*exp(I*(c+d*x^(
1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)-14*b*x^3*polylog(2,-a*e
xp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)-2*I*b*x^(7
/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)^(1/2)+
10080*I*b*polylog(7,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a
/d^7/(-a^2+b^2)^(1/2)-420*b*x^2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+
b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)+420*b*x^2*polylog(4,-a*exp(I*(c+d*x^(1/
2)))/(b+(-a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)-1680*I*b*x^(3/2)*polylog(
5,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^5/(-a^2+b^2)^(1/2)-1008
0*I*b*polylog(7,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^7
/(-a^2+b^2)^(1/2)+5040*b*x*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^
(1/2)))/a/d^6/(-a^2+b^2)^(1/2)-5040*b*x*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(
b+(-a^2+b^2)^(1/2)))/a/d^6/(-a^2+b^2)^(1/2)-10080*b*polylog(8,-a*exp(I*(c+d
*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^8/(-a^2+b^2)^(1/2)+10080*b*polylog(8,-
a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^8/(-a^2+b^2)^(1/2)+84*I*b*
x^(5/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^3/(-a^2
+b^2)^(1/2)+1680*I*b*x^(3/2)*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2
)^(1/2)))/a/d^5/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 1041, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used

$= \{4289, 4276, 3402, 2296, 2221, 2611, 6744, 2320, 6724\}$

$$\int \frac{x^3}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx = \frac{x^4}{4a} + \frac{2ib\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}}+1\right)x^{7/2}}{a\sqrt{b^2-a^2}d} - \frac{2ib\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}}+1\right)x^{7/2}}{a\sqrt{b^2-a^2}d}$$
$$+ \frac{14b\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^3}{a\sqrt{b^2-a^2}d^2}$$
$$- \frac{14b\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^3}{a\sqrt{b^2-a^2}d^2}$$
$$+ \frac{84ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{5/2}}{a\sqrt{b^2-a^2}d^3}$$
$$- \frac{84ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{5/2}}{a\sqrt{b^2-a^2}d^3}$$
$$- \frac{420b\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^2}{a\sqrt{b^2-a^2}d^4}$$
$$+ \frac{420b\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^2}{a\sqrt{b^2-a^2}d^4}$$
$$- \frac{1680ib\,\mathrm{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{3/2}}{a\sqrt{b^2-a^2}d^5}$$
$$+ \frac{1680ib\,\mathrm{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{3/2}}{a\sqrt{b^2-a^2}d^5}$$
$$+ \frac{5040b\,\mathrm{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x}{a\sqrt{b^2-a^2}d^6}$$
$$- \frac{5040b\,\mathrm{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x}{a\sqrt{b^2-a^2}d^6}$$
$$+ \frac{10080ib\,\mathrm{PolyLog}\left(7,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)\sqrt{x}}{a\sqrt{b^2-a^2}d^7}$$
$$- \frac{10080ib\,\mathrm{PolyLog}\left(7,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)\sqrt{x}}{a\sqrt{b^2-a^2}d^7}$$
$$- \frac{10080b\,\mathrm{PolyLog}\left(8,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{a\sqrt{b^2-a^2}d^8}$$
$$+ \frac{10080b\,\mathrm{PolyLog}\left(8,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{a\sqrt{b^2-a^2}d^8}$$

```
[In] Int[x^3/(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] x^4/(4*a) + ((2*I)*b*x^(7/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a
^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - ((2*I)*b*x^(7/2)*Log[1 + (a*E^(I*(c +
 d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (14*b*x^3*P
olyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^
2 + b^2]*d^2) - (14*b*x^3*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[
-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + ((84*I)*b*x^(5/2)*PolyLog[3, -((
a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3)
 - ((84*I)*b*x^(5/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2
+ b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (420*b*x^2*PolyLog[4, -((a*E^(I*(c +
d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (420*b*x^
2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[
-a^2 + b^2]*d^4) - ((1680*I)*b*x^(3/2)*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])
))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5) + ((1680*I)*b*x^(3/2)
*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-
a^2 + b^2]*d^5) + (5040*b*x*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqr
t[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^6) - (5040*b*x*PolyLog[6, -((a*E^(I
*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^6) + ((1
0080*I)*b*Sqrt[x]*PolyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b
^2]))])/(a*Sqrt[-a^2 + b^2]*d^7) - ((10080*I)*b*Sqrt[x]*PolyLog[7, -((a*E^(
I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^7) - (1
0080*b*PolyLog[8, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*
Sqrt[-a^2 + b^2]*d^8) + (10080*b*PolyLog[8, -((a*E^(I*(c + d*Sqrt[x])))/(b
+ Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^8)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^7}{a+b\sec(c+dx)}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{x^7}{a} - \frac{bx^7}{a(b+a\cos(c+dx))}\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^4}{4a} - \frac{(2b)\text{Subst}\left(\int \frac{x^7}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{x^4}{4a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{x^4}{4a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}} + \frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}$$

$$= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{(14ib)\text{Subst}\left(\int x^6\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d} + \frac{(14ib)\text{Subst}\left(\int x^6\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}$$

$$= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{(84b)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(84b)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{84ibx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{(420ib)\text{Subst}\left(\int x^4\,\text{PolyLog}\left(3,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$+ \frac{(420ib)\text{Subst}\left(\int x^4\,\text{PolyLog}\left(3,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{84ibx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$+ \frac{(1680b)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(4,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$- \frac{(1680b)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(4,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}$$

$$= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$- \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$+ \frac{(5040ib)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(5,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$- \frac{(5040ib)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(5,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^5}$$

$$= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$- \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$+ \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} - \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$
$$- \frac{(10080b)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(6,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^6}$$
$$+ \frac{(10080b)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(6,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^6}$$

$$= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$- \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$+ \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} - \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$
$$+ \frac{10080ib\sqrt{x}\operatorname{PolyLog}\left(7,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} - \frac{10080ib\sqrt{x}\operatorname{PolyLog}\left(7,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7}$$
$$- \frac{(10080ib)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(7,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^7}$$
$$+ \frac{(10080ib)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(7,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^7}$$

$$= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{84ibx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$- \frac{1680ibx^{3/2}\,\text{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{1680ibx^{3/2}\,\text{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$+ \frac{5040bx\,\text{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} - \frac{5040bx\,\text{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$
$$+ \frac{10080ib\sqrt{x}\,\text{PolyLog}\left(7,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} - \frac{10080ib\sqrt{x}\,\text{PolyLog}\left(7,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7}$$
$$- \frac{(10080b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(7,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^8}$$
$$+ \frac{(10080b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(7,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^8}$$

$$
\begin{aligned}
&= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
&+ \frac{14bx^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
&+ \frac{84ibx^{5/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
&- \frac{420bx^2\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
&- \frac{1680ibx^{3/2}\,\mathrm{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{1680ibx^{3/2}\,\mathrm{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \\
&+ \frac{5040bx\,\mathrm{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} - \frac{5040bx\,\mathrm{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} \\
&+ \frac{10080ib\sqrt{x}\,\mathrm{PolyLog}\left(7,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} - \frac{10080ib\sqrt{x}\,\mathrm{PolyLog}\left(7,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} \\
&- \frac{10080b\,\mathrm{PolyLog}\left(8,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8} + \frac{10080b\,\mathrm{PolyLog}\left(8,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 802, normalized size of antiderivative = 0.77

$$
\int \frac{x^3}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx
= \frac{\sqrt{-a^2+b^2}d^8x^4 + 8ibd^7x^{7/2}\log\left(1-\frac{ae^{i(c+d\sqrt{x})}}{-b+\sqrt{-a^2+b^2}}\right) - 8ibd^7x^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right) + 56bd^6x^3\,\mathrm{PolyLog}\left(2\right.}{}
$$

```
[In] Integrate[x^3/(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (Sqrt[-a^2 + b^2]*d^8*x^4 + (8*I)*b*d^7*x^(7/2)*Log[1 - (a*E^(I*(c + d*Sqrt
[x])))/(-b + Sqrt[-a^2 + b^2])] - (8*I)*b*d^7*x^(7/2)*Log[1 + (a*E^(I*(c +
d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])] + 56*b*d^6*x^3*PolyLog[2, (a*E^(I*(c +
 d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - 56*b*d^6*x^3*PolyLog[2, -((a*E^(I*
(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] + (336*I)*b*d^5*x^(5/2)*PolyLog[
3, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - (336*I)*b*d^5*x^(5/
2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] - 1680*b
*d^4*x^2*PolyLog[4, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] + 16
80*b*d^4*x^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))
] - (6720*I)*b*d^3*x^(3/2)*PolyLog[5, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[
```

```
-a^2 + b^2])] + (6720*I)*b*d^3*x^(3/2)*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])
))/(b + Sqrt[-a^2 + b^2]))] + 20160*b*d^2*x*PolyLog[6, (a*E^(I*(c + d*Sqrt[
x])))/(-b + Sqrt[-a^2 + b^2])] - 20160*b*d^2*x*PolyLog[6, -((a*E^(I*(c + d*
Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] + (40320*I)*b*d*Sqrt[x]*PolyLog[7, (a*E
^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - (40320*I)*b*d*Sqrt[x]*Poly
Log[7, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] - 40320*b*PolyL
og[8, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] + 40320*b*PolyLog[
8, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(4*a*Sqrt[-a^2 + b
^2]*d^8)
```

Maple [F]

$$\int \frac{x^3}{a + b\sec\left(c + d\sqrt{x}\right)}dx$$

[In] int(x^3/(a+b*sec(c+d*x^(1/2))),x)

[Out] int(x^3/(a+b*sec(c+d*x^(1/2))),x)

Fricas [F]

$$\int \frac{x^3}{a + b\sec\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^3}{b\sec\left(d\sqrt{x} + c\right) + a}\,dx$$

[In] integrate(x^3/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")

[Out] integral(x^3/(b*sec(d*sqrt(x) + c) + a), x)

Sympy [F]

$$\int \frac{x^3}{a + b\sec\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^3}{a + b\sec\left(c + d\sqrt{x}\right)}\,dx$$

[In] integrate(x**3/(a+b*sec(c+d*x**(1/2))),x)

[Out] Integral(x**3/(a + b*sec(c + d*sqrt(x))), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a+b\sec \left(c+d\sqrt{x}\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^3}{a+b\sec \left(c+d\sqrt{x}\right)}\,dx = \int \frac{x^3}{b\sec \left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^3/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*sec(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\sec \left(c+d\sqrt{x}\right)}\,dx = \int \frac{x^3}{a+\frac{b}{\cos\left(c+d\,\sqrt{x}\right)}}\,dx$$

```
[In] int(x^3/(a + b/cos(c + d*x^(1/2))),x)
```

```
[Out] int(x^3/(a + b/cos(c + d*x^(1/2))), x)
```

3.42 $\int \frac{x^2}{a+b\sec(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 781

$$\int \frac{x^2}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx=\frac{x^3}{3a}+\frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{40ibx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{40ibx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}+\frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}-\frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}+\frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}+\frac{240b\,\text{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}-\frac{240b\,\text{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$

```
[Out] 1/3*x^3/a+2*I*b*x^(5/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a
/d/(-a^2+b^2)^(1/2)-2*I*b*x^(5/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)
^(1/2)))/a/d/(-a^2+b^2)^(1/2)+10*b*x^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b
-(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)-10*b*x^2*polylog(2,-a*exp(I*(c+d
*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)+40*I*b*x^(3/2)*poly
log(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)-
40*I*b*x^(3/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^
3/(-a^2+b^2)^(1/2)-120*b*x*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^
(1/2)))/a/d^4/(-a^2+b^2)^(1/2)+120*b*x*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b
```

```
+(-a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)+240*b*polylog(6,-a*exp(I*(c+d*x^
(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^6/(-a^2+b^2)^(1/2)-240*b*polylog(6,-a*exp
(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^6/(-a^2+b^2)^(1/2)-240*I*b*poly
log(5,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^5/(-a^2+b^2
)^(1/2)+240*I*b*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(
1/2)/a/d^5/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {4289, 4276, 3402, 2296, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{x^2}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx = \frac{240b\,\text{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^6\sqrt{b^2-a^2}} - \frac{240b\,\text{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^6\sqrt{b^2-a^2}} - \frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} + \frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} + \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad\sqrt{b^2-a^2}} - \frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad\sqrt{b^2-a^2}} + \frac{x^3}{3a}$$

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[In] Int[x^2/(a + b*Sec[c + d*Sqrt[x]]),x]
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```
[Out] x^3/(3*a) + ((2*I)*b*x^(5/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a
^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - ((2*I)*b*x^(5/2)*Log[1 + (a*E^(I*(c +
 d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (10*b*x^2*P
olyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^
2 + b^2]*d^2) - (10*b*x^2*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[
-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + ((40*I)*b*x^(3/2)*PolyLog[3, -((
a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3)
 - ((40*I)*b*x^(3/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2
+ b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (120*b*x*PolyLog[4, -((a*E^(I*(c + d*
Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (120*b*x*Po
lyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2
 + b^2]*d^4) - ((240*I)*b*Sqrt[x]*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b
 - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5) + ((240*I)*b*Sqrt[x]*PolyL
og[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 +
b^2]*d^5) + (240*b*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 +
b^2]))])/(a*Sqrt[-a^2 + b^2]*d^6) - (240*b*PolyLog[6, -((a*E^(I*(c + d*Sqrt
[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^6)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{x^5}{a+b\sec(c+dx)}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \left(\frac{x^5}{a}-\frac{bx^5}{a(b+a\cos(c+dx))}\right)\,dx, x, \sqrt{x}\right)\end{aligned}$$

$$= \frac{x^3}{3a} - \frac{(2b)\text{Subst}\left(\int \frac{x^5}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{x^3}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{x^3}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}} + \frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}$$

$$= \frac{x^3}{3a} + \frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{(10ib)\text{Subst}\left(\int x^4\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d} + \frac{(10ib)\text{Subst}\left(\int x^4\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}$$

$$= \frac{x^3}{3a} + \frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{(40b)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(40b)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$
\begin{aligned}
&=\frac{x^3}{3a}+\frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\
&+\frac{10bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{10bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}\\
&+\frac{40ibx^{3/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{40ibx^{3/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}\\
&-\frac{(120ib)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(3,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}\\
&+\frac{(120ib)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(3,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{x^3}{3a}+\frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\
&+\frac{10bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{10bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}\\
&+\frac{40ibx^{3/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{40ibx^{3/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}\\
&-\frac{120bx\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}+\frac{120bx\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}\\
&+\frac{(240b)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(4,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}\\
&-\frac{(240b)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(4,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}
\end{aligned}
$$

$$= \frac{x^3}{3a} + \frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{10bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{10bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{40ibx^{3/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{40ibx^{3/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{120bx\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{120bx\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$- \frac{240ib\sqrt{x}\,\mathrm{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{240ib\sqrt{x}\,\mathrm{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$+ \frac{(240ib)\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(5,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$- \frac{(240ib)\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(5,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^5}$$

$$= \frac{x^3}{3a} + \frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{10bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{10bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{40ibx^{3/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{40ibx^{3/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{120bx\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{120bx\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$- \frac{240ib\sqrt{x}\,\mathrm{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{240ib\sqrt{x}\,\mathrm{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$+ \frac{(240b)\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}\left(5,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^6}$$
$$- \frac{(240b)\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}\left(5,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^6}$$

$$
\begin{aligned}
&= \frac{x^3}{3a} + \frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
&+ \frac{10bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{10bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
&+ \frac{40ibx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{40ibx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
&- \frac{120bx\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{120bx\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
&- \frac{240ib\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{240ib\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \\
&+ \frac{240b\operatorname{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} - \frac{240b\operatorname{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 608, normalized size of antiderivative = 0.78

$$
\int \frac{x^2}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx
$$
$$
= \frac{\sqrt{-a^2+b^2}d^6x^3 + 6ibd^5x^{5/2}\log\left(1-\frac{ae^{i(c+d\sqrt{x})}}{-b+\sqrt{-a^2+b^2}}\right) - 6ibd^5x^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right) + 30bd^4x^2\operatorname{PolyLog}\Big(2,}{}
$$

```
[In] Integrate[x^2/(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (Sqrt[-a^2 + b^2]*d^6*x^3 + (6*I)*b*d^5*x^(5/2)*Log[1 - (a*E^(I*(c + d*Sqrt
[x])))/(-b + Sqrt[-a^2 + b^2])] - (6*I)*b*d^5*x^(5/2)*Log[1 + (a*E^(I*(c +
d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])] + 30*b*d^4*x^2*PolyLog[2, (a*E^(I*(c +
 d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - 30*b*d^4*x^2*PolyLog[2, -((a*E^(I*
(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] + (120*I)*b*d^3*x^(3/2)*PolyLog[
3, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - (120*I)*b*d^3*x^(3/
2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] - 360*b*
d^2*x*PolyLog[4, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] + 360*b
*d^2*x*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] - (7
20*I)*b*d*Sqrt[x]*PolyLog[5, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^
2])] + (720*I)*b*d*Sqrt[x]*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt
[-a^2 + b^2]))] + 720*b*PolyLog[6, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^
2 + b^2])] - 720*b*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 +
b^2]))])/(3*a*Sqrt[-a^2 + b^2]*d^6)
```

Maple [F]

$$\int \frac{x^2}{a+b\sec\left(c+d\sqrt{x}\right)}dx$$

```
[In] int(x^2/(a+b*sec(c+d*x^(1/2))),x)
[Out] int(x^2/(a+b*sec(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x^2}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx = \int \frac{x^2}{b\sec\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^2/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(x^2/(b*sec(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x^2}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx = \int \frac{x^2}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x**2/(a+b*sec(c+d*x**(1/2))),x)
[Out] Integral(x**2/(a + b*sec(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^2}{a + b\sec\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^2}{b\sec\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x^2/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(x^2/(b*sec(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + b\sec\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^2}{a + \frac{b}{\cos\left(c + d\sqrt{x}\right)}}\,dx$$

```
[In] int(x^2/(a + b/cos(c + d*x^(1/2))),x)
[Out] int(x^2/(a + b/cos(c + d*x^(1/2))), x)
```

3.43 $\int \frac{x}{a+b\sec(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 521

$$\int \frac{x}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx = \frac{x^2}{2a}+\frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{6bx\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{6bx\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{12ib\sqrt{x}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{12ib\sqrt{x}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{12b\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{12b\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$

```
[Out] 1/2*x^2/a+2*I*b*x^(3/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a
/d/(-a^2+b^2)^(1/2)-2*I*b*x^(3/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)
^(1/2)))/a/d/(-a^2+b^2)^(1/2)+6*b*x*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-(-
a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)-6*b*x*polylog(2,-a*exp(I*(c+d*x^(1/
2)))/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)-12*b*polylog(4,-a*exp(I*(
c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)+12*b*polylog(4,-
```

```
a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)+12*I*b*
polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^3/(-a^2
+b^2)^(1/2)-12*I*b*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*
x^(1/2)/a/d^3/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4289, 4276, 3402, 2296, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{x}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx = -\frac{12b\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}}+\frac{12b\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}}+\frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}-\frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}+\frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}-\frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}+\frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad\sqrt{b^2-a^2}}-\frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad\sqrt{b^2-a^2}}+\frac{x^2}{2a}$$

```
[In] Int[x/(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] x^2/(2*a) + ((2*I)*b*x^(3/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a
^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - ((2*I)*b*x^(3/2)*Log[1 + (a*E^(I*(c +
 d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (6*b*x*Poly
Log[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 +
 b^2]*d^2) - (6*b*x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 +
 b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + ((12*I)*b*Sqrt[x]*PolyLog[3, -((a*E^(I
*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - ((1
2*I)*b*Sqrt[x]*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]
))])/(a*Sqrt[-a^2 + b^2]*d^3) - (12*b*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x]))
)/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (12*b*PolyLog[4, -((
a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
```

```
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^3}{a+b\sec(c+dx)}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^3}{a}-\frac{bx^3}{a(b+a\cos(c+dx))}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{x^2}{2a}-\frac{(2b)\text{Subst}\left(\int \frac{x^3}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{x^2}{2a}-\frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{x^2}{2a}-\frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}\\
&\quad+\frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}\\
&= \frac{x^2}{2a}+\frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\
&\quad-\frac{(6ib)\text{Subst}\left(\int x^2\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}\\
&\quad+\frac{(6ib)\text{Subst}\left(\int x^2\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}
\end{aligned}
$$

$$= \frac{x^2}{2a} + \frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{(12b)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(12b)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{x^2}{2a} + \frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{(12ib)\text{Subst}\left(\int \text{PolyLog}\left(3,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3} + \frac{(12ib)\text{Subst}\left(\int \text{PolyLog}\left(3,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$= \frac{x^2}{2a} + \frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{(12b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(3,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{(12b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(3,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^4}$$

$$= \frac{x^2}{2a} + \frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{12b\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{12b\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.79

$$\int \frac{x}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx$$
$$= \frac{\sqrt{-a^2+b^2}d^4x^2 + 4ibd^3x^{3/2}\log\left(1-\frac{ae^{i(c+d\sqrt{x})}}{-b+\sqrt{-a^2+b^2}}\right) - 4ibd^3x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right) + 12bd^2x\,\text{PolyLog}\left(2,\right.}{}$$

```
[In] Integrate[x/(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (Sqrt[-a^2 + b^2]*d^4*x^2 + (4*I)*b*d^3*x^(3/2)*Log[1 - (a*E^(I*(c + d*Sqrt
[x])))/(-b + Sqrt[-a^2 + b^2])] - (4*I)*b*d^3*x^(3/2)*Log[1 + (a*E^(I*(c +
d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])] + 12*b*d^2*x*PolyLog[2, (a*E^(I*(c + d
*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - 12*b*d^2*x*PolyLog[2, -((a*E^(I*(c +
 d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] + (24*I)*b*d*Sqrt[x]*PolyLog[3, (a*E
^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - (24*I)*b*d*Sqrt[x]*PolyLog
[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] - 24*b*PolyLog[4,
(a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] + 24*b*PolyLog[4, -((a*E
^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(2*a*Sqrt[-a^2 + b^2]*d^4)
```

Maple [F]

$$\int \frac{x}{a+b\sec\left(c+d\sqrt{x}\right)}dx$$

```
[In] int(x/(a+b*sec(c+d*x^(1/2))),x)
```

```
[Out] int(x/(a+b*sec(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x}{b\sec\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(x/(b*sec(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x/(a+b*sec(c+d*x**(1/2))),x)
[Out] Integral(x/(a + b*sec(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x}{b\sec\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(x/(b*sec(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+b\sec \left(c+d\sqrt{x}\right)}\,dx=\int \frac{x}{a+\frac{b}{\cos\left(c+d\,\sqrt{x}\right)}}\,dx$$

```
[In] int(x/(a + b/cos(c + d*x^(1/2))),x)
[Out] int(x/(a + b/cos(c + d*x^(1/2))), x)
```

3.44 $\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\text{Int}\left(\frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sec(c+d*x^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Int[1/(x*(a + b*Sec[c + d*Sqrt[x]])),x]
[Out] Defer[Int][1/(x*(a + b*Sec[c + d*Sqrt[x]])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*Sec[c + d*Sqrt[x]])),x]
[Out] Integrate[1/(x*(a + b*Sec[c + d*Sqrt[x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}dx$$

```
[In] int(1/x/(a+b*sec(c+d*x^(1/2))),x)
[Out] int(1/x/(a+b*sec(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(1/(b*x*sec(d*sqrt(x) + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*sec(c+d*x**(1/2))),x)
[Out] Integral(1/(x*(a + b*sec(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 241, normalized size of antiderivative = 12.05

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] -(2*a*b*integrate((a*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + 2*b*cos(d*
sqrt(x) + c)^2 + a*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 2*b*sin(d*sq
rt(x) + c)^2 + a*cos(d*sqrt(x) + c))/((a^3*cos(2*d*sqrt(x) + 2*c)^2 + 4*a*b
^2*cos(d*sqrt(x) + c)^2 + a^3*sin(2*d*sqrt(x) + 2*c)^2 + 4*a^2*b*sin(2*d*sq
rt(x) + 2*c)*sin(d*sqrt(x) + c) + 4*a*b^2*sin(d*sqrt(x) + c)^2 + 4*a^2*b*co
s(d*sqrt(x) + c) + a^3 + 2*(2*a^2*b*cos(d*sqrt(x) + c) + a^3)*cos(2*d*sqrt(
x) + 2*c))*x), x) - log(x))/a
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*sqrt(x) + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 13.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)}\,dx$$

```
[In] int(1/(x*(a + b/cos(c + d*x^(1/2)))),x)
```

```
[Out] int(1/(x*(a + b/cos(c + d*x^(1/2)))), x)
```

3.45 $\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx=-\frac{a}{x}+b\text{Int}\left(\frac{\sec\left(c+d\sqrt{x}\right)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(sec(c+d*x^(1/2))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] Int[(a + b*Sec[c + d*Sqrt[x]])/x^2,x]
[Out] -(a/x) + b*Defer[Int][Sec[c + d*Sqrt[x]]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x^2}+\frac{b\sec\left(c+d\sqrt{x}\right)}{x^2}\right)\,dx\\&=-\frac{a}{x}+b\int \frac{\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^2,x]
[Out] Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}dx$$

```
[In] int((a+b*sec(c+d*x^(1/2)))/x^2,x)
[Out] int((a+b*sec(c+d*x^(1/2)))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^2,x, algorithm="fricas")
[Out] integral((b*sec(d*sqrt(x) + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] integrate((a+b*sec(c+d*x**(1/2)))/x**2,x)
[Out] Integral((a + b*sec(c + d*sqrt(x)))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.11

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^2,x, algorithm="maxima")
```

```
[Out] (2*b*x*integrate((cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + sin(2*d*sqrt(
x) + 2*c)*sin(d*sqrt(x) + c) + cos(d*sqrt(x) + c))/((cos(2*d*sqrt(x) + 2*c)
^2 + sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1)*x^2), x) - a)
/x
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*sqrt(x) + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+\frac{b}{\cos\left(c+d\,\sqrt{x}\right)}}{x^2}\,dx$$

```
[In] int((a + b/cos(c + d*x^(1/2)))/x^2,x)
```

```
[Out] int((a + b/cos(c + d*x^(1/2)))/x^2, x)
```

3.46 $\int \frac{x^3}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 3123

$$
\begin{aligned}
\int \frac{x^3}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = & -\frac{2ib^2x^{7/2}}{a^2(a^2-b^2)d}+\frac{x^4}{4a^2}+\frac{14b^2x^3\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} \\
& +\frac{14b^2x^3\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}-\frac{2ib^3x^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
& +\frac{4ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
& +\frac{2ib^3x^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
& -\frac{4ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
& -\frac{84ib^2x^{5/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} \\
& -\frac{84ib^2x^{5/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} \\
& -\frac{14b^3x^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} \\
& +\frac{28bx^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
& +\frac{14b^3x^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} \\
& -\frac{28bx^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
& +\frac{420b^2x^2\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^4} \\
& +\frac{420b^2x^2\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^4} \\
& -\frac{84ib^3x^{5/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3} \\
& +\frac{168ibx^{5/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} \\
& +\frac{84ib^3x^{5/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}
\end{aligned}
$$

$168ibx^{5/2}\,\mathrm{PolyLog}\left(3,-ae^{i(c+d\sqrt{x})}\right.$

```
[Out] 2*I*b^3*x^(7/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2
+b^2)^(3/2)/d+84*I*b^3*x^(5/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b
^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+1680*I*b^2*x^(3/2)*polylog(4,-a*exp(I*
(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^5+1680*I*b^2*x^(3/2)*
polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^5+
1680*I*b^3*x^(3/2)*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/
a^2/(-a^2+b^2)^(3/2)/d^5+4*I*b*x^(7/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2
+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+20160*I*b*polylog(7,-a*exp(I*(c+d*x^(1
/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^7/(-a^2+b^2)^(1/2)+168*I*b*x^(5/2
)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2
)^(1/2)+3360*I*b*x^(3/2)*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1
/2)))/a^2/d^5/(-a^2+b^2)^(1/2)+10080*I*b^3*polylog(7,-a*exp(I*(c+d*x^(1/2))
)/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^7+2*b^2*x^(7/2)*sin(
c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cos(c+d*x^(1/2)))-2*I*b^3*x^(7/2)*ln(1+a*ex
p(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-84*I*b^2*x^
(5/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2
)/d^3-84*I*b^2*x^(5/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/
2)))/a^2/(a^2-b^2)/d^3-84*I*b^3*x^(5/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(
b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-1680*I*b^3*x^(3/2)*polylog(5,
-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^5-4*I*
b*x^(7/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2
)^(1/2)-168*I*b*x^(5/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/
2)))/a^2/d^3/(-a^2+b^2)^(1/2)-3360*I*b*x^(3/2)*polylog(5,-a*exp(I*(c+d*x^(1
/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)^(1/2)-10080*I*b^2*polylog(6,
-a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^7-10
080*I*b^2*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))*x^(1/2)/
a^2/(a^2-b^2)/d^7-10080*I*b^3*polylog(7,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^
2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^7-20160*I*b*polylog(7,-a*exp(I*(c
+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^7/(-a^2+b^2)^(1/2)+10080*b
^2*polylog(7,-a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d
^8+10080*b^2*polylog(7,-a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(
a^2-b^2)/d^8+10080*b^3*polylog(8,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2
)))/a^2/(-a^2+b^2)^(3/2)/d^8-10080*b^3*polylog(8,-a*exp(I*(c+d*x^(1/2)))/(b
+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^8-20160*b*polylog(8,-a*exp(I*(c+
d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^8/(-a^2+b^2)^(1/2)+20160*b*polylog(
8,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^8/(-a^2+b^2)^(1/2)+14
*b^2*x^3*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d
^2+10080*b*x*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^
6/(-a^2+b^2)^(1/2)-10080*b*x*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2
)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)-2*I*b^2*x^(7/2)/a^2/(a^2-b^2)/d-840*b*x^
2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^4/(-a^2+b^2
)^(1/2)-5040*b^3*x*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/
a^2/(-a^2+b^2)^(3/2)/d^6+5040*b^3*x*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(b+(-
a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^6+840*b*x^2*polylog(4,-a*exp(I*(c+d
*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^4/(-a^2+b^2)^(1/2)+14*b^2*x^3*ln(1+a
```

```
*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2-14*b^3*x^3*p
olylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)
/d^2+14*b^3*x^3*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2
/(-a^2+b^2)^(3/2)/d^2+420*b^2*x^2*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-I*(a
^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+420*b^2*x^2*polylog(3,-a*exp(I*(c+d*x^(1/
2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+420*b^3*x^2*polylog(4,-a*exp(
I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4-420*b^3*x^2
*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/
2)/d^4-5040*b^2*x*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/
a^2/(a^2-b^2)/d^6-5040*b^2*x*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^
2)^(1/2)))/a^2/(a^2-b^2)/d^6+28*b*x^3*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-
(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)-28*b*x^3*polylog(2,-a*exp(I*(c+
d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+1/4*x^4/a^2
```

Rubi [A] (verified)

Time = 5.09 (sec) , antiderivative size = 3123, normalized size of antiderivative = 1.00, number of steps used = 61, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules

used = {4289, 4276, 3405, 3402, 2296, 2221, 2611, 6744, 2320, 6724, 4618}

$$\int \frac{x^3}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{x^4}{4a^2}+\frac{4ib\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}}+1\right)x^{7/2}}{a^2\sqrt{b^2-a^2}d}$$
$$-\frac{2ib^3\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}}+1\right)x^{7/2}}{a^2\left(b^2-a^2\right)^{3/2}d}$$
$$-\frac{4ib\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}}+1\right)x^{7/2}}{a^2\sqrt{b^2-a^2}d}+\frac{2ib^3\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}}+1\right)x^{7/2}}{a^2\left(b^2-a^2\right)^{3/2}d}$$
$$+\frac{2b^2\sin\left(c+d\sqrt{x}\right)x^{7/2}}{a\left(a^2-b^2\right)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}-\frac{2ib^2x^{7/2}}{a^2\left(a^2-b^2\right)d}$$
$$+\frac{14b^2\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-i\sqrt{a^2-b^2}}+1\right)x^3}{a^2\left(a^2-b^2\right)d^2}+\frac{14b^2\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+i\sqrt{a^2-b^2}}+1\right)x^3}{a^2\left(a^2-b^2\right)d^2}$$
$$+\frac{28b\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^3}{a^2\sqrt{b^2-a^2}d^2}$$
$$-\frac{14b^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$
$$-\frac{28b\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^3}{a^2\sqrt{b^2-a^2}d^2}$$
$$+\frac{14b^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$
$$-\frac{84ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)x^{5/2}}{a^2\left(a^2-b^2\right)d^3}$$
$$-\frac{84ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)x^{5/2}}{a^2\left(a^2-b^2\right)d^3}$$
$$+\frac{168ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\sqrt{b^2-a^2}d^3}$$
$$-\frac{84ib^3\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\left(b^2-a^2\right)^{3/2}d^3}$$
$$-\frac{168ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\sqrt{b^2-a^2}d^3}$$
$$+\frac{84ib^3\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\left(b^2-a^2\right)^{3/2}d^3}$$
$$+\frac{420b^2\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)x^2}{a^2\left(a^2-b^2\right)d^4}$$
$$+\frac{420b^2\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)x^2}{a^2\left(a^2-b^2\right)d^4}$$

```
[In] Int[x^3/(a + b*Sec[c + d*Sqrt[x]])^2,x]

[Out] ((-2*I)*b^2*x^(7/2))/(a^2*(a^2 - b^2)*d) + x^4/(4*a^2) + (14*b^2*x^3*Log[1
+ (a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2)
 + (14*b^2*x^3*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2])])/
(a^2*(a^2 - b^2)*d^2) - ((2*I)*b^3*x^(7/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x]))
)/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((4*I)*b*x^(7/2)*Lo
g[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b
^2]*d) + ((2*I)*b^3*x^(7/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^
2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((4*I)*b*x^(7/2)*Log[1 + (a*E^(I*(
c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((84*I
)*b^2*x^(5/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]
))])/(a^2*(a^2 - b^2)*d^3) - ((84*I)*b^2*x^(5/2)*PolyLog[2, -((a*E^(I*(c +
d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (14*b^3*x^3
*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^
2 + b^2)^(3/2)*d^2) + (28*b*x^3*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b -
 Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (14*b^3*x^3*PolyLog[2, -
((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2
)*d^2) - (28*b*x^3*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 +
b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (420*b^2*x^2*PolyLog[3, -((a*E^(I*(c
+ d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) + (420*b^2*
x^2*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*
(a^2 - b^2)*d^4) - ((84*I)*b^3*x^(5/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])
))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((168*I)*b*x^(5
/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*S
qrt[-a^2 + b^2]*d^3) + ((84*I)*b^3*x^(5/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt
[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((168*I)*b*
x^(5/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a
^2*Sqrt[-a^2 + b^2]*d^3) + ((1680*I)*b^2*x^(3/2)*PolyLog[4, -((a*E^(I*(c +
d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^5) + ((1680*I)*b
^2*x^(3/2)*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))]
)/(a^2*(a^2 - b^2)*d^5) + (420*b^3*x^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])
))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^4) - (840*b*x^2*Poly
Log[4, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2
 + b^2]*d^4) - (420*b^3*x^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqr
t[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^4) + (840*b*x^2*PolyLog[4, -((a
*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^4
) - (5040*b^2*x*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^
2]))])/(a^2*(a^2 - b^2)*d^6) - (5040*b^2*x*PolyLog[5, -((a*E^(I*(c + d*Sqrt
[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^6) + ((1680*I)*b^3*x^(
3/2)*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*
(-a^2 + b^2)^(3/2)*d^5) - ((3360*I)*b*x^(3/2)*PolyLog[5, -((a*E^(I*(c + d*S
qrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^5) - ((1680*I)*
b^3*x^(3/2)*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))]
)/(a^2*(-a^2 + b^2)^(3/2)*d^5) + ((3360*I)*b*x^(3/2)*PolyLog[5, -((a*E^(I*(
```

```
c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^5) - ((1
0080*I)*b^2*Sqrt[x]*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2
- b^2]))])/(a^2*(a^2 - b^2)*d^7) - ((10080*I)*b^2*Sqrt[x]*PolyLog[6, -((a*E
^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^7) - (5
040*b^3*x*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/
(a^2*(-a^2 + b^2)^(3/2)*d^6) + (10080*b*x*PolyLog[6, -((a*E^(I*(c + d*Sqrt[
x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^6) + (5040*b^3*x*Po
lyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 +
 b^2)^(3/2)*d^6) - (10080*b*x*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b + S
qrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^6) + (10080*b^2*PolyLog[7, -((a
*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^8) +
(10080*b^2*PolyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))]
)/(a^2*(a^2 - b^2)*d^8) - ((10080*I)*b^3*Sqrt[x]*PolyLog[7, -((a*E^(I*(c +
d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^7) + ((201
60*I)*b*Sqrt[x]*PolyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2
]))])/(a^2*Sqrt[-a^2 + b^2]*d^7) + ((10080*I)*b^3*Sqrt[x]*PolyLog[7, -((a*E
^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^7
) - ((20160*I)*b*Sqrt[x]*PolyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-
a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^7) + (10080*b^3*PolyLog[8, -((a*E^(I
*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^8) -
 (20160*b*PolyLog[8, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/
(a^2*Sqrt[-a^2 + b^2]*d^8) - (10080*b^3*PolyLog[8, -((a*E^(I*(c + d*Sqrt[x]
)))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^8) + (20160*b*PolyL
og[8, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2
+ b^2]*d^8) + (2*b^2*x^(7/2)*Sin[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b + a*Co
s[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4618

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
```

```
, x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2] + I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a + Rt[-a^2 + b^2,
 2] + I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^7}{(a+b\sec(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^7}{a^2}+\frac{b^2x^7}{a^2(b+a\cos(c+dx))^2}-\frac{2bx^7}{a^2(b+a\cos(c+dx))}\right)dx, x, \sqrt{x}\right)\\
&= \frac{x^4}{4a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^7}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^7}{(b+a\cos(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{x^4}{4a^2}+\frac{2b^2x^{7/2}\sin\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad-\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^7}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}-\frac{(14b^2)\,\text{Subst}\left(\int \frac{x^6\sin(c+dx)}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}
\end{aligned}$$

$$= -\frac{2ib^2x^{7/2}}{a^2(a^2-b^2)d} + \frac{x^4}{4a^2} + \frac{2b^2x^{7/2}\sin(c+d\sqrt{x})}{a(a^2-b^2)d(b+a\cos(c+d\sqrt{x}))}$$
$$-\frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)}$$
$$-\frac{(8b)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$+\frac{(8b)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$-\frac{(14b^2)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^6}{ib-\sqrt{a^2-b^2}+iae^{i(c+dx)}}dx, x, \sqrt{x}\right)}{a(a^2-b^2)d}$$
$$-\frac{(14b^2)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^6}{ib+\sqrt{a^2-b^2}+iae^{i(c+dx)}}dx, x, \sqrt{x}\right)}{a(a^2-b^2)d}$$

$$= -\frac{2ib^2x^{7/2}}{a^2(a^2-b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2x^3\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+\frac{14b^2x^3\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{4ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$-\frac{4ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2b^2x^{7/2}\sin(c+d\sqrt{x})}{a(a^2-b^2)d(b+a\cos(c+d\sqrt{x}))}$$
$$+\frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$-\frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$-\frac{(84b^2)\operatorname{Subst}\left(\int x^5\log\left(1+\frac{iae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$-\frac{(84b^2)\operatorname{Subst}\left(\int x^5\log\left(1+\frac{iae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$-\frac{(28ib)\operatorname{Subst}\left(\int x^6\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+\frac{(28ib)\operatorname{Subst}\left(\int x^6\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$= -\frac{2ib^2x^{7/2}}{a^2(a^2-b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2x^3\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{14b^2x^3\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{4ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{4ibx^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{84ib^2x^{5/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{84ib^2x^{5/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} + \frac{28bx^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{28bx^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2b^2x^{7/2}\sin\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(420ib^2)\,\mathrm{Subst}\left(\int x^4\,\mathrm{PolyLog}\left(2,-\frac{iae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{(420ib^2)\,\mathrm{Subst}\left(\int x^4\,\mathrm{PolyLog}\left(2,-\frac{iae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{(168b)\mathrm{Subst}\left(\int x^5\,\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(168b)\mathrm{Subst}\left(\int x^5\,\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(14ib^3)\,\mathrm{Subst}\left(\int x^6\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{(14ib^3)\,\mathrm{Subst}\left(\int x^6\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

= Too large to display

Mathematica [A] (verified)

Time = 15.37 (sec) , antiderivative size = 3702, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[x^3/(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] (x^4*(b + a*Cos[c + d*Sqrt[x]])^2*Sec[c + d*Sqrt[x]]^2)/(4*a^2*(a + b*Sec[c
 + d*Sqrt[x]])^2) + (2*b*E^(I*c)*(b + a*Cos[c + d*Sqrt[x]])^2*((-2*I)*b*E^(
I*c)*x^(7/2) + ((1 + E^((2*I)*c))*(7*b*d^6*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x
^3*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2
*I)*c)])] + (2*I)*a^2*d^7*E^(I*c)*x^(7/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])
))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - I*b^2*d^7*E^(I*c)*x^(7/2
)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*
I)*c)])] + 7*b*d^6*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x^3*Log[1 + (a*E^(I*(2*c
+ d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (2*I)*a^2*d^
7*E^(I*c)*x^(7/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a
^2 + b^2)*E^((2*I)*c)])] + I*b^2*d^7*E^(I*c)*x^(7/2)*Log[1 + (a*E^(I*(2*c +
 d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 7*d^5*((6*I)*
b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*
Sqrt[x])*x^(5/2)*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt
[(-a^2 + b^2)*E^((2*I)*c)]))] + 7*d^5*((-6*I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*
c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*x^(5/2)*PolyLog[2, -
((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]
 + 210*b*d^4*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x^2*PolyLog[3, -((a*E^(I*(2*c +
 d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (84*I)*a^2*d
^5*E^(I*c)*x^(5/2)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sq
rt[(-a^2 + b^2)*E^((2*I)*c)]))] - (42*I)*b^2*d^5*E^(I*c)*x^(5/2)*PolyLog[3,
 -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])
)] + 210*b*d^4*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x^2*PolyLog[3, -((a*E^(I*(2*c
 + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (84*I)*a^2
*d^5*E^(I*c)*x^(5/2)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) +
Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (42*I)*b^2*d^5*E^(I*c)*x^(5/2)*PolyLog[
3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)
]))] + (840*I)*b*d^3*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x^(3/2)*PolyLog[4, -((a
*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] -
420*a^2*d^4*E^(I*c)*x^2*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c)
 - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 210*b^2*d^4*E^(I*c)*x^2*PolyLog[4, -
((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]
 + (840*I)*b*d^3*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x^(3/2)*PolyLog[4, -((a*E^(
I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 420*
a^2*d^4*E^(I*c)*x^2*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + S
qrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 210*b^2*d^4*E^(I*c)*x^2*PolyLog[4, -((a*
```

```
E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 2
520*b*d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x*PolyLog[5, -((a*E^(I*(2*c + d*Sq
rt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (1680*I)*a^2*d^3*
E^(I*c)*x^(3/2)*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[
(-a^2 + b^2)*E^((2*I)*c)]))] + (840*I)*b^2*d^3*E^(I*c)*x^(3/2)*PolyLog[5, -
((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]
 - 2520*b*d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x*PolyLog[5, -((a*E^(I*(2*c +
d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (1680*I)*a^2*
d^3*E^(I*c)*x^(3/2)*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + S
qrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (840*I)*b^2*d^3*E^(I*c)*x^(3/2)*PolyLog[
5, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)
]))] - (5040*I)*b*d*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*Sqrt[x]*PolyLog[6, -((a*
E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 5
040*a^2*d^2*E^(I*c)*x*PolyLog[6, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) -
 Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 2520*b^2*d^2*E^(I*c)*x*PolyLog[6, -((a
*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] -
(5040*I)*b*d*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*Sqrt[x]*PolyLog[6, -((a*E^(I*(2
*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 5040*a^2
*d^2*E^(I*c)*x*PolyLog[6, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(
-a^2 + b^2)*E^((2*I)*c)]))] + 2520*b^2*d^2*E^(I*c)*x*PolyLog[6, -((a*E^(I*(
2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 5040*b*
Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*PolyLog[7, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*
E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (10080*I)*a^2*d*E^(I*c)*Sqrt[
x]*PolyLog[7, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*
E^((2*I)*c)]))] - (5040*I)*b^2*d*E^(I*c)*Sqrt[x]*PolyLog[7, -((a*E^(I*(2*c
+ d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 5040*b*Sqrt
[(-a^2 + b^2)*E^((2*I)*c)]*PolyLog[7, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I
*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (10080*I)*a^2*d*E^(I*c)*Sqrt[x]*P
olyLog[7, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((
2*I)*c)]))] + (5040*I)*b^2*d*E^(I*c)*Sqrt[x]*PolyLog[7, -((a*E^(I*(2*c + d*
Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 10080*a^2*E^(I*
c)*PolyLog[8, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*
E^((2*I)*c)]))] + 5040*b^2*E^(I*c)*PolyLog[8, -((a*E^(I*(2*c + d*Sqrt[x])))
/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 10080*a^2*E^(I*c)*PolyLog
[8, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c
)]))] - 5040*b^2*E^(I*c)*PolyLog[8, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c
) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]))/(d^7*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((
2*I)*c)]))*Sec[c + d*Sqrt[x]]^2)/(a^2*(a^2 - b^2)*d*(1 + E^((2*I)*c))*(a +
b*Sec[c + d*Sqrt[x]])^2) + (2*(b + a*Cos[c + d*Sqrt[x]])*Sec[c + d*Sqrt[x]]
^2*(b^3*x^(7/2)*Sin[c] - a*b^2*x^(7/2)*Sin[d*Sqrt[x]]))/(a^2*(-a + b)*(a +
b)*d*(a + b*Sec[c + d*Sqrt[x]])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2
]))
```

Maple [F]

$$\int \frac{x^3}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(x^3/(a+b*sec(c+d*x^(1/2)))^2,x)
```

```
[Out] int(x^3/(a+b*sec(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x^3}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^3}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^3/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^3/(b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2), x
)
```

Sympy [F]

$$\int \frac{x^3}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^3}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x**3/(a+b*sec(c+d*x**(1/2)))**2,x)
```

```
[Out] Integral(x**3/(a + b*sec(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^3}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^3}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^3/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(x^3/(b*sec(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^3}{\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x^3/(a + b/cos(c + d*x^(1/2)))^2,x)
[Out] int(x^3/(a + b/cos(c + d*x^(1/2)))^2, x)
```

3.47 $\int \frac{x^2}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 2323

$$\int \frac{x^2}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=-\frac{2ib^2x^{5/2}}{a^2(a^2-b^2)\,d}+\frac{x^3}{3a^2}+\frac{10b^2x^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2}$$
$$+\frac{10b^2x^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2}-\frac{2ib^3x^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d}$$
$$+\frac{4ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d}$$
$$+\frac{2ib^3x^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d}$$
$$-\frac{4ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d}$$
$$-\frac{40ib^2x^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3}$$
$$-\frac{40ib^2x^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3}$$
$$-\frac{10b^3x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2}$$
$$+\frac{20bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^2}$$
$$+\frac{10b^3x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2}$$
$$-\frac{20bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^2}$$
$$+\frac{120b^2x\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^4}$$
$$+\frac{120b^2x\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^4}$$
$$-\frac{40ib^3x^{3/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3}$$
$$+\frac{80ibx^{3/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^3}$$
$$+\frac{40ib^3x^{3/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3}$$

```
[Out] 10*b^2*x^2*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)
/d^2+10*b^2*x^2*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2
-b^2)/d^2-10*b^3*x^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2))
)/a^2/(-a^2+b^2)^(3/2)/d^2+10*b^3*x^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+
(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+120*b^2*x*polylog(3,-a*exp(I*(c
+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+120*b^2*x*polylog(3,-
a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+120*b^3*x*p
olylog(4,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)
/d^4-120*b^3*x*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/
(-a^2+b^2)^(3/2)/d^4+20*b*x^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^
2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)-20*b*x^2*polylog(2,-a*exp(I*(c+d*x^(1/2
)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)-240*b*x*polylog(4,-a*exp
(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^4/(-a^2+b^2)^(1/2)+240*b*x*po
lylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^4/(-a^2+b^2)^(1
/2)-2*I*b^2*x^(5/2)/a^2/(a^2-b^2)/d-40*I*b^2*x^(3/2)*polylog(2,-a*exp(I*(c+
d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-40*I*b^2*x^(3/2)*polyl
og(2,-a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-40*I*
b^3*x^(3/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a
^2+b^2)^(3/2)/d^3-4*I*b*x^(5/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(
1/2)))/a^2/d/(-a^2+b^2)^(1/2)-80*I*b*x^(3/2)*polylog(3,-a*exp(I*(c+d*x^(1/2
)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-240*I*b^3*polylog(5,-a*e
xp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^5-
480*I*b*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2
/d^5/(-a^2+b^2)^(1/2)-2*I*b^3*x^(5/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+
b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+2*b^2*x^(5/2)*sin(c+d*x^(1/2))/a/(a^2-b
^2)/d/(b+a*cos(c+d*x^(1/2)))-240*b^2*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b-I
*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^6-240*b^2*polylog(5,-a*exp(I*(c+d*x^(1/2
)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^6-240*b^3*polylog(6,-a*exp(I*(c+
d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^6+240*b^3*polylog(
6,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^6+48
0*b*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^6/(-a^2+b
^2)^(1/2)-480*b*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2
/d^6/(-a^2+b^2)^(1/2)+2*I*b^3*x^(5/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+
b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+40*I*b^3*x^(3/2)*polylog(3,-a*exp(I*(c+
d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+4*I*b*x^(5/2)*ln
(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+80*I
*b*x^(3/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/
(-a^2+b^2)^(1/2)+240*I*b^2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)
^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^5+240*I*b^2*polylog(4,-a*exp(I*(c+d*x^(1/2
)))/(b+I*(a^2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^5+240*I*b^3*polylog(5,-a
*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^
5+480*I*b*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a
^2/d^5/(-a^2+b^2)^(1/2)+1/3*x^3/a^2
```

Rubi [A] (verified)

Time = 4.03 (sec) , antiderivative size = 2323, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules

used = {4289, 4276, 3405, 3402, 2296, 2221, 2611, 6744, 2320, 6724, 4618}

$$\int \frac{x^2}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ix^{5/2}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}+\frac{2ix^{5/2}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}$$
$$-\frac{10x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$
$$+\frac{10x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$
$$-\frac{40ix^{3/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3}$$
$$+\frac{40ix^{3/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3}$$
$$+\frac{120x\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4}$$
$$-\frac{120x\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4}$$
$$+\frac{240i\sqrt{x}\,\mathrm{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^5}$$
$$-\frac{240i\sqrt{x}\,\mathrm{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^5}$$
$$-\frac{240\,\mathrm{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^6}$$
$$+\frac{240\,\mathrm{PolyLog}\left(6,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^6}-\frac{2ix^{5/2}b^2}{a^2\left(a^2-b^2\right)d}$$
$$+\frac{10x^2\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-i\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2}+\frac{10x^2\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+i\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2}$$
$$-\frac{40ix^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3}$$
$$-\frac{40ix^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3}$$
$$+\frac{120x\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4}$$
$$+\frac{120x\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4}$$
$$+\frac{240i\sqrt{x}\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)b^2}{\cdots}$$

```
[In] Int[x^2/(a + b*Sec[c + d*Sqrt[x]])^2,x]

[Out] ((-2*I)*b^2*x^(5/2))/(a^2*(a^2 - b^2)*d) + x^3/(3*a^2) + (10*b^2*x^2*Log[1
+ (a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2)
 + (10*b^2*x^2*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2])])/
(a^2*(a^2 - b^2)*d^2) - ((2*I)*b^3*x^(5/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x]))
)/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((4*I)*b*x^(5/2)*Lo
g[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b
^2]*d) + ((2*I)*b^3*x^(5/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^
2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((4*I)*b*x^(5/2)*Log[1 + (a*E^(I*(
c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((40*I
)*b^2*x^(3/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]
))])/(a^2*(a^2 - b^2)*d^3) - ((40*I)*b^2*x^(3/2)*PolyLog[2, -((a*E^(I*(c +
d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (10*b^3*x^2
*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^
2 + b^2)^(3/2)*d^2) + (20*b*x^2*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b -
 Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (10*b^3*x^2*PolyLog[2, -
((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2
)*d^2) - (20*b*x^2*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 +
b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (120*b^2*x*PolyLog[3, -((a*E^(I*(c +
d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) + (120*b^2*x*
PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2
 - b^2)*d^4) - ((40*I)*b^3*x^(3/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(
b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((80*I)*b*x^(3/2)*P
olyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-
a^2 + b^2]*d^3) + ((40*I)*b^3*x^(3/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x]))
)/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((80*I)*b*x^(3/2
)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqr
t[-a^2 + b^2]*d^3) + ((240*I)*b^2*Sqrt[x]*PolyLog[4, -((a*E^(I*(c + d*Sqrt[
x])))/(b - I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^5) + ((240*I)*b^2*Sqrt[
x]*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(
a^2 - b^2)*d^5) + (120*b^3*x*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sq
rt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^4) - (240*b*x*PolyLog[4, -((a*
E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^4)
 - (120*b^3*x*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])
)])/(a^2*(-a^2 + b^2)^(3/2)*d^4) + (240*b*x*PolyLog[4, -((a*E^(I*(c + d*Sqr
t[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^4) - (240*b^2*Pol
yLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 -
b^2)*d^6) - (240*b^2*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2
 - b^2]))])/(a^2*(a^2 - b^2)*d^6) + ((240*I)*b^3*Sqrt[x]*PolyLog[5, -((a*E^
(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^5)
 - ((480*I)*b*Sqrt[x]*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2
 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^5) - ((240*I)*b^3*Sqrt[x]*PolyLog[5, -(
(a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)
*d^5) + ((480*I)*b*Sqrt[x]*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt
```

```
[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^5) - (240*b^3*PolyLog[6, -((a*E^(I
*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^6) +
 (480*b*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a
^2*Sqrt[-a^2 + b^2]*d^6) + (240*b^3*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/
(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^6) - (480*b*PolyLog[6,
-((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]
*d^6) + (2*b^2*x^(5/2)*Sin[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b + a*Cos[c +
d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
```

```
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4618

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2] + I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a + Rt[-a^2 + b^2,
 2] + I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^5}{(a+b\sec(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^5}{a^2}+\frac{b^2x^5}{a^2(b+a\cos(c+dx))^2}-\frac{2bx^5}{a^2(b+a\cos(c+dx))}\right)dx, x, \sqrt{x}\right)\\
&= \frac{x^3}{3a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^5}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^5}{(b+a\cos(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{x^3}{3a^2}+\frac{2b^2x^{5/2}\sin\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad-\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^5}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}-\frac{(10b^2)\,\text{Subst}\left(\int \frac{x^4\sin(c+dx)}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}\\
&= -\frac{2ib^2x^{5/2}}{a^2\left(a^2-b^2\right)d}+\frac{x^3}{3a^2}+\frac{2b^2x^{5/2}\sin\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}\\
&\quad-\frac{(4b^3)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}\\
&\quad-\frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}\\
&\quad+\frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}\\
&\quad-\frac{(10b^2)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{ib-\sqrt{a^2-b^2}+iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}\\
&\quad-\frac{(10b^2)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{ib+\sqrt{a^2-b^2}+iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ib^2x^{5/2}}{a^2(a^2-b^2)d} + \frac{x^3}{3a^2} + \frac{10b^2x^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} \\
&+ \frac{10b^2x^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{4ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&- \frac{4ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2b^2x^{5/2}\sin\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)} \\
&+ \frac{(4b^3)\operatorname{Subst}\left(\int\frac{e^{i(c+dx)}x^5}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,\sqrt{x}\right)}{a(-a^2+b^2)^{3/2}} \\
&- \frac{(4b^3)\operatorname{Subst}\left(\int\frac{e^{i(c+dx)}x^5}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,\sqrt{x}\right)}{a(-a^2+b^2)^{3/2}} \\
&- \frac{(40b^2)\operatorname{Subst}\left(\int x^3\log\left(1+\frac{iae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^2} \\
&- \frac{(40b^2)\operatorname{Subst}\left(\int x^3\log\left(1+\frac{iae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^2} \\
&- \frac{(20ib)\operatorname{Subst}\left(\int x^4\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&+ \frac{(20ib)\operatorname{Subst}\left(\int x^4\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}
\end{aligned}
$$

$$= -\frac{2ib^2x^{5/2}}{a^2(a^2-b^2)d} + \frac{x^3}{3a^2} + \frac{10b^2x^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$

$$+ \frac{10b^2x^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$+ \frac{4ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$- \frac{4ibx^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{40ib^2x^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$

$$- \frac{40ib^2x^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} + \frac{20bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$- \frac{20bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2b^2x^{5/2}\sin(c+d\sqrt{x})}{a(a^2-b^2)d(b+a\cos(c+d\sqrt{x}))}$$

$$+ \frac{(120ib^2)\,\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(2,-\frac{iae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$

$$+ \frac{(120ib^2)\,\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(2,-\frac{iae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$

$$- \frac{(80b)\mathrm{Subst}\left(\int x^3\,\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$+ \frac{(80b)\mathrm{Subst}\left(\int x^3\,\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$+ \frac{(10ib^3)\,\mathrm{Subst}\left(\int x^4\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$- \frac{(10ib^3)\,\mathrm{Subst}\left(\int x^4\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

= Too large to display

Mathematica [A] (verified)

Time = 13.74 (sec) , antiderivative size = 2777, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[x^2/(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-4*I)*b^2*E^((2*I)*c)*x^(5/2)*(b + a*Cos[c + d*Sqrt[x]])^2*Sec[c + d*Sqrt
[x]]^2)/(a^2*(a^2 - b^2)*d*(1 + E^((2*I)*c))*(a + b*Sec[c + d*Sqrt[x]])^2)
+ (x^3*(b + a*Cos[c + d*Sqrt[x]])^2*Sec[c + d*Sqrt[x]]^2)/(3*a^2*(a + b*Sec
[c + d*Sqrt[x]])^2) + (2*b*(b + a*Cos[c + d*Sqrt[x]])^2*(5*b*d^4*Sqrt[(-a^2
 + b^2)*E^((2*I)*c)]*x^2*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - S
qrt[(-a^2 + b^2)*E^((2*I)*c)])] + (2*I)*a^2*d^5*E^(I*c)*x^(5/2)*Log[1 + (a*
E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - I*
b^2*d^5*E^(I*c)*x^(5/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sq
rt[(-a^2 + b^2)*E^((2*I)*c)])] + 5*b*d^4*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x^2
*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I
)*c)])] - (2*I)*a^2*d^5*E^(I*c)*x^(5/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))
/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + I*b^2*d^5*E^(I*c)*x^(5/2)*
Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)
*c)])] - 5*d^3*((4*I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sq
rt[x] + b^2*d*E^(I*c)*Sqrt[x])*x^(3/2)*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x
])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 5*d^3*((-4*I)*b*Sqrt[
(-a^2 + b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x]
)*x^(3/2)*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2
+ b^2)*E^((2*I)*c)]))] + 60*b*d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x*PolyLog[
3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)
]))] + (40*I)*a^2*d^3*E^(I*c)*x^(3/2)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x]
)))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (20*I)*b^2*d^3*E^(I*c)
*x^(3/2)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 +
 b^2)*E^((2*I)*c)]))] + 60*b*d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x*PolyLog[3
, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]
))] - (40*I)*a^2*d^3*E^(I*c)*x^(3/2)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])
))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (20*I)*b^2*d^3*E^(I*c)*
x^(3/2)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 +
b^2)*E^((2*I)*c)]))] + (120*I)*b*d*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*Sqrt[x]*P
olyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((
2*I)*c)]))] - 120*a^2*d^2*E^(I*c)*x*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x]))
)/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 60*b^2*d^2*E^(I*c)*x*Pol
yLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*
I)*c)]))] + (120*I)*b*d*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*Sqrt[x]*PolyLog[4, -
((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]
 + 120*a^2*d^2*E^(I*c)*x*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c
```

```
) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 60*b^2*d^2*E^(I*c)*x*PolyLog[4, -((
a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] -
 120*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x]
)))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (240*I)*a^2*d*E^(I*c)*
Sqrt[x]*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 +
b^2)*E^((2*I)*c)]))] + (120*I)*b^2*d*E^(I*c)*Sqrt[x]*PolyLog[5, -((a*E^(I*(
2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 120*b*S
qrt[(-a^2 + b^2)*E^((2*I)*c)]*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E
^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (240*I)*a^2*d*E^(I*c)*Sqrt[x]*
PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^(
(2*I)*c)]))] - (120*I)*b^2*d*E^(I*c)*Sqrt[x]*PolyLog[5, -((a*E^(I*(2*c + d*
Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 240*a^2*E^(I*c)
*PolyLog[6, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^
((2*I)*c)]))] - 120*b^2*E^(I*c)*PolyLog[6, -((a*E^(I*(2*c + d*Sqrt[x])))/(b
*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 240*a^2*E^(I*c)*PolyLog[6, -
((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]
 + 120*b^2*E^(I*c)*PolyLog[6, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sq
rt[(-a^2 + b^2)*E^((2*I)*c)]))])*Sec[c + d*Sqrt[x]]^2)/(a^2*(a^2 - b^2)*d^6
*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*(a + b*Sec[c + d*Sqrt[x]])^2) + (2*(b + a*C
os[c + d*Sqrt[x]])*Sec[c + d*Sqrt[x]]^2*(b^3*x^(5/2)*Sin[c] - a*b^2*x^(5/2)
*Sin[d*Sqrt[x]]))/(a^2*(-a + b)*(a + b)*d*(a + b*Sec[c + d*Sqrt[x]])^2*(Cos
[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]))
```

Maple [F]

$$\int \frac{x^2}{\left(a + b\sec\left(c + d\sqrt{x}\right)\right)^2} dx$$

[In] int(x^2/(a+b*sec(c+d*x^(1/2)))^2,x)

[Out] int(x^2/(a+b*sec(c+d*x^(1/2)))^2,x)

Fricas [F]

$$\int \frac{x^2}{\left(a + b\sec\left(c + d\sqrt{x}\right)\right)^2} dx = \int \frac{x^2}{\left(b\sec\left(d\sqrt{x} + c\right) + a\right)^2} dx$$

[In] integrate(x^2/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(x^2/(b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2), x
)

Sympy [F]

$$\int \frac{x^2}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x**2/(a+b*sec(c+d*x**(1/2)))**2,x)
[Out] Integral(x**2/(a + b*sec(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^2}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^2/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(x^2/(b*sec(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x^2/(a + b/cos(c + d*x^(1/2)))^2,x)
[Out] int(x^2/(a + b/cos(c + d*x^(1/2)))^2, x)
```

3.48 $\int \frac{x}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1523

$$\int \frac{x}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ib^2x^{3/2}}{a^2\left(a^2-b^2\right)d}+\frac{x^2}{2a^2}+\frac{6b^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}$$
$$+\frac{6b^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}-\frac{2ib^3x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}$$
$$+\frac{4ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+\frac{2ib^3x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}$$
$$-\frac{4ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$-\frac{12ib^2\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$
$$-\frac{12ib^2\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$
$$-\frac{6b^3x\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}$$
$$+\frac{12bx\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+\frac{6b^3x\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}$$
$$-\frac{12bx\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+\frac{12b^2\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}$$
$$+\frac{12b^2\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}$$
$$-\frac{12ib^3\sqrt{x}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}$$
$$+\frac{24ib\sqrt{x}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$+\frac{12ib^3\sqrt{x}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}$$

```
[Out] 6*b^3*x*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b
^2)^(3/2)/d^2+12*b*x*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2))
)/a^2/d^2/(-a^2+b^2)^(1/2)-12*b*x*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^
2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+6*b^2*x*ln(1+a*exp(I*(c+d*x^(1/2)))
/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+6*b^2*x*ln(1+a*exp(I*(c+d*x^(1/2)
))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2-6*b^3*x*polylog(2,-a*exp(I*(c+d
*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-2*I*b^2*x^(3/2)/a
^2/(a^2-b^2)/d+2*b^2*x^(3/2)*sin(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cos(c+d*x^
(1/2)))-2*I*b^3*x^(3/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a
^2/(-a^2+b^2)^(3/2)/d-4*I*b*x^(3/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^
2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)-12*I*b^2*polylog(2,-a*exp(I*(c+d*x^(1/2))
)/(b-I*(a^2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^3-12*I*b^2*polylog(2,-a*ex
p(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^3-12*I*b^
3*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2
+b^2)^(3/2)/d^3-24*I*b*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2
)))*x^(1/2)/a^2/d^3/(-a^2+b^2)^(1/2)+12*b^2*polylog(3,-a*exp(I*(c+d*x^(1/2)
))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+12*b^2*polylog(3,-a*exp(I*(c+d*
x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+12*b^3*polylog(4,-a*exp(
I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4-12*b^3*poly
log(4,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^
4-24*b*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^4/(-a^
2+b^2)^(1/2)+24*b*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a
^2/d^4/(-a^2+b^2)^(1/2)+2*I*b^3*x^(3/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^
2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+4*I*b*x^(3/2)*ln(1+a*exp(I*(c+d*x^(1/
2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+12*I*b^3*polylog(3,-a*exp
(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^3+24
*I*b*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^
3/(-a^2+b^2)^(1/2)+1/2*x^2/a^2
```

Rubi [A] (verified)

Time = 2.99 (sec) , antiderivative size = 1523, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules

used = {4289, 4276, 3405, 3402, 2296, 2221, 2611, 6744, 2320, 6724, 4618}

$$\int \frac{x}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ix^{3/2}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} + \frac{2ix^{3/2}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} - \frac{6x\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} + \frac{6x\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} - \frac{12i\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} + \frac{12i\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} + \frac{12\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4} - \frac{12\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4} - \frac{2ix^{3/2}b^2}{a^2\left(a^2-b^2\right)d} + \frac{6x\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-i\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} + \frac{6x\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+i\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} - \frac{12i\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3} - \frac{12i\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3} + \frac{12\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4} + \frac{12\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4} + \frac{2x^{3/2}\sin\left(c+d\sqrt{x}\right)b^2}{a\left(a^2-b^2\right)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)} + \frac{4ix^{3/2}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}}+1\right)b}{a^2\sqrt{b^2-a^2}d} - \frac{4ix^{3/2}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}}+1\right)b}{a^2\sqrt{b^2-a^2}d} + \frac{12x\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2} - \frac{12x\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2} + \frac{24i\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3}$$

```
[In] Int[x/(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x^(3/2))/(a^2*(a^2 - b^2)*d) + x^2/(2*a^2) + (6*b^2*x*Log[1 + (
a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) +
(6*b^2*x*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2])])/(a^2*(
a^2 - b^2)*d^2) - ((2*I)*b^3*x^(3/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b -
 Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((4*I)*b*x^(3/2)*Log[1 +
(a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d)
 + ((2*I)*b^3*x^(3/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^
2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((4*I)*b*x^(3/2)*Log[1 + (a*E^(I*(c + d*
Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((12*I)*b^2*
Sqrt[x]*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))])/(
a^2*(a^2 - b^2)*d^3) - ((12*I)*b^2*Sqrt[x]*PolyLog[2, -((a*E^(I*(c + d*Sqrt
[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (6*b^3*x*PolyLog[
2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^
(3/2)*d^2) + (12*b*x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2
+ b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (6*b^3*x*PolyLog[2, -((a*E^(I*(c +
d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (12*b
*x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sq
rt[-a^2 + b^2]*d^2) + (12*b^2*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b - I
*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) + (12*b^2*PolyLog[3, -((a*E^(I*(
c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) - ((12*I)
*b^3*Sqrt[x]*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))
])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((24*I)*b*Sqrt[x]*PolyLog[3, -((a*E^(I*(c
 + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3) + ((12
*I)*b^3*Sqrt[x]*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2
]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((24*I)*b*Sqrt[x]*PolyLog[3, -((a*E^(I
*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (
12*b^3*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^
2*(-a^2 + b^2)^(3/2)*d^4) - (24*b*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b
 - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^4) - (12*b^3*PolyLog[4, -((
a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*
d^4) + (24*b*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))
])/(a^2*Sqrt[-a^2 + b^2]*d^4) + (2*b^2*x^(3/2)*Sin[c + d*Sqrt[x]])/(a*(a^2
- b^2)*d*(b + a*Cos[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
```

```
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4618

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2] + I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a + Rt[-a^2 + b^2,
 2] + I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^3}{(a+b\sec(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^3}{a^2}+\frac{b^2x^3}{a^2(b+a\cos(c+dx))^2}-\frac{2bx^3}{a^2(b+a\cos(c+dx))}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{x^2}{2a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^3}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^3}{(b+a\cos(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{x^2}{2a^2}+\frac{2b^2x^{3/2}\sin\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad -\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^3}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}-\frac{(6b^2)\,\text{Subst}\left(\int \frac{x^2\sin(c+dx)}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}
\end{aligned}$$

$$= -\frac{2ib^2x^{3/2}}{a^2(a^2-b^2)d} + \frac{x^2}{2a^2} + \frac{2b^2x^{3/2}\sin\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}$$
$$-\frac{(4b^3)\,\text{Subst}\left(\int\frac{e^{i(c+dx)}x^3}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)}$$
$$-\frac{(8b)\text{Subst}\left(\int\frac{e^{i(c+dx)}x^3}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$+\frac{(8b)\text{Subst}\left(\int\frac{e^{i(c+dx)}x^3}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$-\frac{(6b^2)\,\text{Subst}\left(\int\frac{e^{i(c+dx)}x^2}{ib-\sqrt{a^2-b^2}+iae^{i(c+dx)}}\,dx,x,\sqrt{x}\right)}{a(a^2-b^2)d}$$
$$-\frac{(6b^2)\,\text{Subst}\left(\int\frac{e^{i(c+dx)}x^2}{ib+\sqrt{a^2-b^2}+iae^{i(c+dx)}}\,dx,x,\sqrt{x}\right)}{a(a^2-b^2)d}$$

$$= -\frac{2ib^2x^{3/2}}{a^2(a^2-b^2)d} + \frac{x^2}{2a^2} + \frac{6b^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+\frac{6b^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{4ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$-\frac{4ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2b^2x^{3/2}\sin\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}$$
$$+\frac{(4b^3)\,\text{Subst}\left(\int\frac{e^{i(c+dx)}x^3}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,\sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$-\frac{(4b^3)\,\text{Subst}\left(\int\frac{e^{i(c+dx)}x^3}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,\sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$-\frac{(12b^2)\,\text{Subst}\left(\int x\log\left(1+\frac{iae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$-\frac{(12b^2)\,\text{Subst}\left(\int x\log\left(1+\frac{iae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$-\frac{(12ib)\text{Subst}\left(\int x^2\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+\frac{(12ib)\text{Subst}\left(\int x^2\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$= -\frac{2ib^2x^{3/2}}{a^2(a^2-b^2)d} + \frac{x^2}{2a^2} + \frac{6b^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{6b^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{4ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{4ibx^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{12ib^2\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{12ib^2\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} + \frac{12bx\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{12bx\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2b^2x^{3/2}\sin(c+d\sqrt{x})}{a(a^2-b^2)d(b+a\cos(c+d\sqrt{x}))}$$
$$+ \frac{(12ib^2)\,\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(2,-\frac{iae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{(12ib^2)\,\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(2,-\frac{iae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{(24b)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(24b)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(6ib^3)\,\mathrm{Subst}\left(\int x^2\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{(6ib^3)\,\mathrm{Subst}\left(\int x^2\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$= \text{Too large to display}$

Mathematica [A] (verified)

Time = 15.26 (sec) , antiderivative size = 1767, normalized size of antiderivative = 1.16

$$\int \frac{x}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{x^2\left(b+a\cos\left(c+d\sqrt{x}\right)\right)^2\sec^2\left(c+d\sqrt{x}\right)}{2a^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}$$

$$+\frac{2b\left(b+a\cos\left(c+d\sqrt{x}\right)\right)^2\left(-\frac{2ibd^3e^{2ic}x^{3/2}}{1+e^{2ic}}+\frac{3bd^2\sqrt{(-a^2+b^2)e^{2ic}}\,x\log\left(1+\frac{ae^{i(2c+d\sqrt{x})}}{be^{ic}-\sqrt{(-a^2+b^2)e^{2ic}}}\right)+2ia^2d^3e^{ic}x^{3/2}\log\left(1+\frac{\cdots}{be^{i\cdots}}\right.\cdots}{\cdots}\right.\cdots}{\cdots}$$

$$+\frac{2\left(b+a\cos\left(c+d\sqrt{x}\right)\right)\sec^2\left(c+d\sqrt{x}\right)\left(b^3x^{3/2}\sin(c)-ab^2x^{3/2}\sin\left(d\sqrt{x}\right)\right)}{a^2(-a+b)(a+b)d\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

```
[In] Integrate[x/(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] (x^2*(b + a*Cos[c + d*Sqrt[x]])^2*Sec[c + d*Sqrt[x]]^2)/(2*a^2*(a + b*Sec[c
 + d*Sqrt[x]])^2) + (2*b*(b + a*Cos[c + d*Sqrt[x]])^2*(((-2*I)*b*d^3*E^((2*
I)*c)*x^(3/2))/(1 + E^((2*I)*c)) + (3*b*d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*
x*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*
I)*c)])] + (2*I)*a^2*d^3*E^(I*c)*x^(3/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x]))
)/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - I*b^2*d^3*E^(I*c)*x^(3/2)
*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I
)*c)])] + 3*b*d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x*Log[1 + (a*E^(I*(2*c + d
*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (2*I)*a^2*d^3*E
^(I*c)*x^(3/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2
+ b^2)*E^((2*I)*c)])] + I*b^2*d^3*E^(I*c)*x^(3/2)*Log[1 + (a*E^(I*(2*c + d*
Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 3*d*((2*I)*b*Sqr
t[(-a^2 + b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[
x])*Sqrt[x]*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^
2 + b^2)*E^((2*I)*c)]))] + 3*d*((-2*I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - 2
*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*Sqrt[x]*PolyLog[2, -((a*E^(
I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 6*b*
Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*
E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (12*I)*a^2*d*E^(I*c)*Sqrt[x]*
PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^(
(2*I)*c)]))] - (6*I)*b^2*d*E^(I*c)*Sqrt[x]*PolyLog[3, -((a*E^(I*(2*c + d*Sq
rt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 6*b*Sqrt[(-a^2 +
b^2)*E^((2*I)*c)]*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqr
t[(-a^2 + b^2)*E^((2*I)*c)]))] - (12*I)*a^2*d*E^(I*c)*Sqrt[x]*PolyLog[3, -(
(a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]
+ (6*I)*b^2*d*E^(I*c)*Sqrt[x]*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E
^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 12*a^2*E^(I*c)*PolyLog[4, -((a
*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] +
```

```
6*b^2*E^(I*c)*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-
a^2 + b^2)*E^((2*I)*c)]))] + 12*a^2*E^(I*c)*PolyLog[4, -((a*E^(I*(2*c + d*S
qrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 6*b^2*E^(I*c)*Po
lyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2
*I)*c)]))])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)])*Sec[c + d*Sqrt[x]]^2)/(a^2*(a^2
 - b^2)*d^4*(a + b*Sec[c + d*Sqrt[x]])^2) + (2*(b + a*Cos[c + d*Sqrt[x]])*S
ec[c + d*Sqrt[x]]^2*(b^3*x^(3/2)*Sin[c] - a*b^2*x^(3/2)*Sin[d*Sqrt[x]]))/(a
^2*(-a + b)*(a + b)*d*(a + b*Sec[c + d*Sqrt[x]])^2*(Cos[c/2] - Sin[c/2])*(C
os[c/2] + Sin[c/2]))
```

Maple [F]

$$\int \frac{x}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(x/(a+b*sec(c+d*x^(1/2)))^2,x)
[Out] int(x/(a+b*sec(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(x/(b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2), x)
```

Sympy [F]

$$\int \frac{x}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x/(a+b*sec(c+d*x**(1/2)))**2,x)
[Out] Integral(x/(a + b*sec(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate(x/(b*sec(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x}{\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x/(a + b/cos(c + d*x^(1/2)))^2,x)
```

```
[Out] int(x/(a + b/cos(c + d*x^(1/2)))^2, x)
```

3.49 $\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sec(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x*(a + b*Sec[c + d*Sqrt[x]])^2),x]
[Out] Defer[Int][1/(x*(a + b*Sec[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 64.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x*(a + b*Sec[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x*(a + b*Sec[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(1/x/(a+b*sec(c+d*x^(1/2)))^2,x)
[Out] int(1/x/(a+b*sec(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x*sec(d*sqrt(x) + c)^2 + 2*a*b*x*sec(d*sqrt(x) + c) + a^2*x
), x)
```

Sympy [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x/(a+b*sec(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x*(a + b*sec(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 13.79 (sec) , antiderivative size = 4405, normalized size of antiderivative = 220.25

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] ((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4*b^
4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*a
^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*co
s(d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqrt(x))*cos(c) +
(a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*sin(2*d*sqrt(x))^2 + 4*((a^6*b^
2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^
2)*d*sin(d*sqrt(x))^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*sin(d*sqrt(x))*si
n(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*c
os(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*cos(d*sqrt(x)) + (a^6*b^2 -
a^4*b^4)*d*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a
^3*b^5)*cos(2*c)*sin(c))*d*sin(d*sqrt(x)))*cos(2*d*sqrt(x)) - 2*(a^6*b^2*d*
cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c) - 2*(a^7*b
- a^5*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a^7*b - a^5*b^3)*d*sin(d*sqrt(x))*s
in(c) - (a^8 - a^6*b^2)*d)*cos(2*d*sqrt(x) + 2*c) + 2*(2*((a^5*b^3 - a^3*b^
5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x))
- 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin
(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*sin(2*c))*sin(2*d*sqrt(x)) -
2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^6*b^2*d*cos(2*d*sqrt(x))*sin(2*c
) - 2*(a^7*b - a^5*b^3)*d*cos(c)*sin(d*sqrt(x)) - 2*(a^7*b - a^5*b^3)*d*cos
(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x*integrate(-2*(((2*a^5*b - a^3
*b^3)*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + (2*a^5*b - a^3*b^3)*d*s
in(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) - ((2*a^3*b^3 - a*b^5)*d*cos(2*d*s
```

qrt(x))*cos(2*c) - 2*(2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*cos(d*sqrt(x))*cos(c)
- (2*a^3*b^3 - a*b^5)*d*sin(2*d*sqrt(x))*sin(2*c) + 2*(2*a^4*b^2 - 3*a^2*b^
4 + b^6)*d*sin(d*sqrt(x))*sin(c) - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d)*cos(d*s
qrt(x) + c) - ((2*a^3*b^3 - a*b^5)*d*cos(2*c)*sin(2*d*sqrt(x)) - 2*(2*a^4*b
^2 - 3*a^2*b^4 + b^6)*d*cos(c)*sin(d*sqrt(x)) + (2*a^3*b^3 - a*b^5)*d*cos(2
*d*sqrt(x))*sin(2*c) - 2*(2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*cos(d*sqrt(x))*sin
(c))*sin(d*sqrt(x) + c))*x + (a^2*b^4*cos(2*c)*sin(2*d*sqrt(x)) + a^3*b^3*c
os(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a^2*b^4*cos(2*d*sqrt(x))*sin(2*c
) - 2*(a^3*b^3 - a*b^5)*cos(c)*sin(d*sqrt(x)) - 2*(a^3*b^3 - a*b^5)*cos(d*s
qrt(x))*sin(c) + (a*b^5*cos(2*c)*sin(2*d*sqrt(x)) + a*b^5*cos(2*d*sqrt(x))*
sin(2*c) - 2*(a^2*b^4 - b^6)*cos(c)*sin(d*sqrt(x)) - 2*(a^2*b^4 - b^6)*cos(
d*sqrt(x))*sin(c))*cos(d*sqrt(x) + c) - (a^3*b^3*cos(d*sqrt(x) + c) + a^4*b
^2)*sin(2*d*sqrt(x) + 2*c) - (a*b^5*cos(2*d*sqrt(x))*cos(2*c) - a*b^5*sin(2
*d*sqrt(x))*sin(2*c) - a^3*b^3 + a*b^5 - 2*(a^2*b^4 - b^6)*cos(d*sqrt(x))*c
os(c) + 2*(a^2*b^4 - b^6)*sin(d*sqrt(x))*sin(c))*sin(d*sqrt(x) + c))*sqrt(x
))/((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4
*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 -
2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d
*cos(d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqrt(x))*cos(c)
+ (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*sin(2*d*sqrt(x))^2 + 4*((a^6
*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(
c)^2)*d*sin(d*sqrt(x))^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*sin(d*sqrt(x))
*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c
)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*cos(d*sqrt(x)) + (a^6*b^2
- a^4*b^4)*d*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3
- a^3*b^5)*cos(2*c)*sin(c))*d*sin(d*sqrt(x)))*cos(2*d*sqrt(x)) - 2*(a^6*b^2
*d*cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c) - 2*(a^7
*b - a^5*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a^7*b - a^5*b^3)*d*sin(d*sqrt(x)
)*sin(c) - (a^8 - a^6*b^2)*d)*cos(2*d*sqrt(x) + 2*c) + 2*(2*((a^5*b^3 - a^3
*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x
)) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*
sin(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*sin(2*c))*sin(2*d*sqrt(x))
- 2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^6*b^2*d*cos(2*d*sqrt(x))*sin(
2*c) - 2*(a^7*b - a^5*b^3)*d*cos(c)*sin(d*sqrt(x)) - 2*(a^7*b - a^5*b^3)*d*
cos(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x^2), x) + (a^6*d*cos(2*d*sq
rt(x) + 2*c)^2 + a^6*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^2*b^4*cos(2*c)^2 + a^2
*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(
c)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(c)^2)*d*cos(d*sqrt(x))^2 + 4*(a^5*b
- 2*a^3*b^3 + a*b^5)*d*cos(d*sqrt(x))*cos(c) + (a^2*b^4*cos(2*c)^2 + a^2*b^
4*sin(2*c)^2)*d*sin(2*d*sqrt(x))^2 + 4*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(c)^
2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(c)^2)*d*sin(d*sqrt(x))^2 - 4*(a^5*b - 2
*a^3*b^3 + a*b^5)*d*sin(d*sqrt(x))*sin(c) + (a^6 - 2*a^4*b^2 + a^2*b^4)*d -
2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c
))*d*cos(d*sqrt(x)) + (a^4*b^2 - a^2*b^4)*d*cos(2*c) + 2*((a^3*b^3 - a*b^5)
*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*sin(d*sqrt(x)))*cos

```
(2*d*sqrt(x)) - 2*(a^4*b^2*d*cos(2*d*sqrt(x))*cos(2*c) - a^4*b^2*d*sin(2*d*
sqrt(x))*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a^5*b
- a^3*b^3)*d*sin(d*sqrt(x))*sin(c) - (a^6 - a^4*b^2)*d)*cos(2*d*sqrt(x) + 2
*c) + 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*
sin(c))*d*cos(d*sqrt(x)) - 2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3
- a*b^5)*sin(2*c)*sin(c))*d*sin(d*sqrt(x)) + (a^4*b^2 - a^2*b^4)*d*sin(2*c)
)*sin(2*d*sqrt(x)) - 2*(a^4*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^4*b^2*d*cos
(2*d*sqrt(x))*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*cos(c)*sin(d*sqrt(x)) - 2*(a
^5*b - a^3*b^3)*d*cos(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x*log(x) -
 4*(a^2*b^4*cos(2*c)*sin(2*d*sqrt(x)) + a^3*b^3*cos(2*d*sqrt(x) + 2*c)*sin(
d*sqrt(x) + c) + a^2*b^4*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^3*b^3 - a*b^5)*co
s(c)*sin(d*sqrt(x)) - 2*(a^3*b^3 - a*b^5)*cos(d*sqrt(x))*sin(c) + (a*b^5*co
s(2*c)*sin(2*d*sqrt(x)) + a*b^5*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^2*b^4 - b^
6)*cos(c)*sin(d*sqrt(x)) - 2*(a^2*b^4 - b^6)*cos(d*sqrt(x))*sin(c))*cos(d*s
qrt(x) + c) - (a^3*b^3*cos(d*sqrt(x) + c) + a^4*b^2)*sin(2*d*sqrt(x) + 2*c)
 - (a*b^5*cos(2*d*sqrt(x))*cos(2*c) - a*b^5*sin(2*d*sqrt(x))*sin(2*c) - a^3
*b^3 + a*b^5 - 2*(a^2*b^4 - b^6)*cos(d*sqrt(x))*cos(c) + 2*(a^2*b^4 - b^6)*
sin(d*sqrt(x))*sin(c))*sin(d*sqrt(x) + c))*sqrt(x))/((a^8*d*cos(2*d*sqrt(x)
 + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*
sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c
)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*cos(d*sqrt(x))^2 + 4*(a^7
*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqrt(x))*cos(c) + (a^4*b^4*cos(2*c)^2 + a
^4*b^4*sin(2*c)^2)*d*sin(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6
)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*sin(d*sqrt(x))^2 -
 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*sin(d*sqrt(x))*sin(c) + (a^8 - 2*a^6*b^2
 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*
b^5)*sin(2*c)*sin(c))*d*cos(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*cos(2*c) + 2
*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c)
)*d*sin(d*sqrt(x)))*cos(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*d*sqrt(x))*cos(2*
c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt
(x))*cos(c) + 2*(a^7*b - a^5*b^3)*d*sin(d*sqrt(x))*sin(c) - (a^8 - a^6*b^2)
*d)*cos(2*d*sqrt(x) + 2*c) + 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a
^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x)) - 2*((a^5*b^3 - a^3*b^5
)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*sin(d*sqrt(x)) +
 (a^6*b^2 - a^4*b^4)*d*sin(2*c))*sin(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*c)*s
in(2*d*sqrt(x)) + a^6*b^2*d*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^7*b - a^5*b^3)
*d*cos(c)*sin(d*sqrt(x)) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*sin(c))*sin
(2*d*sqrt(x) + 2*c))*x)
```

Giac [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(1/((b*sec(d*sqrt(x) + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 13.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(1/(x*(a + b/cos(c + d*x^(1/2)))^2),x)
[Out] int(1/(x*(a + b/cos(c + d*x^(1/2)))^2), x)
```

3.50 $\int \frac{1}{x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\mathrm{Int}\left(\frac{1}{x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sec(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x^2*(a + b*Sec[c + d*Sqrt[x]])^2),x]
[Out] Defer[Int][1/(x^2*(a + b*Sec[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 49.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2 \left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2 \left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x^2*(a + b*Sec[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x^2*(a + b*Sec[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2 \left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(1/x^2/(a+b*sec(c+d*x^(1/2)))^2,x)
[Out] int(1/x^2/(a+b*sec(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^2 \left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2 x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^2*sec(d*sqrt(x) + c)^2 + 2*a*b*x^2*sec(d*sqrt(x) + c) + a
^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 9.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2 \left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x**2/(a+b*sec(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x**2*(a + b*sec(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 20.36 (sec) , antiderivative size = 4406, normalized size of antiderivative = 220.30

$$\int \frac{1}{x^2 \left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2 x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] ((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4*b^
4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*a
^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*co
s(d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqrt(x))*cos(c) +
(a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*sin(2*d*sqrt(x))^2 + 4*((a^6*b^
2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^
2)*d*sin(d*sqrt(x))^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*sin(d*sqrt(x))*si
n(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*c
os(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*cos(d*sqrt(x)) + (a^6*b^2 -
a^4*b^4)*d*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a
^3*b^5)*cos(2*c)*sin(c))*d*sin(d*sqrt(x)))*cos(2*d*sqrt(x)) - 2*(a^6*b^2*d*
cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c) - 2*(a^7*b
- a^5*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a^7*b - a^5*b^3)*d*sin(d*sqrt(x))*s
in(c) - (a^8 - a^6*b^2)*d)*cos(2*d*sqrt(x) + 2*c) + 2*(2*((a^5*b^3 - a^3*b^
5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x))
- 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin
(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*sin(2*c))*sin(2*d*sqrt(x)) -
2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^6*b^2*d*cos(2*d*sqrt(x))*sin(2*c
) - 2*(a^7*b - a^5*b^3)*d*cos(c)*sin(d*sqrt(x)) - 2*(a^7*b - a^5*b^3)*d*cos
(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x^2*integrate(-2*(((2*a^5*b - a
^3*b^3)*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + (2*a^5*b - a^3*b^3)*d
*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) - ((2*a^3*b^3 - a*b^5)*d*cos(2*d
```

```
*sqrt(x))*cos(2*c) - 2*(2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*cos(d*sqrt(x))*cos(c
) - (2*a^3*b^3 - a*b^5)*d*sin(2*d*sqrt(x))*sin(2*c) + 2*(2*a^4*b^2 - 3*a^2*
b^4 + b^6)*d*sin(d*sqrt(x))*sin(c) - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d)*cos(d
*sqrt(x) + c) - ((2*a^3*b^3 - a*b^5)*d*cos(2*c)*sin(2*d*sqrt(x)) - 2*(2*a^4
*b^2 - 3*a^2*b^4 + b^6)*d*cos(c)*sin(d*sqrt(x)) + (2*a^3*b^3 - a*b^5)*d*cos
(2*d*sqrt(x))*sin(2*c) - 2*(2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*cos(d*sqrt(x))*s
in(c))*sin(d*sqrt(x) + c))*x + 3*(a^2*b^4*cos(2*c)*sin(2*d*sqrt(x)) + a^3*b
^3*cos(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a^2*b^4*cos(2*d*sqrt(x))*sin
(2*c) - 2*(a^3*b^3 - a*b^5)*cos(c)*sin(d*sqrt(x)) - 2*(a^3*b^3 - a*b^5)*cos
(d*sqrt(x))*sin(c) + (a*b^5*cos(2*c)*sin(2*d*sqrt(x)) + a*b^5*cos(2*d*sqrt(
x))*sin(2*c) - 2*(a^2*b^4 - b^6)*cos(c)*sin(d*sqrt(x)) - 2*(a^2*b^4 - b^6)*
cos(d*sqrt(x))*sin(c))*cos(d*sqrt(x) + c) - (a^3*b^3*cos(d*sqrt(x) + c) + a
^4*b^2)*sin(2*d*sqrt(x) + 2*c) - (a*b^5*cos(2*d*sqrt(x))*cos(2*c) - a*b^5*s
in(2*d*sqrt(x))*sin(2*c) - a^3*b^3 + a*b^5 - 2*(a^2*b^4 - b^6)*cos(d*sqrt(x
))*cos(c) + 2*(a^2*b^4 - b^6)*sin(d*sqrt(x))*sin(c))*sin(d*sqrt(x) + c))*sq
rt(x))/((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 +
(a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^
2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^
2)*d*cos(d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqrt(x))*co
s(c) + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*sin(2*d*sqrt(x))^2 + 4*(
(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*
sin(c)^2)*d*sin(d*sqrt(x))^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*sin(d*sqrt
(x))*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*cos
(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*cos(d*sqrt(x)) + (a^6
*b^2 - a^4*b^4)*d*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*
b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*sin(d*sqrt(x)))*cos(2*d*sqrt(x)) - 2*(a^6
*b^2*d*cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c) - 2*
(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a^7*b - a^5*b^3)*d*sin(d*sqr
t(x))*sin(c) - (a^8 - a^6*b^2)*d)*cos(2*d*sqrt(x) + 2*c) + 2*(2*((a^5*b^3 -
 a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*cos(d*sq
rt(x)) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2
*c)*sin(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*sin(2*c))*sin(2*d*sqrt
(x)) - 2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^6*b^2*d*cos(2*d*sqrt(x))*
sin(2*c) - 2*(a^7*b - a^5*b^3)*d*cos(c)*sin(d*sqrt(x)) - 2*(a^7*b - a^5*b^3
)*d*cos(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x^3), x) - ((a^6 - a^4*b
^2)*d*cos(2*d*sqrt(x) + 2*c)^2 - (a^4*b^2 - a^2*b^4)*d*cos(2*d*sqrt(x))*cos
(2*c) + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*sqrt(x))*cos(c) + (a^6 - a^4*
b^2)*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4*b^2 - a^2*b^4)*d*sin(2*d*sqrt(x))*si
n(2*c) - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*sin(d*sqrt(x))*sin(c) + (a^6 - 2*a
^4*b^2 + a^2*b^4)*d - ((a^4*b^2 - a^2*b^4)*d*cos(2*d*sqrt(x))*cos(2*c) - 2*
(a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*sqrt(x))*cos(c) - (a^4*b^2 - a^2*b^4)*d
*sin(2*d*sqrt(x))*sin(2*c) + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*sin(d*sqrt(x))
*sin(c) - 2*(a^5*b - a^3*b^3)*d*cos(d*sqrt(x) + c) - (2*a^6 - 3*a^4*b^2 + a
^2*b^4)*d)*cos(2*d*sqrt(x) + 2*c) - 2*((a^3*b^3 - a*b^5)*d*cos(2*d*sqrt(x))
*cos(2*c) - 2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*cos(d*sqrt(x))*cos(c) - (a^3*b^
```

```
3 - a*b^5)*d*sin(2*d*sqrt(x))*sin(2*c) + 2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*si
n(d*sqrt(x))*sin(c) - (a^5*b - 2*a^3*b^3 + a*b^5)*d)*cos(d*sqrt(x) + c) - (
(a^4*b^2 - a^2*b^4)*d*cos(2*c)*sin(2*d*sqrt(x)) - 2*(a^5*b - 2*a^3*b^3 + a*
b^5)*d*cos(c)*sin(d*sqrt(x)) + (a^4*b^2 - a^2*b^4)*d*cos(2*d*sqrt(x))*sin(2
*c) - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*sqrt(x))*sin(c) - 2*(a^5*b - a^
3*b^3)*d*sin(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c) - 2*((a^3*b^3 - a*b^5)*
d*cos(2*c)*sin(2*d*sqrt(x)) - 2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*cos(c)*sin(d*
sqrt(x)) + (a^3*b^3 - a*b^5)*d*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^4*b^2 - 2*a
^2*b^4 + b^6)*d*cos(d*sqrt(x))*sin(c))*sin(d*sqrt(x) + c))*x - 4*(a^2*b^4*c
os(2*c)*sin(2*d*sqrt(x)) + a^3*b^3*cos(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c
) + a^2*b^4*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^3*b^3 - a*b^5)*cos(c)*sin(d*sq
rt(x)) - 2*(a^3*b^3 - a*b^5)*cos(d*sqrt(x))*sin(c) + (a*b^5*cos(2*c)*sin(2*
d*sqrt(x)) + a*b^5*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^2*b^4 - b^6)*cos(c)*sin
(d*sqrt(x)) - 2*(a^2*b^4 - b^6)*cos(d*sqrt(x))*sin(c))*cos(d*sqrt(x) + c) -
 (a^3*b^3*cos(d*sqrt(x) + c) + a^4*b^2)*sin(2*d*sqrt(x) + 2*c) - (a*b^5*cos
(2*d*sqrt(x))*cos(2*c) - a*b^5*sin(2*d*sqrt(x))*sin(2*c) - a^3*b^3 + a*b^5
- 2*(a^2*b^4 - b^6)*cos(d*sqrt(x))*cos(c) + 2*(a^2*b^4 - b^6)*sin(d*sqrt(x)
)*sin(c))*sin(d*sqrt(x) + c))*sqrt(x))/((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a
^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d
*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^
2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*cos(d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^
3 + a^3*b^5)*d*cos(d*sqrt(x))*cos(c) + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*
c)^2)*d*sin(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 +
(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*sin(d*sqrt(x))^2 - 4*(a^7*b - 2
*a^5*b^3 + a^3*b^5)*d*sin(d*sqrt(x))*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d
 - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)
*sin(c))*d*cos(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*cos(2*c) + 2*((a^5*b^3 -
a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*sin(d*sqr
t(x)))*cos(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*
d*sin(2*d*sqrt(x))*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*cos(c) +
 2*(a^7*b - a^5*b^3)*d*sin(d*sqrt(x))*sin(c) - (a^8 - a^6*b^2)*d)*cos(2*d*s
qrt(x) + 2*c) + 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*
b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x)) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*co
s(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a
^4*b^4)*d*sin(2*c))*sin(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x
)) + a^6*b^2*d*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*cos(c)*sin
(d*sqrt(x)) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x)
+ 2*c))*x^2)
```

Giac [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(1/((b*sec(d*sqrt(x) + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 13.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(1/(x^2*(a + b/cos(c + d*x^(1/2)))^2),x)
[Out] int(1/(x^2*(a + b/cos(c + d*x^(1/2)))^2), x)
```

3.51 $\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 284

$$\begin{aligned}\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx &= \frac{2}{5}ax^{5/2}-\frac{4ibx^2\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}\\
&+\frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\
&-\frac{24bx\operatorname{PolyLog}\left(3,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{24bx\operatorname{PolyLog}\left(3,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\
&-\frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}\\
&+\frac{48b\operatorname{PolyLog}\left(5,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{48b\operatorname{PolyLog}\left(5,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}\end{aligned}$$

```
[Out] 2/5*a*x^(5/2)-4*I*b*x^2*arctan(exp(I*(c+d*x^(1/2))))/d+8*I*b*x^(3/2)*polylo
g(2,-I*exp(I*(c+d*x^(1/2))))/d^2-8*I*b*x^(3/2)*polylog(2,I*exp(I*(c+d*x^(1/
2))))/d^2-24*b*x*polylog(3,-I*exp(I*(c+d*x^(1/2))))/d^3+24*b*x*polylog(3,I*
exp(I*(c+d*x^(1/2))))/d^3+48*b*polylog(5,-I*exp(I*(c+d*x^(1/2))))/d^5-48*b*
polylog(5,I*exp(I*(c+d*x^(1/2))))/d^5-48*I*b*polylog(4,-I*exp(I*(c+d*x^(1/2
))))*x^(1/2)/d^4+48*I*b*polylog(4,I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {14, 4289, 4266, 2611, 6744, 2320, 6724}

$$\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2}{5}ax^{5/2}-\frac{4ibx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{48b\,\text{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{48b\,\text{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{48ib\sqrt{x}\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{48ib\sqrt{x}\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}-\frac{24bx\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{24bx\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{8ibx^{3/2}\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{8ibx^{3/2}\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}$$

```
[In] Int[x^(3/2)*(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (2*a*x^(5/2))/5 - ((4*I)*b*x^2*ArcTan[E^(I*(c + d*Sqrt[x]))])/d + ((8*I)*b*
x^(3/2)*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((8*I)*b*x^(3/2)*Poly
Log[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - (24*b*x*PolyLog[3, (-I)*E^(I*(c + d*
Sqrt[x]))])/d^3 + (24*b*x*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 - ((48*I
)*b*Sqrt[x]*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((48*I)*b*Sqrt[x]
*PolyLog[4, I*E^(I*(c + d*Sqrt[x]))])/d^4 + (48*b*PolyLog[5, (-I)*E^(I*(c +
 d*Sqrt[x]))])/d^5 - (48*b*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))])/d^5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^{3/2}+bx^{3/2}\sec\left(c+d\sqrt{x}\right)\right)\,dx\\
&= \frac{2}{5}ax^{5/2}+b\int x^{3/2}\sec\left(c+d\sqrt{x}\right)\,dx\\
&= \frac{2}{5}ax^{5/2}+(2b)\text{Subst}\left(\int x^4\sec(c+dx)\,dx,x,\sqrt{x}\right)\\
&= \frac{2}{5}ax^{5/2}-\frac{4ibx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{(8b)\text{Subst}\left(\int x^3\log\left(1-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d}\\
&\quad+\frac{(8b)\text{Subst}\left(\int x^3\log\left(1+ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d}
\end{aligned}$$

$$= \frac{2}{5}ax^{5/2} - \frac{4ibx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8ibx^{3/2}\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8ibx^{3/2}\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{(24ib)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(2,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2} + \frac{(24ib)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(2,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= \frac{2}{5}ax^{5/2} - \frac{4ibx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8ibx^{3/2}\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8ibx^{3/2}\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{24bx\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24bx\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{(48b)\text{Subst}\left(\int x\,\text{PolyLog}\left(3,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3} - \frac{(48b)\text{Subst}\left(\int x\,\text{PolyLog}\left(3,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= \frac{2}{5}ax^{5/2} - \frac{4ibx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8ibx^{3/2}\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8ibx^{3/2}\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{24bx\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24bx\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{48ib\sqrt{x}\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{48ib\sqrt{x}\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(48ib)\text{Subst}\left(\int \text{PolyLog}\left(4,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(48ib)\text{Subst}\left(\int \text{PolyLog}\left(4,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$
\begin{aligned}
&= \frac{2}{5}ax^{5/2} - \frac{4ibx^2 \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8ibx^{3/2} \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
&\quad - \frac{8ibx^{3/2} \operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{24bx \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
&\quad + \frac{24bx \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{48ib\sqrt{x} \operatorname{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
&\quad + \frac{48ib\sqrt{x} \operatorname{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(48b)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(4,-ix)}{x}\, dx, x, e^{i(c+d\sqrt{x})}\right)}{d^5} \\
&\quad - \frac{(48b)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(4,ix)}{x}\, dx, x, e^{i(c+d\sqrt{x})}\right)}{d^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2}{5}ax^{5/2} - \frac{4ibx^2 \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8ibx^{3/2} \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
&\quad - \frac{8ibx^{3/2} \operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{24bx \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
&\quad + \frac{24bx \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{48ib\sqrt{x} \operatorname{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
&\quad + \frac{48ib\sqrt{x} \operatorname{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
&\quad + \frac{48b \operatorname{PolyLog}\left(5, -ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{48b \operatorname{PolyLog}\left(5, ie^{i(c+d\sqrt{x})}\right)}{d^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.99

$$
\int x^{3/2}\left(a + b\sec\left(c+d\sqrt{x}\right)\right)\, dx = \frac{2\left(ad^5x^{5/2} - 10ibd^4x^2 \arctan\left(e^{i(c+d\sqrt{x})}\right) + 20ibd^3x^{3/2} \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right) - 20i\right.}{}
$$

```
[In] Integrate[x^(3/2)*(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (2*(a*d^5*x^(5/2) - (10*I)*b*d^4*x^2*ArcTan[E^(I*(c + d*Sqrt[x]))] + (20*I)
*b*d^3*x^(3/2)*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (20*I)*b*d^3*x^(3/2
)*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))] - 60*b*d^2*x*PolyLog[3, (-I)*E^(I*(c
+ d*Sqrt[x]))] + 60*b*d^2*x*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))] - (120*I)*b
*d*Sqrt[x]*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))] + (120*I)*b*d*Sqrt[x]*Pol
yLog[4, I*E^(I*(c + d*Sqrt[x]))] + 120*b*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x
]))] - 120*b*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))]))/(5*d^5)
```

Maple [F]

$$\int x^{\frac{3}{2}}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)dx$$

```
[In] int(x^(3/2)*(a+b*sec(c+d*x^(1/2))),x)
[Out] int(x^(3/2)*(a+b*sec(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\sec\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(b*x^(3/2)*sec(d*sqrt(x) + c) + a*x^(3/2), x)
```

Sympy [F]

$$\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int x^{\frac{3}{2}}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate(x**(3/2)*(a+b*sec(c+d*x**(1/2))),x)
[Out] Integral(x**(3/2)*(a + b*sec(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 738 vs. 2(208) = 416.

Time = 0.44 (sec) , antiderivative size = 738, normalized size of antiderivative = 2.60

$$\int x^{3/2}(a+b\sec\left(c+d\sqrt{x}\right))\,dx=\frac{2\left(d\sqrt{x}+c\right)^{5}a-10\left(d\sqrt{x}+c\right)^{4}ac+20\left(d\sqrt{x}+c\right)^{3}ac^{2}-20\left(d\sqrt{x}+c\right)^{2}ac^{3}+10\left(d\right.}{}$$

```
[In] integrate(x^(3/2)*(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")
[Out] 1/5*(2*(d*sqrt(x) + c)^5*a - 10*(d*sqrt(x) + c)^4*a*c + 20*(d*sqrt(x) + c)^
3*a*c^2 - 20*(d*sqrt(x) + c)^2*a*c^3 + 10*(d*sqrt(x) + c)*a*c^4 + 10*b*c^4*
log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 10*(I*(d*sqrt(x) + c)^4*b -
```

```
4*I*(d*sqrt(x) + c)^3*b*c + 6*I*(d*sqrt(x) + c)^2*b*c^2 - 4*I*(d*sqrt(x) +
c)*b*c^3)*arctan2(cos(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) - 10*(I*(d*sq
rt(x) + c)^4*b - 4*I*(d*sqrt(x) + c)^3*b*c + 6*I*(d*sqrt(x) + c)^2*b*c^2 -
4*I*(d*sqrt(x) + c)*b*c^3)*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c)
+ 1) - 40*(I*(d*sqrt(x) + c)^3*b - 3*I*(d*sqrt(x) + c)^2*b*c + 3*I*(d*sqrt(
x) + c)*b*c^2 - I*b*c^3)*dilog(I*e^(I*d*sqrt(x) + I*c)) - 40*(-I*(d*sqrt(x)
 + c)^3*b + 3*I*(d*sqrt(x) + c)^2*b*c - 3*I*(d*sqrt(x) + c)*b*c^2 + I*b*c^3
)*dilog(-I*e^(I*d*sqrt(x) + I*c)) + 5*((d*sqrt(x) + c)^4*b - 4*(d*sqrt(x) +
 c)^3*b*c + 6*(d*sqrt(x) + c)^2*b*c^2 - 4*(d*sqrt(x) + c)*b*c^3)*log(cos(d*
sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*sqrt(x) + c) + 1) - 5*((d*s
qrt(x) + c)^4*b - 4*(d*sqrt(x) + c)^3*b*c + 6*(d*sqrt(x) + c)^2*b*c^2 - 4*(
d*sqrt(x) + c)*b*c^3)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*s
in(d*sqrt(x) + c) + 1) - 240*b*polylog(5, I*e^(I*d*sqrt(x) + I*c)) + 240*b*
polylog(5, -I*e^(I*d*sqrt(x) + I*c)) - 240*(-I*(d*sqrt(x) + c)*b + I*b*c)*p
olylog(4, I*e^(I*d*sqrt(x) + I*c)) - 240*(I*(d*sqrt(x) + c)*b - I*b*c)*poly
log(4, -I*e^(I*d*sqrt(x) + I*c)) + 120*((d*sqrt(x) + c)^2*b - 2*(d*sqrt(x)
+ c)*b*c + b*c^2)*polylog(3, I*e^(I*d*sqrt(x) + I*c)) - 120*((d*sqrt(x) + c
)^2*b - 2*(d*sqrt(x) + c)*b*c + b*c^2)*polylog(3, -I*e^(I*d*sqrt(x) + I*c))
)/d^5
```

Giac [F]

$$\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\sec\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*sec(d*sqrt(x) + c) + a)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int x^{3/2}\left(a+\frac{b}{\cos\left(c+d\,\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x^(3/2)*(a + b/cos(c + d*x^(1/2))),x)
[Out] int(x^(3/2)*(a + b/cos(c + d*x^(1/2))), x)
```

3.52 $\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 158

$$\begin{aligned}\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx =& \frac{2}{3}ax^{3/2}-\frac{4ibx\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}\\ &+\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}\\ &-\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}\\ &-\frac{4b\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}\\ &+\frac{4b\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}\end{aligned}$$

```
[Out] 2/3*a*x^(3/2)-4*I*b*x*arctan(exp(I*(c+d*x^(1/2))))/d-4*b*polylog(3,-I*exp(I
*(c+d*x^(1/2))))/d^3+4*b*polylog(3,I*exp(I*(c+d*x^(1/2))))/d^3+4*I*b*polylo
g(2,-I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^2-4*I*b*polylog(2,I*exp(I*(c+d*x^(1/
2))))*x^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {14, 4289, 4266, 2611, 2320, 6724}

$$\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2}{3}ax^{3/2}-\frac{4ibx\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}-\frac{4b\,\text{PolyLog}\left(3,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{4b\,\text{PolyLog}\left(3,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}$$

```
[In] Int[Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (2*a*x^(3/2))/3 - ((4*I)*b*x*ArcTan[E^(I*(c + d*Sqrt[x]))])/d + ((4*I)*b*Sq
rt[x]*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((4*I)*b*Sqrt[x]*PolyLo
g[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - (4*b*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[
x]))])/d^3 + (4*b*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a\sqrt{x}+b\sqrt{x}\sec\left(c+d\sqrt{x}\right)\right)\,dx\\
&= \frac{2}{3}ax^{3/2}+b\int \sqrt{x}\sec\left(c+d\sqrt{x}\right)\,dx\\
&= \frac{2}{3}ax^{3/2}+(2b)\text{Subst}\left(\int x^2\sec(c+dx)\,dx,x,\sqrt{x}\right)\\
&= \frac{2}{3}ax^{3/2}-\frac{4ibx\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}-\frac{(4b)\text{Subst}\left(\int x\log\left(1-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d}\\
&\quad+\frac{(4b)\text{Subst}\left(\int x\log\left(1+ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d}\\
&= \frac{2}{3}ax^{3/2}-\frac{4ibx\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}+\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\
&\quad-\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\
&\quad-\frac{(4ib)\text{Subst}\left(\int \text{PolyLog}\left(2,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}\\
&\quad+\frac{(4ib)\text{Subst}\left(\int \text{PolyLog}\left(2,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2}{3}ax^{3/2} - \frac{4ibx\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{4ib\sqrt{x}\,\mathrm{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
&\quad - \frac{4ib\sqrt{x}\,\mathrm{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{(4b)\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(2,-ix)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^3} \\
&\quad + \frac{(4b)\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(2,ix)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^3} \\
&= \frac{2}{3}ax^{3/2} - \frac{4ibx\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{4ib\sqrt{x}\,\mathrm{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
&\quad - \frac{4ib\sqrt{x}\,\mathrm{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
&\quad - \frac{4b\,\mathrm{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{4b\,\mathrm{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98

$$
\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx
$$

$$
= \frac{2\left(ad^3x^{3/2} - 6ibd^2x\arctan\left(e^{i(c+d\sqrt{x})}\right) + 6ibd\sqrt{x}\,\mathrm{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right) - 6ibd\sqrt{x}\,\mathrm{PolyLog}\left(2,ie^{i(c+d\sqrt{x}
}{3d^3}
$$

```
[In] Integrate[Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (2*(a*d^3*x^(3/2) - (6*I)*b*d^2*x*ArcTan[E^(I*(c + d*Sqrt[x]))] + (6*I)*b*d
*Sqrt[x]*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (6*I)*b*d*Sqrt[x]*PolyLog
[2, I*E^(I*(c + d*Sqrt[x]))] - 6*b*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))] +
 6*b*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))]))/(3*d^3)
```

Maple [F]

$$
\int \left(a+b\sec\left(c+d\sqrt{x}\right)\right)\sqrt{x}dx
$$

```
[In] int((a+b*sec(c+d*x^(1/2)))*x^(1/2),x)
```

```
[Out] int((a+b*sec(c+d*x^(1/2)))*x^(1/2),x)
```

Fricas [F]

$$\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int \left(b\sec\left(d\sqrt{x}+c\right)+a\right)\sqrt{x}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))*x^(1/2),x, algorithm="fricas")
[Out] integral(b*sqrt(x)*sec(d*sqrt(x) + c) + a*sqrt(x), x)
```

Sympy [F]

$$\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx=\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate((a+b*sec(c+d*x**(1/2)))*x**(1/2),x)
[Out] Integral(sqrt(x)*(a + b*sec(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(114) = 228$.

Time = 0.41 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.37

$$\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx$$

$$=\frac{2\left(d\sqrt{x}+c\right)^{3}a-6\left(d\sqrt{x}+c\right)^{2}ac+6\left(d\sqrt{x}+c\right)ac^{2}+6\,bc^{2}\log\left(\sec\left(d\sqrt{x}+c\right)+\tan\left(d\sqrt{x}+c\right)\right)-6\left(i\left(\right.\right.}{}$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))*x^(1/2),x, algorithm="maxima")
[Out] 1/3*(2*(d*sqrt(x) + c)^3*a - 6*(d*sqrt(x) + c)^2*a*c + 6*(d*sqrt(x) + c)*a*
c^2 + 6*b*c^2*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 6*(I*(d*sqrt(x
) + c)^2*b - 2*I*(d*sqrt(x) + c)*b*c)*arctan2(cos(d*sqrt(x) + c), sin(d*sqr
t(x) + c) + 1) - 6*(I*(d*sqrt(x) + c)^2*b - 2*I*(d*sqrt(x) + c)*b*c)*arctan
2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) - 12*(I*(d*sqrt(x) + c)*b -
I*b*c)*dilog(I*e^(I*d*sqrt(x) + I*c)) - 12*(-I*(d*sqrt(x) + c)*b + I*b*c)*d
ilog(-I*e^(I*d*sqrt(x) + I*c)) + 3*((d*sqrt(x) + c)^2*b - 2*(d*sqrt(x) + c)
*b*c)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*sqrt(x) + c
) + 1) - 3*((d*sqrt(x) + c)^2*b - 2*(d*sqrt(x) + c)*b*c)*log(cos(d*sqrt(x)
+ c)^2 + sin(d*sqrt(x) + c)^2 - 2*sin(d*sqrt(x) + c) + 1) + 12*b*polylog(3,
 I*e^(I*d*sqrt(x) + I*c)) - 12*b*polylog(3, -I*e^(I*d*sqrt(x) + I*c)))/d^3
```

Giac [F]

$$\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx = \int \left(b\sec\left(d\sqrt{x}+c\right)+a\right)\sqrt{x}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))*x^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*sqrt(x) + c) + a)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)\,dx = \int \sqrt{x}\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x^(1/2)*(a + b/cos(c + d*x^(1/2))),x)
[Out] int(x^(1/2)*(a + b/cos(c + d*x^(1/2))), x)
```

3.53 $\int \frac{a+b\sec(c+d\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx=2a\sqrt{x}+\frac{2b\text{arctanh}\left(\sin\left(c+d\sqrt{x}\right)\right)}{d}$$

```
[Out] 2*b*arctanh(sin(c+d*x^(1/2)))/d+2*a*x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {14, 4289, 3855}

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx=2a\sqrt{x}+\frac{2b\text{arctanh}\left(\sin\left(c+d\sqrt{x}\right)\right)}{d}$$

```
[In] Int[(a + b*Sec[c + d*Sqrt[x]])/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*ArcTanh[Sin[c + d*Sqrt[x]]])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{\sqrt{x}} + \frac{b\sec\left(c+d\sqrt{x}\right)}{\sqrt{x}}\right) dx \\
&= 2a\sqrt{x} + b\int \frac{\sec\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx \\
&= 2a\sqrt{x} + (2b)\text{Subst}\left(\int \sec(c+dx)\,dx, x, \sqrt{x}\right) \\
&= 2a\sqrt{x} + \frac{2b\text{arctanh}\left(\sin\left(c+d\sqrt{x}\right)\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = 2a\sqrt{x} + \frac{2b\text{arctanh}\left(\sin\left(c+d\sqrt{x}\right)\right)}{d}$$

```
[In] Integrate[(a + b*Sec[c + d*Sqrt[x]])/Sqrt[x],x]
[Out] 2*a*Sqrt[x] + (2*b*ArcTanh[Sin[c + d*Sqrt[x]]])/d
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$2a\sqrt{x}+\frac{2b\ln\left(\sec\left(c+d\sqrt{x}\right)+\tan\left(c+d\sqrt{x}\right)\right)}{d}$	32
default	$2a\sqrt{x}+\frac{2b\ln\left(\sec\left(c+d\sqrt{x}\right)+\tan\left(c+d\sqrt{x}\right)\right)}{d}$	32
parts	$2a\sqrt{x}+\frac{2b\ln\left(\sec\left(c+d\sqrt{x}\right)+\tan\left(c+d\sqrt{x}\right)\right)}{d}$	32

```
[In] int((a+b*sec(c+d*x^(1/2)))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*a*x^(1/2)+2*b/d*ln(sec(c+d*x^(1/2))+tan(c+d*x^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\,ad\sqrt{x}+b\log\left(\sin\left(d\sqrt{x}+c\right)+1\right)-b\log\left(-\sin\left(d\sqrt{x}+c\right)+1\right)}{d}$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*d*sqrt(x) + b*log(sin(d*sqrt(x) + c) + 1) - b*log(-sin(d*sqrt(x) + c)
+ 1))/d
```

Sympy [A] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = 2a\sqrt{x}+2b\left(\begin{cases}\frac{\sqrt{x}\left(\tan\left(c\right)\sec\left(c\right)+\sec^{2}\left(c\right)\right)}{\tan\left(c\right)+\sec\left(c\right)} & \text{for}\ d=0\\ \frac{\log\left(\tan\left(c+d\sqrt{x}\right)+\sec\left(c+d\sqrt{x}\right)\right)}{d} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a+b*sec(c+d*x**(1/2)))/x**(1/2),x)
```

```
[Out] 2*a*sqrt(x) + 2*b*Piecewise((sqrt(x)*(tan(c)*sec(c) + sec(c)**2)/(tan(c) +
sec(c)), Eq(d, 0)), (log(tan(c + d*sqrt(x)) + sec(c + d*sqrt(x)))/d, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,a\sqrt{x}+\frac{2\,b\log\left(\sec\left(d\sqrt{x}+c\right)+\tan\left(d\sqrt{x}+c\right)\right)}{d}$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*a*sqrt(x) + 2*b*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(22) = 44.

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\left(\left(d\sqrt{x}+c\right)a+b\log\left(\left|\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)+1\right|\right)-b\log\left(\left|\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)-1\right|\right)\right)}{d}$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*((d*sqrt(x) + c)*a + b*log(abs(tan(1/2*d*sqrt(x) + 1/2*c) + 1)) - b*log(a
bs(tan(1/2*d*sqrt(x) + 1/2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 15.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.73

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,a\,\sqrt{x}-\frac{2\,b\,\ln\left(\frac{b\,2\mathrm{i}-2\,b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}}{\sqrt{x}}\right)}{d}+\frac{2\,b\,\ln\left(\frac{b\,2\mathrm{i}+2\,b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}}{\sqrt{x}}\right)}{d}$$

```
[In] int((a + b/cos(c + d*x^(1/2)))/x^(1/2),x)
```

```
[Out] 2*a*x^(1/2) - (2*b*log((b*2i - 2*b*exp(d*x^(1/2)*1i)*exp(c*1i))/x^(1/2)))/d
 + (2*b*log((b*2i + 2*b*exp(d*x^(1/2)*1i)*exp(c*1i))/x^(1/2)))/d
```

3.54 $\int \frac{a+b\sec(c+d\sqrt{x})}{x^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=-\frac{2a}{\sqrt{x}}+b\text{Int}\left(\frac{\sec\left(c+d\sqrt{x}\right)}{x^{3/2}},x\right)$$

```
[Out] -2*a/x^(1/2)+b*Unintegrable(sec(c+d*x^(1/2))/x^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx$$

```
[In] Int[(a + b*Sec[c + d*Sqrt[x]])/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + b*Defer[Int][Sec[c + d*Sqrt[x]]/x^(3/2), x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x^{3/2}}+\frac{b\sec\left(c+d\sqrt{x}\right)}{x^{3/2}}\right)\,dx\\&=-\frac{2a}{\sqrt{x}}+b\int \frac{\sec\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 22.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^(3/2),x]
[Out] Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{\frac{3}{2}}}dx$$

```
[In] int((a+b*sec(c+d*x^(1/2)))/x^(3/2),x)
[Out] int((a+b*sec(c+d*x^(1/2)))/x^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^(3/2),x, algorithm="fricas")
[Out] integral((b*sqrt(x)*sec(d*sqrt(x) + c) + a*sqrt(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x**(1/2)))/x**(3/2),x)
[Out] Integral((a + b*sec(c + d*sqrt(x)))/x**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 112, normalized size of antiderivative = 5.60

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(b*sqrt(x)*integrate((cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + sin(2*d
*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + cos(d*sqrt(x) + c))/((cos(2*d*sqrt(x)
+ 2*c)^2 + sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1)*x^(3/2)
), x) - a)/sqrt(x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*sqrt(x) + c) + a)/x^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 13.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+\frac{b}{\cos\left(c+d\,\sqrt{x}\right)}}{x^{3/2}}\,dx$$

```
[In] int((a + b/cos(c + d*x^(1/2)))/x^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x^(1/2)))/x^(3/2), x)
```

3.55 $\int \frac{a+b\sec(c+d\sqrt{x})}{x^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=-\frac{2a}{3x^{3/2}}+b\text{Int}\left(\frac{\sec\left(c+d\sqrt{x}\right)}{x^{5/2}},x\right)$$

```
[Out] -2/3*a/x^(3/2)+b*Unintegrable(sec(c+d*x^(1/2))/x^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx$$

```
[In] Int[(a + b*Sec[c + d*Sqrt[x]])/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) + b*Defer[Int][Sec[c + d*Sqrt[x]]/x^(5/2), x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x^{5/2}}+\frac{b\sec\left(c+d\sqrt{x}\right)}{x^{5/2}}\right)dx\\&=-\frac{2a}{3x^{3/2}}+b\int \frac{\sec\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 22.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^(5/2),x]
[Out] Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{\frac{5}{2}}}dx$$

```
[In] int((a+b*sec(c+d*x^(1/2)))/x^(5/2),x)
[Out] int((a+b*sec(c+d*x^(1/2)))/x^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^(5/2),x, algorithm="fricas")
[Out] integral((b*sqrt(x)*sec(d*sqrt(x) + c) + a*sqrt(x))/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 3.86 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x**(1/2)))/x**(5/2),x)
[Out] Integral((a + b*sec(c + d*sqrt(x)))/x**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 113, normalized size of antiderivative = 5.65

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*b*x^(3/2)*integrate((cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + sin
(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + cos(d*sqrt(x) + c))/((cos(2*d*sqrt
(x) + 2*c)^2 + sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1)*x^(
5/2)), x) - a)/x^(3/2)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{b\sec\left(d\sqrt{x}+c\right)+a}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*sqrt(x) + c) + a)/x^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 13.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}}{x^{5/2}}\,dx$$

```
[In] int((a + b/cos(c + d*x^(1/2)))/x^(5/2),x)
```

```
[Out] int((a + b/cos(c + d*x^(1/2)))/x^(5/2), x)
```

3.56 $\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 451

$$\begin{aligned}\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx &= -\frac{2ib^2x^2}{d}+\frac{2}{5}a^2x^{5/2} \\ &- \frac{8iabx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{8b^2x^{3/2}\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2} \\ &+ \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} \\ &- \frac{12ib^2x\operatorname{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3}-\frac{48abx\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} \\ &+ \frac{48abx\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4} \\ &- \frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} \\ &+ \frac{6ib^2\operatorname{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)}{d^5}+\frac{96ab\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} \\ &- \frac{96ab\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}+\frac{2b^2x^2\tan\left(c+d\sqrt{x}\right)}{d}\end{aligned}$$

```
[Out] 96*I*a*b*polylog(4,I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^4+2/5*a^2*x^(5/2)-96*I
*a*b*polylog(4,-I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^4+8*b^2*x^(3/2)*ln(1+exp(
2*I*(c+d*x^(1/2))))/d^2-16*I*a*b*x^(3/2)*polylog(2,I*exp(I*(c+d*x^(1/2))))/
d^2-12*I*b^2*x*polylog(2,-exp(2*I*(c+d*x^(1/2))))/d^3+16*I*a*b*x^(3/2)*poly
log(2,-I*exp(I*(c+d*x^(1/2))))/d^2-48*a*b*x*polylog(3,-I*exp(I*(c+d*x^(1/2)
)))/d^3+48*a*b*x*polylog(3,I*exp(I*(c+d*x^(1/2))))/d^3+6*I*b^2*polylog(4,-e
```

```
xp(2*I*(c+d*x^(1/2))))/d^5+96*a*b*polylog(5,-I*exp(I*(c+d*x^(1/2))))/d^5-96
*a*b*polylog(5,I*exp(I*(c+d*x^(1/2))))/d^5+12*b^2*polylog(3,-exp(2*I*(c+d*x
^(1/2))))*x^(1/2)/d^4-2*I*b^2*x^2/d-8*I*a*b*x^2*arctan(exp(I*(c+d*x^(1/2)))
)/d+2*b^2*x^2*tan(c+d*x^(1/2))/d
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {4289, 4275, 4266, 2611, 6744, 2320, 6724, 4269, 3800, 2221}

$$\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{2}{5}a^2x^{5/2} -\frac{8iabx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{96ab\,\text{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} -\frac{96ab\,\text{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} +\frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}-\frac{48abx\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} +\frac{48abx\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{16iabx^{3/2}\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} -\frac{16iabx^{3/2}\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}+\frac{6ib^2\,\text{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)}{d^5} +\frac{12b^2\sqrt{x}\,\text{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4}-\frac{12ib^2x\,\text{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} +\frac{8b^2x^{3/2}\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}+\frac{2b^2x^2\tan\left(c+d\sqrt{x}\right)}{d}-\frac{2ib^2x^2}{d}$$

```
[In] Int[x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x^2)/d + (2*a^2*x^(5/2))/5 - ((8*I)*a*b*x^2*ArcTan[E^(I*(c + d*
Sqrt[x]))])/d + (8*b^2*x^(3/2)*Log[1 + E^((2*I)*(c + d*Sqrt[x]))])/d^2 + ((
16*I)*a*b*x^(3/2)*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((16*I)*a*b
*x^(3/2)*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - ((12*I)*b^2*x*PolyLog[2
, -E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (48*a*b*x*PolyLog[3, (-I)*E^(I*(c + d*
Sqrt[x]))])/d^3 + (48*a*b*x*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 + (12*
b^2*Sqrt[x]*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))])/d^4 - ((96*I)*a*b*Sqrt[
x]*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((96*I)*a*b*Sqrt[x]*PolyLo
g[4, I*E^(I*(c + d*Sqrt[x]))])/d^4 + ((6*I)*b^2*PolyLog[4, -E^((2*I)*(c + d
*Sqrt[x]))])/d^5 + (96*a*b*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))])/d^5 - (9
```

```
6*a*b*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))])/d^5 + (2*b^2*x^2*Tan[c + d*Sqrt[
x]])/d
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^4(a+b\sec(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(a^2x^4+2abx^4\sec(c+dx)+b^2x^4\sec^2(c+dx)\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{5}a^2x^{5/2}+(4ab)\text{Subst}\left(\int x^4\sec(c+dx)\,dx, x, \sqrt{x}\right)\\
&\quad +\left(2b^2\right)\text{Subst}\left(\int x^4\sec^2(c+dx)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{5}a^2x^{5/2}-\frac{8iabx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{2b^2x^2\tan\left(c+d\sqrt{x}\right)}{d}\\
&\quad -\frac{(16ab)\text{Subst}\left(\int x^3\log\left(1-ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad +\frac{(16ab)\text{Subst}\left(\int x^3\log\left(1+ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad -\frac{\left(8b^2\right)\text{Subst}\left(\int x^3\tan(c+dx)\,dx, x, \sqrt{x}\right)}{d}
\end{aligned}
$$

$$= -\frac{2ib^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8iabx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}$$
$$+ \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{2b^2x^2\tan\left(c+d\sqrt{x}\right)}{d} - \frac{(48iab)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(2, -ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$
$$+ \frac{(48iab)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(2, ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$
$$+ \frac{(16ib^2)\operatorname{Subst}\left(\int \frac{e^{2i(c+dx)}x^3}{1+e^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{2ib^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8iabx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8b^2x^{3/2}\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{48abx\operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{48abx\operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{2b^2x^2\tan\left(c+d\sqrt{x}\right)}{d} + \frac{(96ab)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(3, -ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3}$$
$$- \frac{(96ab)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(3, ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3}$$
$$- \frac{(24b^2)\operatorname{Subst}\left(\int x^2\log\left(1+e^{2i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$

$$= -\frac{2ib^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8iabx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8b^2x^{3/2}\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{16iabx^{3/2}\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12ib^2x\,\text{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{48abx\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{48abx\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{2b^2x^2\tan\left(c+d\sqrt{x}\right)}{d} + \frac{(96iab)\text{Subst}\left(\int \text{PolyLog}\left(4,-ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(96iab)\text{Subst}\left(\int \text{PolyLog}\left(4,ie^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4} + \frac{(24ib^2)\,\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= -\frac{2ib^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8iabx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8b^2x^{3/2}\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{16iabx^{3/2}\,\text{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\,\text{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12ib^2x\,\text{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{48abx\,\text{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{48abx\,\text{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b^2\sqrt{x}\,\text{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{2b^2x^2\tan\left(c+d\sqrt{x}\right)}{d} + \frac{(96ab)\text{Subst}\left(\int \frac{\text{PolyLog}(4,-ix)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{(96ab)\text{Subst}\left(\int \frac{\text{PolyLog}(4,ix)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{(12b^2)\,\text{Subst}\left(\int \text{PolyLog}\left(3,-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$= -\frac{2ib^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8iabx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8b^2x^{3/2}\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{12ib^2x\operatorname{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{48abx\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{48abx\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4}$$
$$- \frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+ \frac{96ab\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{96ab\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5}$$
$$+ \frac{2b^2x^2\tan\left(c+d\sqrt{x}\right)}{d} + \frac{(6ib^2)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-x)}{x}\,dx, x, e^{2i(c+d\sqrt{x})}\right)}{d^5}$$

$$= -\frac{2ib^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8iabx^2\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8b^2x^{3/2}\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{12ib^2x\operatorname{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{48abx\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{48abx\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,-e^{2i(c+d\sqrt{x})}\right)}{d^4}$$
$$- \frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,-ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,ie^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+ \frac{6ib^2\operatorname{PolyLog}\left(4,-e^{2i(c+d\sqrt{x})}\right)}{d^5} + \frac{96ab\operatorname{PolyLog}\left(5,-ie^{i(c+d\sqrt{x})}\right)}{d^5}$$
$$- \frac{96ab\operatorname{PolyLog}\left(5,ie^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{2b^2x^2\tan\left(c+d\sqrt{x}\right)}{d}$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.98

$$\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{2\left(-5ib^2d^4x^2+a^2d^5x^{5/2}-20iabd^4x^2\arctan\left(e^{i(c+d\sqrt{x})}\right)+20b^2d^3x^{3/2}\log\left(1+e^{2i(c+\cdots\right.\right.}{\cdots}$$

```
[In] Integrate[x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*((-5*I)*b^2*d^4*x^2 + a^2*d^5*x^(5/2) - (20*I)*a*b*d^4*x^2*ArcTan[E^(I*(
c + d*Sqrt[x]))] + 20*b^2*d^3*x^(3/2)*Log[1 + E^((2*I)*(c + d*Sqrt[x]))] +
(40*I)*a*b*d^3*x^(3/2)*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (40*I)*a*b*
d^3*x^(3/2)*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))] - (30*I)*b^2*d^2*x*PolyLog[
2, -E^((2*I)*(c + d*Sqrt[x]))] - 120*a*b*d^2*x*PolyLog[3, (-I)*E^(I*(c + d*
Sqrt[x]))] + 120*a*b*d^2*x*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))] + 30*b^2*d*S
qrt[x]*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))] - (240*I)*a*b*d*Sqrt[x]*PolyL
og[4, (-I)*E^(I*(c + d*Sqrt[x]))] + (240*I)*a*b*d*Sqrt[x]*PolyLog[4, I*E^(I
*(c + d*Sqrt[x]))] + (15*I)*b^2*PolyLog[4, -E^((2*I)*(c + d*Sqrt[x]))] + 24
0*a*b*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))] - 240*a*b*PolyLog[5, I*E^(I*(c
 + d*Sqrt[x]))] + 5*b^2*d^4*x^2*Tan[c + d*Sqrt[x]]))/(5*d^5)
```

Maple [F]

$$\int x^{\frac{3}{2}}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2dx$$

```
[In] int(x^(3/2)*(a+b*sec(c+d*x^(1/2)))^2,x)
```

```
[Out] int(x^(3/2)*(a+b*sec(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^(3/2)*sec(d*sqrt(x) + c)^2 + 2*a*b*x^(3/2)*sec(d*sqrt(x) + c
) + a^2*x^(3/2), x)
```

Sympy [F]

$$\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x^{\frac{3}{2}}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx$$

```
[In] integrate(x**(3/2)*(a+b*sec(c+d*x**(1/2)))**2,x)
[Out] Integral(x**(3/2)*(a + b*sec(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2869 vs. 2(346) = 692.

Time = 0.54 (sec) , antiderivative size = 2869, normalized size of antiderivative = 6.36

$$\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(x^(3/2)*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] 2/5*((d*sqrt(x) + c)^5*a^2 - 5*(d*sqrt(x) + c)^4*a^2*c + 10*(d*sqrt(x) + c)
^3*a^2*c^2 - 10*(d*sqrt(x) + c)^2*a^2*c^3 + 5*(d*sqrt(x) + c)*a^2*c^4 + 10*
a*b*c^4*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) + 5*(6*b^2*c^4 - 6*((d
*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c
^2 - 4*(d*sqrt(x) + c)*a*b*c^3 + ((d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x) + c)
^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^3)*cos(2*d
*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^4*a*b - 4*I*(d*sqrt(x) + c)^3*a*b*c +
6*I*(d*sqrt(x) + c)^2*a*b*c^2 - 4*I*(d*sqrt(x) + c)*a*b*c^3)*sin(2*d*sqrt(x
) + 2*c))*arctan2(cos(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) - 6*((d*sqrt(
x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4
*(d*sqrt(x) + c)*a*b*c^3 + ((d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b
*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^3)*cos(2*d*sqrt(
x) + 2*c) + (I*(d*sqrt(x) + c)^4*a*b - 4*I*(d*sqrt(x) + c)^3*a*b*c + 6*I*(d
*sqrt(x) + c)^2*a*b*c^2 - 4*I*(d*sqrt(x) + c)*a*b*c^3)*sin(2*d*sqrt(x) + 2*
c))*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) + 4*(4*(d*sqrt(x)
+ c)^3*b^2 - 9*(d*sqrt(x) + c)^2*b^2*c + 9*(d*sqrt(x) + c)*b^2*c^2 - 3*b^2*
c^3 + (4*(d*sqrt(x) + c)^3*b^2 - 9*(d*sqrt(x) + c)^2*b^2*c + 9*(d*sqrt(x) +
 c)*b^2*c^2 - 3*b^2*c^3)*cos(2*d*sqrt(x) + 2*c) - (-4*I*(d*sqrt(x) + c)^3*b
^2 + 9*I*(d*sqrt(x) + c)^2*b^2*c - 9*I*(d*sqrt(x) + c)*b^2*c^2 + 3*I*b^2*c^
3)*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(2*d*sqrt(x) + 2*c), cos(2*d*sqrt(x)
+ 2*c) + 1) - 6*((d*sqrt(x) + c)^4*b^2 - 4*(d*sqrt(x) + c)^3*b^2*c + 6*(d*s
qrt(x) + c)^2*b^2*c^2 - 4*(d*sqrt(x) + c)*b^2*c^3)*cos(2*d*sqrt(x) + 2*c) -
 6*(4*(d*sqrt(x) + c)^2*b^2 - 6*(d*sqrt(x) + c)*b^2*c + 3*b^2*c^2 + (4*(d*s
qrt(x) + c)^2*b^2 - 6*(d*sqrt(x) + c)*b^2*c + 3*b^2*c^2)*cos(2*d*sqrt(x) +
```

2*c) + (4*I*(d*sqrt(x) + c)^2*b^2 - 6*I*(d*sqrt(x) + c)*b^2*c + 3*I*b^2*c^2
)*sin(2*d*sqrt(x) + 2*c))*dilog(-e^(2*I*d*sqrt(x) + 2*I*c)) - 24*((d*sqrt(x
) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3*(d*sqrt(x) + c)*a*b*c^2 - a*b*
c^3 + ((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3*(d*sqrt(x) + c
)*a*b*c^2 - a*b*c^3)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^3*a*b - 3*
I*(d*sqrt(x) + c)^2*a*b*c + 3*I*(d*sqrt(x) + c)*a*b*c^2 - I*a*b*c^3)*sin(2*
d*sqrt(x) + 2*c))*dilog(I*e^(I*d*sqrt(x) + I*c)) + 24*((d*sqrt(x) + c)^3*a*
b - 3*(d*sqrt(x) + c)^2*a*b*c + 3*(d*sqrt(x) + c)*a*b*c^2 - a*b*c^3 + ((d*s
qrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3*(d*sqrt(x) + c)*a*b*c^2 -
 a*b*c^3)*cos(2*d*sqrt(x) + 2*c) - (-I*(d*sqrt(x) + c)^3*a*b + 3*I*(d*sqrt(
x) + c)^2*a*b*c - 3*I*(d*sqrt(x) + c)*a*b*c^2 + I*a*b*c^3)*sin(2*d*sqrt(x)
+ 2*c))*dilog(-I*e^(I*d*sqrt(x) + I*c)) - 2*(4*I*(d*sqrt(x) + c)^3*b^2 - 9*
I*(d*sqrt(x) + c)^2*b^2*c + 9*I*(d*sqrt(x) + c)*b^2*c^2 - 3*I*b^2*c^3 + (4*
I*(d*sqrt(x) + c)^3*b^2 - 9*I*(d*sqrt(x) + c)^2*b^2*c + 9*I*(d*sqrt(x) + c)
*b^2*c^2 - 3*I*b^2*c^3)*cos(2*d*sqrt(x) + 2*c) - (4*(d*sqrt(x) + c)^3*b^2 -
 9*(d*sqrt(x) + c)^2*b^2*c + 9*(d*sqrt(x) + c)*b^2*c^2 - 3*b^2*c^3)*sin(2*d
*sqrt(x) + 2*c))*log(cos(2*d*sqrt(x) + 2*c)^2 + sin(2*d*sqrt(x) + 2*c)^2 +
2*cos(2*d*sqrt(x) + 2*c) + 1) - 3*(I*(d*sqrt(x) + c)^4*a*b - 4*I*(d*sqrt(x)
 + c)^3*a*b*c + 6*I*(d*sqrt(x) + c)^2*a*b*c^2 - 4*I*(d*sqrt(x) + c)*a*b*c^3
 + (I*(d*sqrt(x) + c)^4*a*b - 4*I*(d*sqrt(x) + c)^3*a*b*c + 6*I*(d*sqrt(x)
+ c)^2*a*b*c^2 - 4*I*(d*sqrt(x) + c)*a*b*c^3)*cos(2*d*sqrt(x) + 2*c) - ((d*
sqrt(x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^
2 - 4*(d*sqrt(x) + c)*a*b*c^3)*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) +
c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*sqrt(x) + c) + 1) - 3*(-I*(d*sqrt(x)
+ c)^4*a*b + 4*I*(d*sqrt(x) + c)^3*a*b*c - 6*I*(d*sqrt(x) + c)^2*a*b*c^2 +
4*I*(d*sqrt(x) + c)*a*b*c^3 + (-I*(d*sqrt(x) + c)^4*a*b + 4*I*(d*sqrt(x) +
c)^3*a*b*c - 6*I*(d*sqrt(x) + c)^2*a*b*c^2 + 4*I*(d*sqrt(x) + c)*a*b*c^3)*c
os(2*d*sqrt(x) + 2*c) + ((d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b*c
+ 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^3)*sin(2*d*sqrt(x)
+ 2*c))*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*sin(d*sqrt(x) +
 c) + 1) - 144*(-I*a*b*cos(2*d*sqrt(x) + 2*c) + a*b*sin(2*d*sqrt(x) + 2*c)
- I*a*b)*polylog(5, I*e^(I*d*sqrt(x) + I*c)) - 144*(I*a*b*cos(2*d*sqrt(x) +
 2*c) - a*b*sin(2*d*sqrt(x) + 2*c) + I*a*b)*polylog(5, -I*e^(I*d*sqrt(x) +
I*c)) + 12*(b^2*cos(2*d*sqrt(x) + 2*c) + I*b^2*sin(2*d*sqrt(x) + 2*c) + b^2
)*polylog(4, -e^(2*I*d*sqrt(x) + 2*I*c)) + 144*((d*sqrt(x) + c)*a*b - a*b*c
 + ((d*sqrt(x) + c)*a*b - a*b*c)*cos(2*d*sqrt(x) + 2*c) - (-I*(d*sqrt(x) +
c)*a*b + I*a*b*c)*sin(2*d*sqrt(x) + 2*c))*polylog(4, I*e^(I*d*sqrt(x) + I*c
)) - 144*((d*sqrt(x) + c)*a*b - a*b*c + ((d*sqrt(x) + c)*a*b - a*b*c)*cos(2
*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)*a*b - I*a*b*c)*sin(2*d*sqrt(x) + 2*c
))*polylog(4, -I*e^(I*d*sqrt(x) + I*c)) - 6*(4*I*(d*sqrt(x) + c)*b^2 - 3*I*
b^2*c + (4*I*(d*sqrt(x) + c)*b^2 - 3*I*b^2*c)*cos(2*d*sqrt(x) + 2*c) - (4*(
d*sqrt(x) + c)*b^2 - 3*b^2*c)*sin(2*d*sqrt(x) + 2*c))*polylog(3, -e^(2*I*d*
sqrt(x) + 2*I*c)) - 72*(I*(d*sqrt(x) + c)^2*a*b - 2*I*(d*sqrt(x) + c)*a*b*c
 + I*a*b*c^2 + (I*(d*sqrt(x) + c)^2*a*b - 2*I*(d*sqrt(x) + c)*a*b*c + I*a*b
*c^2)*cos(2*d*sqrt(x) + 2*c) - ((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a

```
*b*c + a*b*c^2)*sin(2*d*sqrt(x) + 2*c))*polylog(3, I*e^(I*d*sqrt(x) + I*c))
 - 72*(-I*(d*sqrt(x) + c)^2*a*b + 2*I*(d*sqrt(x) + c)*a*b*c - I*a*b*c^2 + (
-I*(d*sqrt(x) + c)^2*a*b + 2*I*(d*sqrt(x) + c)*a*b*c - I*a*b*c^2)*cos(2*d*s
qrt(x) + 2*c) + ((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c + a*b*c^2)
*sin(2*d*sqrt(x) + 2*c))*polylog(3, -I*e^(I*d*sqrt(x) + I*c)) - 6*(I*(d*sqr
t(x) + c)^4*b^2 - 4*I*(d*sqrt(x) + c)^3*b^2*c + 6*I*(d*sqrt(x) + c)^2*b^2*c
^2 - 4*I*(d*sqrt(x) + c)*b^2*c^3)*sin(2*d*sqrt(x) + 2*c))/(-3*I*cos(2*d*sqr
t(x) + 2*c) + 3*sin(2*d*sqrt(x) + 2*c) - 3*I))/d^5
```

Giac [F]

$$\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*sec(d*sqrt(x) + c) + a)^2*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x^{3/2}\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2\,dx$$

```
[In] int(x^(3/2)*(a + b/cos(c + d*x^(1/2)))^2,x)
[Out] int(x^(3/2)*(a + b/cos(c + d*x^(1/2)))^2, x)
```

3.57 $\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 255

$$
\begin{aligned}
\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx = &-\frac{2ib^2x}{d}+\frac{2}{3}a^2x^{3/2}-\frac{8iabx\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}\\
&+\frac{4b^2\sqrt{x}\log\left(1+e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\
&+\frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\
&-\frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\
&-\frac{2ib^2\operatorname{PolyLog}\left(2,-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\
&-\frac{8ab\operatorname{PolyLog}\left(3,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\
&+\frac{8ab\operatorname{PolyLog}\left(3,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{2b^2x\tan\left(c+d\sqrt{x}\right)}{d}
\end{aligned}
$$

```
[Out] -2*I*b^2*x/d+2/3*a^2*x^(3/2)-8*I*a*b*x*arctan(exp(I*(c+d*x^(1/2))))/d-2*I*b
^2*polylog(2,-exp(2*I*(c+d*x^(1/2))))/d^3-8*a*b*polylog(3,-I*exp(I*(c+d*x^(
1/2))))/d^3+8*a*b*polylog(3,I*exp(I*(c+d*x^(1/2))))/d^3+4*b^2*ln(1+exp(2*I*
(c+d*x^(1/2))))*x^(1/2)/d^2+8*I*a*b*polylog(2,-I*exp(I*(c+d*x^(1/2))))*x^(1
/2)/d^2-8*I*a*b*polylog(2,I*exp(I*(c+d*x^(1/2))))*x^(1/2)/d^2+2*b^2*x*tan(c
+d*x^(1/2))/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4289, 4275, 4266, 2611, 2320, 6724, 4269, 3800, 2221, 2317, 2438}

$$\begin{aligned}\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=&\frac{2}{3}a^2x^{3/2}-\frac{8iabx\arctan\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}\\&-\frac{8ab\,\text{PolyLog}\left(3,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\&+\frac{8ab\,\text{PolyLog}\left(3,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\&+\frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,-ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\&-\frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,ie^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\&-\frac{2ib^2\,\text{PolyLog}\left(2,-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\&+\frac{4b^2\sqrt{x}\log\left(1+e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\&+\frac{2b^2x\tan\left(c+d\sqrt{x}\right)}{d}-\frac{2ib^2x}{d}\end{aligned}$$

```
[In] Int[Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x)/d + (2*a^2*x^(3/2))/3 - ((8*I)*a*b*x*ArcTan[E^(I*(c + d*Sqrt
[x]))])/d + (4*b^2*Sqrt[x]*Log[1 + E^((2*I)*(c + d*Sqrt[x]))])/d^2 + ((8*I)
*a*b*Sqrt[x]*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((8*I)*a*b*Sqrt[
x]*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - ((2*I)*b^2*PolyLog[2, -E^((2*
I)*(c + d*Sqrt[x]))])/d^3 - (8*a*b*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))])/
d^3 + (8*a*b*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 + (2*b^2*x*Tan[c + d*
Sqrt[x]])/d
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^2(a+b\sec(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(a^2x^2+2abx^2\sec(c+dx)+b^2x^2\sec^2(c+dx)\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{3}a^2x^{3/2}+(4ab)\text{Subst}\left(\int x^2\sec(c+dx)\,dx, x, \sqrt{x}\right)\\
&\quad +\left(2b^2\right)\text{Subst}\left(\int x^2\sec^2(c+dx)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{3}a^2x^{3/2}-\frac{8iabx\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{2b^2x\tan\left(c+d\sqrt{x}\right)}{d}\\
&\quad -\frac{(8ab)\text{Subst}\left(\int x\log\left(1-ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad +\frac{(8ab)\text{Subst}\left(\int x\log\left(1+ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad -\frac{\left(4b^2\right)\text{Subst}\left(\int x\tan(c+dx)\,dx, x, \sqrt{x}\right)}{d}
\end{aligned}
$$

$$= -\frac{2ib^2x}{d} + \frac{2}{3}a^2x^{3/2} - \frac{8iabx\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} + \frac{2b^2x\tan\left(c+d\sqrt{x}\right)}{d}$$
$$- \frac{(8iab)\text{Subst}\left(\int \text{PolyLog}\left(2, -ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$
$$+ \frac{(8iab)\text{Subst}\left(\int \text{PolyLog}\left(2, ie^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$
$$+ \frac{(8ib^2)\,\text{Subst}\left(\int \frac{e^{2i(c+dx)}x}{1+e^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{2ib^2x}{d} + \frac{2}{3}a^2x^{3/2} - \frac{8iabx\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{4b^2\sqrt{x}\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{2b^2x\tan\left(c+d\sqrt{x}\right)}{d} - \frac{(8ab)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{(8ab)\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$- \frac{(4b^2)\,\text{Subst}\left(\int \log\left(1+e^{2i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$

$$= -\frac{2ib^2x}{d} + \frac{2}{3}a^2x^{3/2} - \frac{8iabx\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{4b^2\sqrt{x}\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{8ab\,\text{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{8ab\,\text{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{2b^2x\tan\left(c+d\sqrt{x}\right)}{d} + \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i(c+d\sqrt{x})}\right)}{d^3}$$

$$= -\frac{2ib^2x}{d} + \frac{2}{3}a^2x^{3/2} - \frac{8iabx\arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{4b^2\sqrt{x}\log\left(1+e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,-ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{2ib^2\operatorname{PolyLog}\left(2,-e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{8ab\operatorname{PolyLog}\left(3,-ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{8ab\operatorname{PolyLog}\left(3,ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{2b^2x\tan\left(c+d\sqrt{x}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.97

$$\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx$$
$$= \frac{2\left(-3ib^2d^2x + a^2d^3x^{3/2} - 12iabd^2x\arctan\left(e^{i(c+d\sqrt{x})}\right) + 6b^2d\sqrt{x}\log\left(1+e^{2i(c+d\sqrt{x})}\right) + 12iabd\sqrt{x}\operatorname{PolyLo}\right.}{}$$

```
[In] Integrate[Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*((-3*I)*b^2*d^2*x + a^2*d^3*x^(3/2) - (12*I)*a*b*d^2*x*ArcTan[E^(I*(c +
d*Sqrt[x]))] + 6*b^2*d*Sqrt[x]*Log[1 + E^((2*I)*(c + d*Sqrt[x]))] + (12*I)*
a*b*d*Sqrt[x]*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (12*I)*a*b*d*Sqrt[x]
*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))] - (3*I)*b^2*PolyLog[2, -E^((2*I)*(c +
d*Sqrt[x]))] - 12*a*b*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))] + 12*a*b*PolyL
og[3, I*E^(I*(c + d*Sqrt[x]))] + 3*b^2*d^2*x*Tan[c + d*Sqrt[x]]))/(3*d^3)
```

Maple [F]

$$\int \left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\sqrt{x}dx$$

```
[In] int((a+b*sec(c+d*x^(1/2)))^2*x^(1/2),x)
```

```
[Out] int((a+b*sec(c+d*x^(1/2)))^2*x^(1/2),x)
```

Fricas [F]

$$\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2\sqrt{x}\,dx$$

[In] integrate((a+b*sec(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="fricas")

[Out] integral(b^2*sqrt(x)*sec(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*sec(d*sqrt(x) + c
) + a^2*sqrt(x), x)

Sympy [F]

$$\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx$$

[In] integrate((a+b*sec(c+d*x**(1/2)))**2*x**(1/2),x)

[Out] Integral(sqrt(x)*(a + b*sec(c + d*sqrt(x)))**2, x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1272 vs. $2(194) = 388$.

Time = 0.45 (sec) , antiderivative size = 1272, normalized size of antiderivative = 4.99

$$\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\text{Too large to display}$$

[In] integrate((a+b*sec(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="maxima")

[Out] 2/3*((d*sqrt(x) + c)^3*a^2 - 3*(d*sqrt(x) + c)^2*a^2*c + 3*(d*sqrt(x) + c)*
a^2*c^2 + 6*a*b*c^2*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) + 3*(2*b^2
c^2 - 2((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c + ((d*sqrt(x) + c
)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) +
 c)^2*a*b - 2*I*(d*sqrt(x) + c)*a*b*c)*sin(2*d*sqrt(x) + 2*c))*arctan2(cos(
d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) - 2*((d*sqrt(x) + c)^2*a*b - 2*(d*s
qrt(x) + c)*a*b*c + ((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c)*cos(2
*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^2*a*b - 2*I*(d*sqrt(x) + c)*a*b*c)*s
in(2*d*sqrt(x) + 2*c))*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1)
 + 2*((d*sqrt(x) + c)*b^2 - b^2*c + ((d*sqrt(x) + c)*b^2 - b^2*c)*cos(2*d*s
qrt(x) + 2*c) - (-I*(d*sqrt(x) + c)*b^2 + I*b^2*c)*sin(2*d*sqrt(x) + 2*c))*
arctan2(sin(2*d*sqrt(x) + 2*c), cos(2*d*sqrt(x) + 2*c) + 1) - 2*((d*sqrt(x)
 + c)^2*b^2 - 2*(d*sqrt(x) + c)*b^2*c)*cos(2*d*sqrt(x) + 2*c) - (b^2*cos(2*

```
d*sqrt(x) + 2*c) + I*b^2*sin(2*d*sqrt(x) + 2*c) + b^2)*dilog(-e^(2*I*d*sqrt
(x) + 2*I*c)) - 4*((d*sqrt(x) + c)*a*b - a*b*c + ((d*sqrt(x) + c)*a*b - a*b
*c)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)*a*b - I*a*b*c)*sin(2*d*sqrt
(x) + 2*c))*dilog(I*e^(I*d*sqrt(x) + I*c)) + 4*((d*sqrt(x) + c)*a*b - a*b*c
 + ((d*sqrt(x) + c)*a*b - a*b*c)*cos(2*d*sqrt(x) + 2*c) - (-I*(d*sqrt(x) +
c)*a*b + I*a*b*c)*sin(2*d*sqrt(x) + 2*c))*dilog(-I*e^(I*d*sqrt(x) + I*c)) +
 (-I*(d*sqrt(x) + c)*b^2 + I*b^2*c + (-I*(d*sqrt(x) + c)*b^2 + I*b^2*c)*cos
(2*d*sqrt(x) + 2*c) + ((d*sqrt(x) + c)*b^2 - b^2*c)*sin(2*d*sqrt(x) + 2*c))
*log(cos(2*d*sqrt(x) + 2*c)^2 + sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*sqrt(x
) + 2*c) + 1) + (-I*(d*sqrt(x) + c)^2*a*b + 2*I*(d*sqrt(x) + c)*a*b*c + (-I
*(d*sqrt(x) + c)^2*a*b + 2*I*(d*sqrt(x) + c)*a*b*c)*cos(2*d*sqrt(x) + 2*c)
+ ((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c)*sin(2*d*sqrt(x) + 2*c))
*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*sqrt(x) + c) + 1
) + (I*(d*sqrt(x) + c)^2*a*b - 2*I*(d*sqrt(x) + c)*a*b*c + (I*(d*sqrt(x) +
c)^2*a*b - 2*I*(d*sqrt(x) + c)*a*b*c)*cos(2*d*sqrt(x) + 2*c) - ((d*sqrt(x)
+ c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c)*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqr
t(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*sin(d*sqrt(x) + c) + 1) - 4*(I*a*b*c
os(2*d*sqrt(x) + 2*c) - a*b*sin(2*d*sqrt(x) + 2*c) + I*a*b)*polylog(3, I*e^
(I*d*sqrt(x) + I*c)) - 4*(-I*a*b*cos(2*d*sqrt(x) + 2*c) + a*b*sin(2*d*sqrt(
x) + 2*c) - I*a*b)*polylog(3, -I*e^(I*d*sqrt(x) + I*c)) - 2*(I*(d*sqrt(x) +
 c)^2*b^2 - 2*I*(d*sqrt(x) + c)*b^2*c)*sin(2*d*sqrt(x) + 2*c))/(-I*cos(2*d*
sqrt(x) + 2*c) + sin(2*d*sqrt(x) + 2*c) - I))/d^3
```

Giac [F]

$$\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\int \left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2\sqrt{x}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*sqrt(x) + c) + a)^2*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2\,dx=\int \sqrt{x}\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2\,dx$$

```
[In] int(x^(1/2)*(a + b/cos(c + d*x^(1/2)))^2,x)
```

```
[Out] int(x^(1/2)*(a + b/cos(c + d*x^(1/2)))^2, x)
```

3.58 $\int \frac{(a+b\sec(c+d\sqrt{x}))^2}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{(a+b\sec(c+d\sqrt{x}))^2}{\sqrt{x}}\,dx = 2a^2\sqrt{x}+\frac{4ab\operatorname{arctanh}(\sin(c+d\sqrt{x}))}{d}+\frac{2b^2\tan(c+d\sqrt{x})}{d}$$

```
[Out] 4*a*b*arctanh(sin(c+d*x^(1/2)))/d+2*a^2*x^(1/2)+2*b^2*tan(c+d*x^(1/2))/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4289, 3858, 3855, 3852, 8}

$$\int \frac{(a+b\sec(c+d\sqrt{x}))^2}{\sqrt{x}}\,dx = 2a^2\sqrt{x}+\frac{4ab\operatorname{arctanh}(\sin(c+d\sqrt{x}))}{d}+\frac{2b^2\tan(c+d\sqrt{x})}{d}$$

```
[In] Int[(a + b*Sec[c + d*Sqrt[x]])^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*ArcTanh[Sin[c + d*Sqrt[x]]])/d + (2*b^2*Tan[c + d*Sq
rt[x]])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int (a+b\sec(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2a^2\sqrt{x}+(4ab)\text{Subst}\left(\int \sec(c+dx)\,dx, x, \sqrt{x}\right)+\left(2b^2\right)\text{Subst}\left(\int \sec^2(c+dx)\,dx, x, \sqrt{x}\right)\\
&= 2a^2\sqrt{x}+\frac{4ab\operatorname{arctanh}\left(\sin\left(c+d\sqrt{x}\right)\right)}{d}-\frac{\left(2b^2\right)\text{Subst}\left(\int 1\,dx, x, -\tan\left(c+d\sqrt{x}\right)\right)}{d}\\
&= 2a^2\sqrt{x}+\frac{4ab\operatorname{arctanh}\left(\sin\left(c+d\sqrt{x}\right)\right)}{d}+\frac{2b^2\tan\left(c+d\sqrt{x}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$
\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx=\frac{2\left(a^2d\sqrt{x}+2ab\operatorname{arctanh}\left(\sin\left(c+d\sqrt{x}\right)\right)+b^2\tan\left(c+d\sqrt{x}\right)\right)}{d}
$$

```
[In] Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/Sqrt[x],x]
[Out] (2*(a^2*d*Sqrt[x] + 2*a*b*ArcTanh[Sin[c + d*Sqrt[x]]] + b^2*Tan[c + d*Sqrt[
x]]))/d
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

method	result	size
parts	$2a^2\sqrt{x}+\frac{2b^2\tan(c+d\sqrt{x})}{d}+\frac{4ba\ln(\sec(c+d\sqrt{x})+\tan(c+d\sqrt{x}))}{d}$	51
derivativedivides	$\frac{2a^2(c+d\sqrt{x})+4ba\ln(\sec(c+d\sqrt{x})+\tan(c+d\sqrt{x}))+2b^2\tan(c+d\sqrt{x})}{d}$	52
default	$\frac{2a^2(c+d\sqrt{x})+4ba\ln(\sec(c+d\sqrt{x})+\tan(c+d\sqrt{x}))+2b^2\tan(c+d\sqrt{x})}{d}$	52

```
[In] int((a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*a^2*x^(1/2)+2*b^2*tan(c+d*x^(1/2))/d+4*b*a/d*ln(sec(c+d*x^(1/2))+tan(c+d*
x^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(41) = 82.

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.94

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx$$
$$=\frac{2\left(a^2d\sqrt{x}\cos\left(d\sqrt{x}+c\right)+ab\cos\left(d\sqrt{x}+c\right)\log\left(\sin\left(d\sqrt{x}+c\right)+1\right)-ab\cos\left(d\sqrt{x}+c\right)\log\left(-\sin\left(d\sqrt{x}+\right.\right.\right.}{d\cos\left(d\sqrt{x}+c\right)}$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="fricas")
[Out] 2*(a^2*d*sqrt(x)*cos(d*sqrt(x) + c) + a*b*cos(d*sqrt(x) + c)*log(sin(d*sqrt
(x) + c) + 1) - a*b*cos(d*sqrt(x) + c)*log(-sin(d*sqrt(x) + c) + 1) + b^2*s
in(d*sqrt(x) + c))/(d*cos(d*sqrt(x) + c))
```

Sympy [A] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx$$
$$=\begin{cases}\frac{2a^2(c+d\sqrt{x})+4ab\log\left(\tan\left(c+d\sqrt{x}\right)+\sec\left(c+d\sqrt{x}\right)\right)+2b^2\tan\left(c+d\sqrt{x}\right)}{d} & \text{for } d\neq 0\\ -\sqrt{x}(-2a^2-4ab\sec\left(c\right)-2b^2\sec^2\left(c\right)) & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*sec(c+d*x**(1/2)))**2/x**(1/2),x)
[Out] Piecewise(((2*a**2*(c + d*sqrt(x)) + 4*a*b*log(tan(c + d*sqrt(x)) + sec(c +
 d*sqrt(x))) + 2*b**2*tan(c + d*sqrt(x)))/d, Ne(d, 0)), (-sqrt(x)*(-2*a**2
- 4*a*b*sec(c) - 2*b**2*sec(c)**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = 2\,a^2\sqrt{x} + \frac{4\,ab\log\left(\sec\left(d\sqrt{x}+c\right)+\tan\left(d\sqrt{x}+c\right)\right)}{d} + \frac{2\,b^2\tan\left(d\sqrt{x}+c\right)}{d}$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*a^2*sqrt(x) + 4*a*b*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c))/d + 2*b^
2*tan(d*sqrt(x) + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(41) = 82.

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = \frac{2\left(\left(d\sqrt{x}+c\right)a^2 + 2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)+1\right|\right) - 2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,b^2\tan\left(\frac{1}{2}\,d\sqrt{x}+\right.}{\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)}\right.}{d}$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*((d*sqrt(x) + c)*a^2 + 2*a*b*log(abs(tan(1/2*d*sqrt(x) + 1/2*c) + 1)) - 2
*a*b*log(abs(tan(1/2*d*sqrt(x) + 1/2*c) - 1)) - 2*b^2*tan(1/2*d*sqrt(x) + 1
/2*c)/(tan(1/2*d*sqrt(x) + 1/2*c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 14.97 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.32

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = 2\,a^2\,\sqrt{x} + \frac{b^2\,4\mathrm{i}}{d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,\sqrt{x}\,2\mathrm{i}}+1\right)} + \frac{4\,a\,b\,\ln\left(-\frac{a\,b\,4\mathrm{i}}{\sqrt{x}}-\frac{4\,a\,b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}}{\sqrt{x}}\right)}{d} - \frac{4\,a\,b\,\ln\left(\frac{a\,b\,4\mathrm{i}}{\sqrt{x}}-\frac{4\,a\,b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}}{\sqrt{x}}\right)}{d}$$

```
[In] int((a + b/cos(c + d*x^(1/2)))^2/x^(1/2),x)
```

```
[Out] 2*a^2*x^(1/2) + (b^2*4i)/(d*(exp(c*2i + d*x^(1/2)*2i) + 1)) + (4*a*b*log(-
(a*b*4i)/x^(1/2) - (4*a*b*exp(d*x^(1/2)*1i)*exp(c*1i))/x^(1/2)))/d - (4*a*b
*log((a*b*4i)/x^(1/2) - (4*a*b*exp(d*x^(1/2)*1i)*exp(c*1i))/x^(1/2)))/d
```

3.59 $\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}} \, dx = \text{Int}\left(\frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}, x\right)$$

```
[Out] Unintegrable((a+b*sec(c+d*x^(1/2)))^2/x^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}} \, dx = \int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}} \, dx$$

```
[In] Int[(a + b*Sec[c + d*Sqrt[x]])^2/x^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*Sqrt[x]])^2/x^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}} \, dx$$

Mathematica [N/A]

Not integrable

Time = 72.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x^(3/2),x]
[Out] Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{3}{2}}}dx$$

```
[In] int((a+b*sec(c+d*x^(1/2)))^2/x^(3/2),x)
[Out] int((a+b*sec(c+d*x^(1/2)))^2/x^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="fricas")
[Out] integral((b^2*sqrt(x)*sec(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*sec(d*sqrt(x) +
c) + a^2*sqrt(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x**(1/2)))**2/x**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*sqrt(x)))**2/x**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 718, normalized size of antiderivative = 32.64

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] (4*b^2*sin(2*d*sqrt(x) + 2*c) + (d*cos(2*d*sqrt(x) + 2*c)^2*integrate(4*(b^
2*sin(2*d*sqrt(x) + 2*c) + (a*b*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c)
 + a*b*d*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*cos(d*sqrt(x) +
c))*sqrt(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*
d*cos(2*d*sqrt(x) + 2*c) + d)*x^2), x) + d*integrate(4*(b^2*sin(2*d*sqrt(x)
 + 2*c) + (a*b*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + a*b*d*sin(2*d*
sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*cos(d*sqrt(x) + c))*sqrt(x))/((d*
cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x)
 + 2*c) + d)*x^2), x)*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c)
*integrate(4*(b^2*sin(2*d*sqrt(x) + 2*c) + (a*b*d*cos(2*d*sqrt(x) + 2*c)*co
s(d*sqrt(x) + c) + a*b*d*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*
cos(d*sqrt(x) + c))*sqrt(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(
x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^2), x) + d*integrate(4*(b^2
*sin(2*d*sqrt(x) + 2*c) + (a*b*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c)
+ a*b*d*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*cos(d*sqrt(x) + c
))*sqrt(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d
*cos(2*d*sqrt(x) + 2*c) + d)*x^2), x))*x - 2*(a^2*d*cos(2*d*sqrt(x) + 2*c)^
2 + a^2*d*sin(2*d*sqrt(x) + 2*c)^2 + 2*a^2*d*cos(2*d*sqrt(x) + 2*c) + a^2*d
)*sqrt(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*
cos(2*d*sqrt(x) + 2*c) + d)*x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*sqrt(x) + c) + a)^2/x^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 13.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+\frac{b}{\cos\left(c+d\,\sqrt{x}\right)}\right)^2}{x^{3/2}}\,dx$$

```
[In] int((a + b/cos(c + d*x^(1/2)))^2/x^(3/2),x)
[Out] int((a + b/cos(c + d*x^(1/2)))^2/x^(3/2), x)
```

3.60 $\int \frac{(a+b\sec(c+d\sqrt{x}))^2}{x^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx = \text{Int}\left(\frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*sec(c+d*x^(1/2)))^2/x^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx = \int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx$$

```
[In] Int[(a + b*Sec[c + d*Sqrt[x]])^2/x^(5/2),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*Sqrt[x]])^2/x^(5/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 72.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x^(5/2),x]
[Out] Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{5}{2}}}dx$$

```
[In] int((a+b*sec(c+d*x^(1/2)))^2/x^(5/2),x)
[Out] int((a+b*sec(c+d*x^(1/2)))^2/x^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sqrt(x)*sec(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*sec(d*sqrt(x) +
c) + a^2*sqrt(x))/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 5.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x**(1/2)))**2/x**(5/2),x)
[Out] Integral((a + b*sec(c + d*sqrt(x)))**2/x**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sec(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*sqrt(x) + c) + a)^2/x^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 14.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2}{x^{5/2}}\,dx$$

```
[In] int((a + b/cos(c + d*x^(1/2)))^2/x^(5/2),x)
[Out] int((a + b/cos(c + d*x^(1/2)))^2/x^(5/2), x)
```

3.61 $\int \frac{x^{3/2}}{a+b\sec(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 653

$$\begin{aligned}\int \frac{x^{3/2}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx &= \frac{2x^{5/2}}{5a}+\frac{2ibx^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\ &- \frac{2ibx^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{8bx^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\ &- \frac{8bx^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{24ibx\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\ &- \frac{24ibx\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{48b\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\ &+ \frac{48b\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\ &- \frac{48ib\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}+\frac{48ib\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}\end{aligned}$$

```
[Out] 2/5*x^(5/2)/a+2*I*b*x^2*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a
/d/(-a^2+b^2)^(1/2)-2*I*b*x^2*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/
2)))/a/d/(-a^2+b^2)^(1/2)+8*b*x^(3/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-
(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)-8*b*x^(3/2)*polylog(2,-a*exp(I*(c
+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)+24*I*b*x*polylog(
3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)-24*I
*b*x*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^
2)^(1/2)-48*I*b*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d
^5/(-a^2+b^2)^(1/2)+48*I*b*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^
```

```
(1/2)))/a/d^5/(-a^2+b^2)^(1/2)-48*b*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b-(-
a^2+b^2)^(1/2)))*x^(1/2)/a/d^4/(-a^2+b^2)^(1/2)+48*b*polylog(4,-a*exp(I*(c+
d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^4/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {4289, 4276, 3402, 2296, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{x^{3/2}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx = -\frac{48ib\,\text{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} + \frac{48ib\,\text{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{24ibx\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{24ibx\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} + \frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{2ibx^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad\sqrt{b^2-a^2}} - \frac{2ibx^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad\sqrt{b^2-a^2}} + \frac{2x^{5/2}}{5a}$$

```
[In] Int[x^(3/2)/(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (2*x^(5/2))/(5*a) + ((2*I)*b*x^2*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqr
t[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - ((2*I)*b*x^2*Log[1 + (a*E^(I*(c +
 d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (8*b*x^(3/2
)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[
-a^2 + b^2]*d^2) - (8*b*x^(3/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b +
 Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + ((24*I)*b*x*PolyLog[3, -((
a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3)
 - ((24*I)*b*x*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]
))])/(a*Sqrt[-a^2 + b^2]*d^3) - (48*b*Sqrt[x]*PolyLog[4, -((a*E^(I*(c + d*S
qrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (48*b*Sqrt[x
]*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[
-a^2 + b^2]*d^4) - ((48*I)*b*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sq
rt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5) + ((48*I)*b*PolyLog[5, -((a*E^(
I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^4}{a+b\sec(c+dx)}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^4}{a}-\frac{bx^4}{a(b+a\cos(c+dx))}\right)dx, x, \sqrt{x}\right)\\
&= \frac{2x^{5/2}}{5a}-\frac{(2b)\text{Subst}\left(\int \frac{x^4}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{2x^{5/2}}{5a}-\frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{2x^{5/2}}{5a}-\frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}\\
&\quad+\frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}\\
&= \frac{2x^{5/2}}{5a}+\frac{2ibx^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\
&\quad-\frac{(8ib)\text{Subst}\left(\int x^3\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}\\
&\quad+\frac{(8ib)\text{Subst}\left(\int x^3\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}
\end{aligned}
$$

$$= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{8bx^{3/2}\,\mathrm{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{8bx^{3/2}\,\mathrm{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$- \frac{(24b)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(2, -\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(24b)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(2, -\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{8bx^{3/2}\,\mathrm{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{8bx^{3/2}\,\mathrm{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{24ibx\,\mathrm{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{24ibx\,\mathrm{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{(48ib)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(3, -\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$+ \frac{(48ib)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(3, -\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{8bx^{3/2}\,\mathrm{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{8bx^{3/2}\,\mathrm{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{24ibx\,\mathrm{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{24ibx\,\mathrm{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{48b\sqrt{x}\,\mathrm{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{48b\sqrt{x}\,\mathrm{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$+ \frac{(48b)\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(4, -\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$- \frac{(48b)\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(4, -\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}$$

$$
\begin{aligned}
&= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
&+ \frac{8bx^{3/2}\operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{8bx^{3/2}\operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
&+ \frac{24ibx\operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{24ibx\operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
&- \frac{48b\sqrt{x}\operatorname{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{48b\sqrt{x}\operatorname{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
&- \frac{(48ib)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(4, \frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^5} \\
&+ \frac{(48ib)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(4, -\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
&+ \frac{8bx^{3/2}\operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{8bx^{3/2}\operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
&+ \frac{24ibx\operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{24ibx\operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
&- \frac{48b\sqrt{x}\operatorname{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{48b\sqrt{x}\operatorname{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
&- \frac{48ib\operatorname{PolyLog}\left(5, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{48ib\operatorname{PolyLog}\left(5, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.79

$$
\int \frac{x^{3/2}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx = \frac{2\left(\sqrt{-a^2+b^2}d^5x^{5/2} + 5ibd^4x^2\log\left(1 - \frac{ae^{i(c+d\sqrt{x})}}{-b+\sqrt{-a^2+b^2}}\right) - 5ibd^4x^2\log\left(1 + \frac{ae^{i(c+d}}{b+\sqrt{-a^2}}\right.\right.}{}
$$

```
[In] Integrate[x^(3/2)/(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (2*(Sqrt[-a^2 + b^2]*d^5*x^(5/2) + (5*I)*b*d^4*x^2*Log[1 - (a*E^(I*(c + d*S
qrt[x]))/(-b + Sqrt[-a^2 + b^2])] - (5*I)*b*d^4*x^2*Log[1 + (a*E^(I*(c + d
```

```
*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])] + 20*b*d^3*x^(3/2)*PolyLog[2, (a*E^(I*(
c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - 20*b*d^3*x^(3/2)*PolyLog[2, -((
a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] + (60*I)*b*d^2*x*PolyLog[
3, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - (60*I)*b*d^2*x*Poly
Log[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] - 120*b*d*Sqrt[
x]*PolyLog[4, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] + 120*b*d*
Sqrt[x]*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] - (
120*I)*b*PolyLog[5, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] + (1
20*I)*b*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))]))/(
5*a*Sqrt[-a^2 + b^2]*d^5)
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{a+b\sec\left(c+d\sqrt{x}\right)}dx$$

[In] int(x^(3/2)/(a+b*sec(c+d*x^(1/2))),x)

[Out] int(x^(3/2)/(a+b*sec(c+d*x^(1/2))),x)

Fricas [F]

$$\int \frac{x^{3/2}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{b\sec\left(d\sqrt{x}+c\right)+a}\,dx$$

[In] integrate(x^(3/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")

[Out] integral(x^(3/2)/(b*sec(d*sqrt(x) + c) + a), x)

Sympy [F]

$$\int \frac{x^{3/2}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx$$

[In] integrate(x**(3/2)/(a+b*sec(c+d*x**(1/2))),x)

[Out] Integral(x**(3/2)/(a + b*sec(c + d*sqrt(x))), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{3/2}}{a + b\sec\left(c + d\sqrt{x}\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^(3/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^{3/2}}{a + b\sec\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^{\frac{3}{2}}}{b\sec\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(b*sec(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{a + b\sec\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^{3/2}}{a + \frac{b}{\cos\left(c + d\sqrt{x}\right)}}\,dx$$

```
[In] int(x^(3/2)/(a + b/cos(c + d*x^(1/2))),x)
```

```
[Out] int(x^(3/2)/(a + b/cos(c + d*x^(1/2))), x)
```

3.62 $\int \frac{\sqrt{x}}{a+b\sec(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 393

$$\begin{aligned}\int \frac{\sqrt{x}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx &= \frac{2x^{3/2}}{3a}+\frac{2ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\&\quad+\frac{4b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}\\&\quad-\frac{4b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}\\&\quad+\frac{4ib\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{4ib\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}\end{aligned}$$

```
[Out] 2/3*x^(3/2)/a+2*I*b*x*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d
/(-a^2+b^2)^(1/2)-2*I*b*x*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))
/a/d/(-a^2+b^2)^(1/2)+4*I*b*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)
^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)-4*I*b*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+
(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)+4*b*polylog(2,-a*exp(I*(c+d*x^(1/
2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^2/(-a^2+b^2)^(1/2)-4*b*polylog(2,-a*
exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^2/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4289, 4276, 3402, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{\sqrt{x}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx=\frac{4ib\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}-\frac{4ib\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}+\frac{4b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}-\frac{4b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}+\frac{2ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad\sqrt{b^2-a^2}}-\frac{2ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad\sqrt{b^2-a^2}}+\frac{2x^{3/2}}{3a}$$

```
[In] Int[Sqrt[x]/(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (2*x^(3/2))/(3*a) + ((2*I)*b*x*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[
-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - ((2*I)*b*x*Log[1 + (a*E^(I*(c + d*S
qrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (4*b*Sqrt[x]*Po
lyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2
 + b^2]*d^2) - (4*b*Sqrt[x]*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqr
t[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + ((4*I)*b*PolyLog[3, -((a*E^(I*
(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - ((4*
I)*b*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sq
rt[-a^2 + b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^2}{a+b\sec(c+dx)}\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int \left(\frac{x^2}{a} - \frac{bx^2}{a(b+a\cos(c+dx))}\right)\,dx, x, \sqrt{x}\right)$$
$$= \frac{2x^{3/2}}{3a} - \frac{(2b)\text{Subst}\left(\int \frac{x^2}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{2x^{3/2}}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{2x^{3/2}}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}} + \frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}$$
$$= \frac{2x^{3/2}}{3a} + \frac{2ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{(4ib)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d} + \frac{(4ib)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}$$
$$= \frac{2x^{3/2}}{3a} + \frac{2ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{(4b)\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(4b)\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{2x^{3/2}}{3a} + \frac{2ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(4ib)\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{(4ib)\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$= \frac{2x^{3/2}}{3a} + \frac{2ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{4ib\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{4ib\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

Mathematica [A] (verified)

Time = 3.03 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{x}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx$$

$$= \frac{2\left(\sqrt{-a^2+b^2}d^3x^{3/2} + 3ibd^2x\log\left(1-\frac{ae^{i(c+d\sqrt{x})}}{-b+\sqrt{-a^2+b^2}}\right) - 3ibd^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right) + 6bd\sqrt{x}\,\text{PolyLog}\left(2,\frac{a}{-b}\right.\right.}{3a\sqrt{-}}$$

```
[In] Integrate[Sqrt[x]/(a + b*Sec[c + d*Sqrt[x]]),x]
```

```
[Out] (2*(Sqrt[-a^2 + b^2]*d^3*x^(3/2) + (3*I)*b*d^2*x*Log[1 - (a*E^(I*(c + d*Sqr
t[x])))/(-b + Sqrt[-a^2 + b^2])] - (3*I)*b*d^2*x*Log[1 + (a*E^(I*(c + d*Sqr
t[x])))/(b + Sqrt[-a^2 + b^2])] + 6*b*d*Sqrt[x]*PolyLog[2, (a*E^(I*(c + d*S
qrt[x])))/(-b + Sqrt[-a^2 + b^2])] - 6*b*d*Sqrt[x]*PolyLog[2, -((a*E^(I*(c
+ d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] + (6*I)*b*PolyLog[3, (a*E^(I*(c + d
*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - (6*I)*b*PolyLog[3, -((a*E^(I*(c + d*
Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))]))/(3*a*Sqrt[-a^2 + b^2]*d^3)
```

Maple [F]

$$\int \frac{\sqrt{x}}{a+b\sec\left(c+d\sqrt{x}\right)}dx$$

```
[In] int(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x)
[Out] int(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{\sqrt{x}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx = \int \frac{\sqrt{x}}{b\sec\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(sqrt(x)/(b*sec(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx = \int \frac{\sqrt{x}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x**(1/2)/(a+b*sec(c+d*x**(1/2))),x)
[Out] Integral(sqrt(x)/(a + b*sec(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{\sqrt{x}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx=\int \frac{\sqrt{x}}{b\sec\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(sqrt(x)/(b*sec(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{a+b\sec\left(c+d\sqrt{x}\right)}\,dx=\int \frac{\sqrt{x}}{a+\frac{b}{\cos\left(c+d\,\sqrt{x}\right)}}\,dx$$

```
[In] int(x^(1/2)/(a + b/cos(c + d*x^(1/2))),x)
[Out] int(x^(1/2)/(a + b/cos(c + d*x^(1/2))), x)
```

3.63 $\int \frac{1}{\sqrt{x}(a+b\sec(c+d\sqrt{x}))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\frac{2\sqrt{x}}{a}-\frac{4b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}$$

```
[Out] -4*b*arctanh((a-b)^(1/2)*tan(1/2*c+1/2*d*x^(1/2))/(a+b)^(1/2))/a/d/(a-b)^(1
/2)/(a+b)^(1/2)+2*x^(1/2)/a
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4289, 3868, 2738, 214}

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\frac{2\sqrt{x}}{a}-\frac{4b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[In] Int[1/(Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]])),x]
```

```
[Out] (2*Sqrt[x])/a - (4*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*Sqrt[x])/2])/Sqrt[a +
b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1}{a+b\sec(c+dx)}\,dx, x, \sqrt{x}\right) \\
&= \frac{2\sqrt{x}}{a} - \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx, x, \sqrt{x}\right)}{a} \\
&= \frac{2\sqrt{x}}{a} - \frac{4\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{ad} \\
&= \frac{2\sqrt{x}}{a} - \frac{4b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx = \frac{2\left(\frac{c}{d}+\sqrt{x}+\frac{2b\,\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}\right)}{a}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]])),x]
[Out] (2*(c/d + Sqrt[x] + (2*b*ArcTanh[((-a + b)*Tan[(c + d*Sqrt[x])/2])/Sqrt[a^2
 - b^2]])/(Sqrt[a^2 - b^2]*d)))/a
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{\frac{4\arctan\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{a}-\frac{4b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}}{d}$	70
default	$\frac{\frac{4\arctan\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{a}-\frac{4b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}}{d}$	70

```
[In] int(1/(a+b*sec(c+d*x^(1/2)))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d*(2/a*arctan(tan(1/2*c+1/2*d*x^(1/2)))-2*b/a/((a-b)*(a+b))^(1/2)*arctanh
((a-b)*tan(1/2*c+1/2*d*x^(1/2))/((a-b)*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 274, normalized size of antiderivative = 4.03

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$$
$$=\left[\frac{2\left(a^2-b^2\right)d\sqrt{x}+\sqrt{a^2-b^2}b\log\left(\frac{2\,ab\cos\left(d\sqrt{x}+c\right)-\left(a^2-2\,b^2\right)\cos\left(d\sqrt{x}+c\right)^2+2\,a^2-b^2-2\left(\sqrt{a^2-b^2}b\cos\left(d\sqrt{x}+c\right)+\sqrt{a^2-b^2}a\right)\sin}{a^2\cos\left(d\sqrt{x}+c\right)^2+2\,ab\cos\left(d\sqrt{x}+c\right)+b^2}\right.}{\left(a^3-ab^2\right)d}\right.$$

```
[In] integrate(1/(a+b*sec(c+d*x^(1/2)))/x^(1/2),x, algorithm="fricas")
[Out] [(2*(a^2 - b^2)*d*sqrt(x) + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*sqrt(x) + c)
 - (a^2 - 2*b^2)*cos(d*sqrt(x) + c)^2 + 2*a^2 - b^2 - 2*(sqrt(a^2 - b^2)*b*
cos(d*sqrt(x) + c) + sqrt(a^2 - b^2)*a)*sin(d*sqrt(x) + c))/(a^2*cos(d*sqrt
(x) + c)^2 + 2*a*b*cos(d*sqrt(x) + c) + b^2)))/((a^3 - a*b^2)*d), 2*((a^2 -
 b^2)*d*sqrt(x) - sqrt(-a^2 + b^2)*b*arctan(-(sqrt(-a^2 + b^2)*b*cos(d*sqrt
(x) + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*sin(d*sqrt(x) + c))))/((a^3 - a
*b^2)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] integrate(1/(a+b*sec(c+d*x**(1/2)))/x**(1/2),x)
[Out] Integral(1/(sqrt(x)*(a + b*sec(c + d*sqrt(x)))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*sec(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(55)=110$.

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 4.09

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx = \frac{2\left(\sqrt{-a^2+b^2}(a-2\,b)d|-a+b|-\sqrt{-a^2+b^2}|a||-a+b||d|\right)\left(\pi\left\lfloor\frac{d\sqrt{x}+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)}{\sqrt{-\frac{bd+\sqrt{b^2d^2+(ad+bd)(a}}{ad-bd}}}\right.\right.}{(a^2-2\,ab+b^2)a^2d^2+(a^2b-2\,ab^2+b^3)d|a||d|} + \frac{2\left(ad-2\,bd+|a||d|\right)\left(\pi\left\lfloor\frac{d\sqrt{x}+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)}{\sqrt{-\frac{bd-\sqrt{b^2d^2+(ad+bd)(ad-bd)}}{ad-bd}}}\right)\right)}{a^2d^2-bd|a||d|}$$

```
[In] integrate(1/(a+b*sec(c+d*x^(1/2)))/x^(1/2),x, algorithm="giac")
[Out] 2*(sqrt(-a^2 + b^2)*(a - 2*b)*d*abs(-a + b) - sqrt(-a^2 + b^2)*abs(a)*abs(-
a + b)*abs(d))*(pi*floor(1/2*(d*sqrt(x) + c)/pi + 1/2) + arctan(tan(1/2*d*s
```

```
qrt(x) + 1/2*c)/sqrt(-(b*d + sqrt(b^2*d^2 + (a*d + b*d)*(a*d - b*d)))/(a*d
- b*d))))/((a^2 - 2*a*b + b^2)*a^2*d^2 + (a^2*b - 2*a*b^2 + b^3)*d*abs(a)*a
bs(d)) + 2*(a*d - 2*b*d + abs(a)*abs(d))*(pi*floor(1/2*(d*sqrt(x) + c)/pi +
 1/2) + arctan(tan(1/2*d*sqrt(x) + 1/2*c)/sqrt(-(b*d - sqrt(b^2*d^2 + (a*d
+ b*d)*(a*d - b*d)))/(a*d - b*d))))/(a^2*d^2 - b*d*abs(a)*abs(d))
```

Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{x}\,(a+b\sec(c+d\sqrt{x}))}\,dx = \frac{2\sqrt{x}}{a} + \frac{2\,b\,\ln\left(2\,b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}} - \frac{b\,\left(a+b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)\,2\mathrm{i}}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{a-b}} - \frac{2\,b\,\ln\left(2\,b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}} + \frac{b\,\left(a+b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)\,2\mathrm{i}}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{a-b}}$$

```
[In] int(1/(x^(1/2)*(a + b/cos(c + d*x^(1/2)))),x)
```

```
[Out] (2*x^(1/2))/a + (2*b*log(2*b*exp(d*x^(1/2)*1i)*exp(c*1i) - (b*(a + b*exp(d*
x^(1/2)*1i)*exp(c*1i))*2i)/((a + b)^(1/2)*(a - b)^(1/2))))/(a*d*(a + b)^(1/
2)*(a - b)^(1/2)) - (2*b*log(2*b*exp(d*x^(1/2)*1i)*exp(c*1i) + (b*(a + b*ex
p(d*x^(1/2)*1i)*exp(c*1i))*2i)/((a + b)^(1/2)*(a - b)^(1/2))))/(a*d*(a + b)
^(1/2)*(a - b)^(1/2))
```

3.64 $\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\text{Int}\left(\frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^(3/2)/(a+b*sec(c+d*x^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Int[1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])),x]
[Out] Defer[Int][1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Integrate[1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])),x]
[Out] Integrate[1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{3}{2}}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}dx$$

```
[In] int(1/x^(3/2)/(a+b*sec(c+d*x^(1/2))),x)
[Out] int(1/x^(3/2)/(a+b*sec(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(sqrt(x)/(b*x^2*sec(d*sqrt(x) + c) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{\frac{3}{2}}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] integrate(1/x**(3/2)/(a+b*sec(c+d*x**(1/2))),x)
[Out] Integral(1/(x**(3/2)*(a + b*sec(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 243, normalized size of antiderivative = 11.05

$$\int \frac{1}{x^{3/2} \left(a + b \sec \left(c + d\sqrt{x}\right)\right)} \, dx = \int \frac{1}{\left(b \sec \left(d\sqrt{x} + c\right) + a\right) x^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/x^(3/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")
[Out] -2*(a*b*sqrt(x)*integrate((a*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + 2*
b*cos(d*sqrt(x) + c)^2 + a*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 2*b*
sin(d*sqrt(x) + c)^2 + a*cos(d*sqrt(x) + c))/((a^3*cos(2*d*sqrt(x) + 2*c)^2
 + 4*a*b^2*cos(d*sqrt(x) + c)^2 + a^3*sin(2*d*sqrt(x) + 2*c)^2 + 4*a^2*b*si
n(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 4*a*b^2*sin(d*sqrt(x) + c)^2 + 4*
a^2*b*cos(d*sqrt(x) + c) + a^3 + 2*(2*a^2*b*cos(d*sqrt(x) + c) + a^3)*cos(2
*d*sqrt(x) + 2*c))*x^(3/2)), x) + 1)/(a*sqrt(x))
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2} \left(a + b \sec \left(c + d\sqrt{x}\right)\right)} \, dx = \int \frac{1}{\left(b \sec \left(d\sqrt{x} + c\right) + a\right) x^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/x^(3/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*sqrt(x) + c) + a)*x^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 13.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2} \left(a + b \sec \left(c + d\sqrt{x}\right)\right)} \, dx = \int \frac{1}{x^{3/2} \left(a + \frac{b}{\cos\left(c + d\sqrt{x}\right)}\right)} \, dx$$

```
[In] int(1/(x^(3/2)*(a + b/cos(c + d*x^(1/2)))),x)
[Out] int(1/(x^(3/2)*(a + b/cos(c + d*x^(1/2)))), x)
```

3.65 $\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\text{Int}\left(\frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^(5/2)/(a+b*sec(c+d*x^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Int[1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])),x]
[Out] Defer[Int][1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2} \left(a+b \sec \left(c+d\sqrt{x}\right)\right)} \, dx=\int \frac{1}{x^{5/2} \left(a+b \sec \left(c+d\sqrt{x}\right)\right)} \, dx$$

```
[In] Integrate[1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])),x]
[Out] Integrate[1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{5}{2}} \left(a+b \sec \left(c+d\sqrt{x}\right)\right)}dx$$

```
[In] int(1/x^(5/2)/(a+b*sec(c+d*x^(1/2))),x)
[Out] int(1/x^(5/2)/(a+b*sec(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{5/2} \left(a+b \sec \left(c+d\sqrt{x}\right)\right)} \, dx=\int \frac{1}{\left(b \sec \left(d\sqrt{x}+c\right)+a\right) x^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/x^(5/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(sqrt(x)/(b*x^3*sec(d*sqrt(x) + c) + a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 6.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2} \left(a+b \sec \left(c+d\sqrt{x}\right)\right)} \, dx=\int \frac{1}{x^{\frac{5}{2}} \left(a+b \sec \left(c+d\sqrt{x}\right)\right)} \, dx$$

```
[In] integrate(1/x**(5/2)/(a+b*sec(c+d*x**(1/2))),x)
[Out] Integral(1/(x**(5/2)*(a + b*sec(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 11.09

$$\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx = \int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] -2/3*(3*a*b*x^(3/2)*integrate((a*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c)
+ 2*b*cos(d*sqrt(x) + c)^2 + a*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) +
2*b*sin(d*sqrt(x) + c)^2 + a*cos(d*sqrt(x) + c))/((a^3*cos(2*d*sqrt(x) + 2*
c)^2 + 4*a*b^2*cos(d*sqrt(x) + c)^2 + a^3*sin(2*d*sqrt(x) + 2*c)^2 + 4*a^2*
b*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 4*a*b^2*sin(d*sqrt(x) + c)^2
+ 4*a^2*b*cos(d*sqrt(x) + c) + a^3 + 2*(2*a^2*b*cos(d*sqrt(x) + c) + a^3)*c
os(2*d*sqrt(x) + 2*c))*x^(5/2)), x) + 1)/(a*x^(3/2))
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx = \int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*sqrt(x) + c) + a)*x^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 13.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}\,dx = \int \frac{1}{x^{5/2}\,\left(a+\frac{b}{\cos\left(c+d\,\sqrt{x}\right)}\right)}\,dx$$

```
[In] int(1/(x^(5/2)*(a + b/cos(c + d*x^(1/2)))),x)
```

```
[Out] int(1/(x^(5/2)*(a + b/cos(c + d*x^(1/2)))), x)
```

3.66 $\int \frac{x^{3/2}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 1925

$$
\begin{aligned}
&\int \frac{x^{3/2}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ib^2x^2}{a^2\left(a^2-b^2\right)d}+\frac{2x^{5/2}}{5a^2} \\
&+\frac{8b^2x^{3/2}\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}+\frac{8b^2x^{3/2}\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2} \\
&-\frac{2ib^3x^2\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}+\frac{4ibx^2\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&+\frac{2ib^3x^2\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}-\frac{4ibx^2\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&-\frac{24ib^2x\,\mathrm{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}-\frac{24ib^2x\,\mathrm{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3} \\
&-\frac{8b^3x^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}+\frac{16bx^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&+\frac{8b^3x^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}-\frac{16bx^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&+\frac{48b^2\sqrt{x}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}+\frac{48b^2\sqrt{x}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^4} \\
&-\frac{24ib^3x\,\mathrm{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}+\frac{48ibx\,\mathrm{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} \\
&+\frac{24ib^3x\,\mathrm{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}-\frac{48ibx\,\mathrm{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} \\
&+\frac{48ib^2\,\mathrm{PolyLog}\left(4,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^5}+\frac{48ib^2\,\mathrm{PolyLog}\left(4,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^5} \\
&+\frac{48b^3\sqrt{x}\,\mathrm{PolyLog}\left(4,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^4}-\frac{96b\sqrt{x}\,\mathrm{PolyLog}\left(4,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^4} \\
&-\frac{48b^3\sqrt{x}\,\mathrm{PolyLog}\left(4,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^4} \\
&+\frac{96b\sqrt{x}\,\mathrm{PolyLog}\left(4,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^4}+\frac{48ib^3\,\mathrm{PolyLog}\left(5,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^5} \\
&-\frac{96ib\,\mathrm{PolyLog}\left(5,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^5}-\frac{48ib^3\,\mathrm{PolyLog}\left(5,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^5} \\
&+\frac{96ib\,\mathrm{PolyLog}\left(5,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^5}+\frac{2b^2x^2\sin\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}
\end{aligned}
$$

```
[Out] 8*b^2*x^(3/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b
^2)/d^2+8*b^2*x^(3/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^
2/(a^2-b^2)/d^2-8*b^3*x^(3/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^
2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+8*b^3*x^(3/2)*polylog(2,-a*exp(I*(c+d*x
^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+16*b*x^(3/2)*polylo
g(2,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)-
16*b*x^(3/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^
2/(-a^2+b^2)^(1/2)+48*b^2*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^
(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^4+48*b^2*polylog(3,-a*exp(I*(c+d*x^(1/2)))/
(b+I*(a^2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^4+48*b^3*polylog(4,-a*exp(I*
(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^4-48*b^
3*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2
+b^2)^(3/2)/d^4-96*b*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2))
)*x^(1/2)/a^2/d^4/(-a^2+b^2)^(1/2)+96*b*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(
b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^4/(-a^2+b^2)^(1/2)-48*I*b^3*polylog(5,-a
*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^5-96*I*b
*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)
^(1/2)-2*I*b^2*x^2/a^2/(a^2-b^2)/d+2/5*x^(5/2)/a^2+2*b^2*x^2*sin(c+d*x^(1/2
))/a/(a^2-b^2)/d/(b+a*cos(c+d*x^(1/2)))-2*I*b^3*x^2*ln(1+a*exp(I*(c+d*x^(1/
2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-24*I*b^2*x*polylog(2,-a*e
xp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-24*I*b^2*x*pol
ylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-24*
I*b^3*x*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b
^2)^(3/2)/d^3-4*I*b*x^2*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a
^2/d/(-a^2+b^2)^(1/2)-48*I*b*x*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b
^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)+2*I*b^3*x^2*ln(1+a*exp(I*(c+d*x^(1/2))
)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+24*I*b^3*x*polylog(3,-a*exp(
I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+4*I*b*x^2*l
n(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+48*
I*b*x*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2
+b^2)^(1/2)+48*I*b^2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)
))/a^2/(a^2-b^2)/d^5+48*I*b^2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b
^2)^(1/2)))/a^2/(a^2-b^2)/d^5+48*I*b^3*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b
-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^5+96*I*b*polylog(5,-a*exp(I*(c+d
*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 3.58 (sec) , antiderivative size = 1925, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules

used = {4289, 4276, 3405, 3402, 2296, 2221, 2611, 6744, 2320, 6724, 4618}

$$
\begin{aligned}
\int \frac{x^{3/2}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx =& -\frac{2ix^2\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} \\
&+\frac{2ix^2\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}-\frac{8x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} \\
&+\frac{8x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}-\frac{24ix\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} \\
&+\frac{24ix\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3}+\frac{48\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4} \\
&-\frac{48\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4}+\frac{48i\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^5} \\
&-\frac{48i\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^5}-\frac{2ix^2b^2}{a^2\left(a^2-b^2\right)d} \\
&+\frac{8x^{3/2}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-i\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2}+\frac{8x^{3/2}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+i\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} \\
&-\frac{24ix\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3}-\frac{24ix\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3} \\
&+\frac{48\sqrt{x}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4}+\frac{48\sqrt{x}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4} \\
&+\frac{48i\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^5}+\frac{48i\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^5} \\
&+\frac{2x^2\sin\left(c+d\sqrt{x}\right)b^2}{a\left(a^2-b^2\right)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}+\frac{4ix^2\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}}+1\right)b}{a^2\sqrt{b^2-a^2}d} \\
&-\frac{4ix^2\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}}+1\right)b}{a^2\sqrt{b^2-a^2}d}+\frac{16x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2} \\
&-\frac{16x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2}+\frac{48ix\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3} \\
&-\frac{48ix\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3}-\frac{96\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^4} \\
&+\frac{96\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^4}-\frac{96i\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^5} \\
&+\frac{96i\operatorname{PolyLog}\left(5,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^5}+\frac{2x^{5/2}}{5a^2}
\end{aligned}
$$

[In] Int[x^(3/2)/(a + b*Sec[c + d*Sqrt[x]])^2,x]

[Out] ((-2*I)*b^2*x^2)/(a^2*(a^2 - b^2)*d) + (2*x^(5/2))/(5*a^2) + (8*b^2*x^(3/2)
*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^
2)*d^2) + (8*b^2*x^(3/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2
- b^2])])/(a^2*(a^2 - b^2)*d^2) - ((2*I)*b^3*x^2*Log[1 + (a*E^(I*(c + d*Sqr
t[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((4*I)*b*x^2*
Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 +
b^2]*d) + ((2*I)*b^3*x^2*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2
+ b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((4*I)*b*x^2*Log[1 + (a*E^(I*(c + d*
Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((24*I)*b^2*
x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))])/(a^2*(a
^2 - b^2)*d^3) - ((24*I)*b^2*x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b +
I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (8*b^3*x^(3/2)*PolyLog[2, -((
a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*
d^2) + (16*b*x^(3/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2
+ b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (8*b^3*x^(3/2)*PolyLog[2, -((a*E^(I
*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) -
(16*b*x^(3/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]
))])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (48*b^2*Sqrt[x]*PolyLog[3, -((a*E^(I*(c +
d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) + (48*b^2*Sq
rt[x]*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^
2*(a^2 - b^2)*d^4) - ((24*I)*b^3*x*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(
b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((48*I)*b*x*PolyLog
[3, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 +
b^2]*d^3) + ((24*I)*b^3*x*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[
-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((48*I)*b*x*PolyLog[3, -((a*
E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3)
+ ((48*I)*b^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2
]))])/(a^2*(a^2 - b^2)*d^5) + ((48*I)*b^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[
x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^5) + (48*b^3*Sqrt[x]*Po
lyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 +
b^2)^(3/2)*d^4) - (96*b*Sqrt[x]*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b
- Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^4) - (48*b^3*Sqrt[x]*PolyLog
[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)
^(3/2)*d^4) + (96*b*Sqrt[x]*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqr
t[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^4) + ((48*I)*b^3*PolyLog[5, -((a*
E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^
5) - ((96*I)*b*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]
))])/(a^2*Sqrt[-a^2 + b^2]*d^5) - ((48*I)*b^3*PolyLog[5, -((a*E^(I*(c + d*S
qrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^5) + ((96*I)*
b*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqr
t[-a^2 + b^2]*d^5) + (2*b^2*x^2*Sin[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b + a
*Cos[c + d*Sqrt[x]]))

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
```

```
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4618

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2] + I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a + Rt[-a^2 + b^2,
 2] + I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^4}{(a+b\sec(c+dx))^2}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{x^4}{a^2}+\frac{b^2x^4}{a^2(b+a\cos(c+dx))^2}-\frac{2bx^4}{a^2(b+a\cos(c+dx))}\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{2x^{5/2}}{5a^2} - \frac{(4b)\text{Subst}\left(\int \frac{x^4}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{x^4}{(b+a\cos(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}$$

$$= \frac{2x^{5/2}}{5a^2} + \frac{2b^2x^2\sin\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)} - \frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}$$
$$- \frac{(2b^3)\,\text{Subst}\left(\int \frac{x^4}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)} - \frac{(8b^2)\,\text{Subst}\left(\int \frac{x^3\sin(c+dx)}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}$$

$$= -\frac{2ib^2x^2}{a^2\left(a^2-b^2\right)d} + \frac{2x^{5/2}}{5a^2} + \frac{2b^2x^2\sin\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{(4b^3)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}$$
$$- \frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$+ \frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$- \frac{(8b^2)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{ib-\sqrt{a^2-b^2}+iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}$$
$$- \frac{(8b^2)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{ib+\sqrt{a^2-b^2}+iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}$$

$$= -\frac{2ib^2x^2}{a^2(a^2-b^2)d} + \frac{2x^{5/2}}{5a^2} + \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$

$$+ \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{4ibx^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$- \frac{4ibx^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2b^2x^2\sin\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}$$

$$+ \frac{(4b^3)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$

$$- \frac{(4b^3)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$

$$- \frac{(24b^2)\,\text{Subst}\left(\int x^2\log\left(1+\frac{iae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$

$$- \frac{(24b^2)\,\text{Subst}\left(\int x^2\log\left(1+\frac{iae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$

$$- \frac{(16ib)\text{Subst}\left(\int x^3\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$+ \frac{(16ib)\text{Subst}\left(\int x^3\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$
\begin{aligned}
&= -\frac{2ib^2x^2}{a^2(a^2-b^2)d} + \frac{2x^{5/2}}{5a^2} + \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} \\
&\quad + \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&\quad + \frac{4ibx^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&\quad - \frac{4ibx^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{24ib^2x\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} \\
&\quad - \frac{24ib^2x\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} + \frac{16bx^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&\quad - \frac{16bx^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2b^2x^2\sin\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)} \\
&\quad + \frac{(48ib^2)\,\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2,-\frac{iae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3} \\
&\quad + \frac{(48ib^2)\,\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2,-\frac{iae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3} \\
&\quad - \frac{(48b)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&\quad + \frac{(48b)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&\quad + \frac{(8ib^3)\,\mathrm{Subst}\left(\int x^3\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&\quad - \frac{(8ib^3)\,\mathrm{Subst}\left(\int x^3\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}
\end{aligned}
$$

$= \text{Too large to display}$

Mathematica [A] (verified)

Time = 14.04 (sec) , antiderivative size = 2254, normalized size of antiderivative = 1.17

$$\int \frac{x^{3/2}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[x^(3/2)/(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*x^(5/2)*(b + a*Cos[c + d*Sqrt[x]])^2*Sec[c + d*Sqrt[x]]^2)/(5*a^2*(a + b
*Sec[c + d*Sqrt[x]])^2) + (2*b*E^(I*c)*(b + a*Cos[c + d*Sqrt[x]])^2*((-2*I)
*b*E^(I*c)*x^2 + ((1 + E^((2*I)*c))*(4*b*d^3*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]
*x^(3/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)
*E^((2*I)*c)])] + (2*I)*a^2*d^4*E^(I*c)*x^2*Log[1 + (a*E^(I*(2*c + d*Sqrt[x
])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - I*b^2*d^4*E^(I*c)*x^2*
Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)
*c)])] + 4*b*d^3*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x^(3/2)*Log[1 + (a*E^(I*(2*
c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (2*I)*a^2*
d^4*E^(I*c)*x^2*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2
 + b^2)*E^((2*I)*c)])] + I*b^2*d^4*E^(I*c)*x^2*Log[1 + (a*E^(I*(2*c + d*Sqr
t[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 4*d^2*((3*I)*b*Sqrt
[(-a^2 + b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x
])*x*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2
)*E^((2*I)*c)]))] + 4*d^2*((-3*I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - 2*a^2*
d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*x*PolyLog[2, -((a*E^(I*(2*c + d*
Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 24*b*d*Sqrt[(-a
^2 + b^2)*E^((2*I)*c)]*Sqrt[x]*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*
E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (24*I)*a^2*d^2*E^(I*c)*x*Poly
Log[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I
)*c)]))] - (12*I)*b^2*d^2*E^(I*c)*x*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x]))
)/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 24*b*d*Sqrt[(-a^2 + b^2)
*E^((2*I)*c)]*Sqrt[x]*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) +
 Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (24*I)*a^2*d^2*E^(I*c)*x*PolyLog[3, -(
(a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]
+ (12*I)*b^2*d^2*E^(I*c)*x*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I
*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (24*I)*b*Sqrt[(-a^2 + b^2)*E^((2*
I)*c)]*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b
^2)*E^((2*I)*c)]))] - 48*a^2*d*E^(I*c)*Sqrt[x]*PolyLog[4, -((a*E^(I*(2*c +
d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 24*b^2*d*E^(I
*c)*Sqrt[x]*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^
2 + b^2)*E^((2*I)*c)]))] + (24*I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*PolyLog[
4, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)
]))] + 48*a^2*d*E^(I*c)*Sqrt[x]*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(b
*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - 24*b^2*d*E^(I*c)*Sqrt[x]*Pol
yLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*
```

```
I)*c)]))] - (48*I)*a^2*E^(I*c)*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*
E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (24*I)*b^2*E^(I*c)*PolyLog[5,
 -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])
)] + (48*I)*a^2*E^(I*c)*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c)
 + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (24*I)*b^2*E^(I*c)*PolyLog[5, -((a*E
^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]))/(d
^4*E^(I*c)*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))*Sec[c + d*Sqrt[x]]^2)/(a^2*(a^2
 - b^2)*d*(1 + E^((2*I)*c))*(a + b*Sec[c + d*Sqrt[x]])^2) + (2*(b + a*Cos[c
 + d*Sqrt[x]])*Sec[c + d*Sqrt[x]]^2*(b^3*x^2*Sin[c] - a*b^2*x^2*Sin[d*Sqrt[
x]]))/(a^2*(-a + b)*(a + b)*d*(a + b*Sec[c + d*Sqrt[x]])^2*(Cos[c/2] - Sin[
c/2])*(Cos[c/2] + Sin[c/2]))
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{\left(a+b\sec \left(c+d\sqrt{x}\right)\right)^{2}}dx$$

```
[In] int(x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x)
[Out] int(x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x^{3/2}}{\left(a+b\sec \left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(b\sec \left(d\sqrt{x}+c\right)+a\right)^{2}}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(x^(3/2)/(b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2
), x)
```

Sympy [F]

$$\int \frac{x^{3/2}}{\left(a+b\sec \left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(a+b\sec \left(c+d\sqrt{x}\right)\right)^{2}}\,dx$$

```
[In] integrate(x**(3/2)/(a+b*sec(c+d*x**(1/2)))**2,x)
[Out] Integral(x**(3/2)/(a + b*sec(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{3/2}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^{3/2}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^{\frac{3}{2}}}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(b*sec(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^{3/2}}{\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x^(3/2)/(a + b/cos(c + d*x^(1/2)))^2,x)
```

```
[Out] int(x^(3/2)/(a + b/cos(c + d*x^(1/2)))^2, x)
```

3.67 $\int \frac{\sqrt{x}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1125

$$\int \frac{\sqrt{x}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ib^2x}{a^2\left(a^2-b^2\right)d}+\frac{2x^{3/2}}{3a^2}+\frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}$$
$$+\frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}-\frac{2ib^3x\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}$$
$$+\frac{4ibx\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}+\frac{2ib^3x\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}$$
$$-\frac{4ibx\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}-\frac{4ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$
$$-\frac{4ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$
$$-\frac{4b^3\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}$$
$$+\frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+\frac{4b^3\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}$$
$$-\frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$-\frac{4ib^3\,\mathrm{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}$$
$$+\frac{8ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$+\frac{4ib^3\,\mathrm{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}$$
$$-\frac{8ib\,\mathrm{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$+\frac{2b^2x\sin\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}$$

```
[Out] 8*I*b*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2
+b^2)^(1/2)+2/3*x^(3/2)/a^2-4*I*b^3*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-
a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-4*I*b^2*polylog(2,-a*exp(I*(c+d*x
^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-4*I*b*x*ln(1+a*exp(I*(c+d
*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)-2*I*b^3*x*ln(1+a*ex
p(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+4*I*b*x*ln(
1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)-2*I*b
^2*x/a^2/(a^2-b^2)/d+2*b^2*x*sin(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cos(c+d*x^
(1/2)))-4*I*b^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^
2/(a^2-b^2)/d^3+2*I*b^3*x*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))
/a^2/(-a^2+b^2)^(3/2)/d-8*I*b*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^
2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)+4*I*b^3*polylog(3,-a*exp(I*(c+d*x^(1/2)
))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+4*b^2*ln(1+a*exp(I*(c+d*x
^(1/2)))/(b-I*(a^2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^2+4*b^2*ln(1+a*exp(
I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^2-4*b^3*pol
ylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)
^(3/2)/d^2+4*b^3*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^
(1/2)/a^2/(-a^2+b^2)^(3/2)/d^2+8*b*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-(-a
^2+b^2)^(1/2)))*x^(1/2)/a^2/d^2/(-a^2+b^2)^(1/2)-8*b*polylog(2,-a*exp(I*(c+
d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^2/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 1125, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules

used = {4289, 4276, 3405, 3402, 2296, 2221, 2611, 2320, 6724, 4618, 2317, 2438}

$$\int \frac{\sqrt{x}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ix\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} + \frac{2ix\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}$$
$$-\frac{4\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$
$$+\frac{4\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$
$$-\frac{4i\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3}$$
$$+\frac{4i\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} - \frac{2ixb^2}{a^2\left(a^2-b^2\right)d}$$
$$+\frac{4\sqrt{x}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-i\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} + \frac{4\sqrt{x}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+i\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2}$$
$$-\frac{4i\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3}$$
$$-\frac{4i\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3}$$
$$+\frac{2x\sin\left(c+d\sqrt{x}\right)b^2}{a\left(a^2-b^2\right)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}$$
$$+\frac{4ix\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}}+1\right)b}{a^2\sqrt{b^2-a^2}d} - \frac{4ix\log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}}+1\right)b}{a^2\sqrt{b^2-a^2}d}$$
$$+\frac{8\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2}$$
$$-\frac{8\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2}$$
$$+\frac{8i\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3}$$
$$-\frac{8i\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3} + \frac{2x^{3/2}}{3a^2}$$

```
[In] Int[Sqrt[x]/(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x)/(a^2*(a^2 - b^2)*d) + (2*x^(3/2))/(3*a^2) + (4*b^2*Sqrt[x]*L
og[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)
*d^2) + (4*b^2*Sqrt[x]*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 -
b^2])])/(a^2*(a^2 - b^2)*d^2) - ((2*I)*b^3*x*Log[1 + (a*E^(I*(c + d*Sqrt[x]
)))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((4*I)*b*x*Log[1
+ (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*
d) + ((2*I)*b^3*x*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])]
)/(a^2*(-a^2 + b^2)^(3/2)*d) - ((4*I)*b*x*Log[1 + (a*E^(I*(c + d*Sqrt[x])))
/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((4*I)*b^2*PolyLog[2,
-((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3
) - ((4*I)*b^2*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2
]))])/(a^2*(a^2 - b^2)*d^3) - (4*b^3*Sqrt[x]*PolyLog[2, -((a*E^(I*(c + d*Sq
rt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (8*b*Sqrt
[x]*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*S
qrt[-a^2 + b^2]*d^2) + (4*b^3*Sqrt[x]*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x]))
)/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (8*b*Sqrt[x]*Pol
yLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^
2 + b^2]*d^2) - ((4*I)*b^3*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt
[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((8*I)*b*PolyLog[3, -((a*E^
(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3) +
 ((4*I)*b^3*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))]
)/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((8*I)*b*PolyLog[3, -((a*E^(I*(c + d*Sqrt[
x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (2*b^2*x*Sin[c
 + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4618

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2] + I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a + Rt[-a^2 + b^2,
 2] + I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^2}{(a+b\sec(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^2}{a^2}+\frac{b^2x^2}{a^2(b+a\cos(c+dx))^2}-\frac{2bx^2}{a^2(b+a\cos(c+dx))}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2x^{3/2}}{3a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^2}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{(b+a\cos(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{2x^{3/2}}{3a^2}+\frac{2b^2x\sin\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad -\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^2}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}-\frac{(4b^2)\,\text{Subst}\left(\int \frac{x\sin(c+dx)}{b+a\cos(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}
\end{aligned}
$$

$$= -\frac{2ib^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{2b^2x\sin(c+d\sqrt{x})}{a(a^2-b^2)d(b+a\cos(c+d\sqrt{x}))} - \frac{(4b^3)\,\mathrm{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)} - \frac{(8b)\mathrm{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}} + \frac{(8b)\mathrm{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}} - \frac{(4b^2)\,\mathrm{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib-\sqrt{a^2-b^2}+iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(a^2-b^2)d} - \frac{(4b^2)\,\mathrm{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+\sqrt{a^2-b^2}+iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(a^2-b^2)d}$$

$$= -\frac{2ib^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{4ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{4ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2b^2x\sin(c+d\sqrt{x})}{a(a^2-b^2)d(b+a\cos(c+d\sqrt{x}))} + \frac{(4b^3)\,\mathrm{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}} - \frac{(4b^3)\,\mathrm{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}} - \frac{(4b^2)\,\mathrm{Subst}\left(\int \log\left(1+\frac{iae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2} - \frac{(4b^2)\,\mathrm{Subst}\left(\int \log\left(1+\frac{iae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2} - \frac{(8ib)\mathrm{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{(8ib)\mathrm{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$
\begin{aligned}
&= -\frac{2ib^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} \\
&+ \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&+ \frac{4ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&- \frac{4ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{8b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&- \frac{8b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2b^2x\sin\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)} \\
&+ \frac{(4ib^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{iax}{ib-\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{a^2(a^2-b^2)d^3} \\
&+ \frac{(4ib^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{iax}{ib+\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{a^2(a^2-b^2)d^3} \\
&- \frac{(8b)\text{Subst}\left(\int\text{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&+ \frac{(8b)\text{Subst}\left(\int\text{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&+ \frac{(4ib^3)\,\text{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&- \frac{(4ib^3)\,\text{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}
\end{aligned}
$$

$$= -\frac{2ib^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{4ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{4ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{4ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{4ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{4b^3\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$+ \frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{4b^3\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$- \frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2b^2x\sin\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(8ib)\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$- \frac{(8ib)\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$+ \frac{(4b^3)\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$- \frac{(4b^3)\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$

$$= -\frac{2ib^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$

$$+ \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$+ \frac{4ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$- \frac{4ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{4ib^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$

$$- \frac{4ib^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$

$$+ \frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$

$$- \frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{8ib\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$

$$- \frac{8ib\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{2b^2x\sin(c+d\sqrt{x})}{a(a^2-b^2)d(b+a\cos(c+d\sqrt{x}))}$$

$$- \frac{(4ib^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{i(c+d\sqrt{x})}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$

$$+ \frac{(4ib^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{i(c+d\sqrt{x})}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$

$$= -\frac{2ib^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{4ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{4ibx\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{4ib^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{4ib^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$+ \frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$- \frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{4ib^3\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$+ \frac{8ib\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{4ib^3\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$- \frac{8ib\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{2b^2x\sin\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cos\left(c+d\sqrt{x}\right)\right)}$$

Mathematica [A] (verified)

Time = 9.27 (sec) , antiderivative size = 1210, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{x}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$= \frac{2\left(b+a\cos\left(c+d\sqrt{x}\right)\right)\sec^2\left(c+d\sqrt{x}\right)\left(x^{3/2}\left(b+a\cos\left(c+d\sqrt{x}\right)\right)+\frac{3b\left(b+a\cos\left(c+d\sqrt{x}\right)\right)\left(-\frac{2ibd^2e^{2ic}x}{1+e^{2ic}}+\frac{2bd\sqrt{\left(-a^2\right.}}{}\right.}{}\right.}{}$$

```
[In] Integrate[Sqrt[x]/(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

[Out] (2*(b + a*Cos[c + d*Sqrt[x]])*Sec[c + d*Sqrt[x]]^2*(x^(3/2)*(b + a*Cos[c + d*Sqrt[x]]) + (3*b*(b + a*Cos[c + d*Sqrt[x]])*(((-2*I)*b*d^2*E^((2*I)*c)*x)/(1 + E^((2*I)*c)) + (2*b*d*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*Sqrt[x]*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (2*I)*a^2*d^2*E^(I*c)*x*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - I*b^2*d^2*E^(I*c)*x*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 2*b*d*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*Sqrt[x]*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (2*I)*a^2*d^2*E^(I*c)*x*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + I*b^2*d^2*E^(I*c)*x*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + 2*((-I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] + 2*a^2*d*E^(I*c)*Sqrt[x] - b^2*d*E^(I*c)*Sqrt[x])*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 2*((-I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (4*I)*a^2*E^(I*c)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (2*I)*b^2*E^(I*c)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (4*I)*a^2*E^(I*c)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (2*I)*b^2*E^(I*c)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))])/Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))/((a^2 - b^2)*d^3) + (3*b^2*x*(-(b*Sin[c]) + a*Sin[d*Sqrt[x]]))/((a - b)*(a + b)*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]))))/(3*a^2*(a + b*Sec[c + d*Sqrt[x]])^2)

Maple [F]

$$\int \frac{\sqrt{x}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}dx$$

[In] int(x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x)

[Out] int(x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x)

Fricas [F]

$$\int \frac{\sqrt{x}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{\sqrt{x}}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

[In] integrate(x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(sqrt(x)/(b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2), x)

Sympy [F]

$$\int \frac{\sqrt{x}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{\sqrt{x}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x**(1/2)/(a+b*sec(c+d*x**(1/2)))**2,x)
[Out] Integral(sqrt(x)/(a + b*sec(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{\sqrt{x}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{\sqrt{x}}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(sqrt(x)/(b*sec(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{\sqrt{x}}{\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x^(1/2)/(a + b/cos(c + d*x^(1/2)))^2,x)
[Out] int(x^(1/2)/(a + b/cos(c + d*x^(1/2)))^2, x)
```

3.68 $\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2} dx = \frac{2\sqrt{x}}{a^2} - \frac{4b(2a^2-b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{2b^2\tan\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\sec\left(c+d\sqrt{x}\right)\right)}$$

```
[Out] -4*b*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*c+1/2*d*x^(1/2))/(a+b)^(1/2))/
a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+2*x^(1/2)/a^2+2*b^2*tan(c+d*x^(1/2))/a/(a^2-b
^2)/d/(a+b*sec(c+d*x^(1/2)))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4289, 3870, 4004, 3916, 2738, 214}

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2} dx = -\frac{4b(2a^2-b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}} + \frac{2b^2\tan\left(c+d\sqrt{x}\right)}{ad\left(a^2-b^2\right)\left(a+b\sec\left(c+d\sqrt{x}\right)\right)} + \frac{2\sqrt{x}}{a^2}$$

```
[In] Int[1/(Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]])^2),x]
```

```
[Out] (2*Sqrt[x])/a^2 - (4*b*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*Sqrt[x
])/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (2*b^2*Tan[c + d
*Sqrt[x]])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*Sqrt[x]]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{(a+b\sec(c+dx))^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2b^2 \tan\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right) d \left(a+b \sec\left(c+d\sqrt{x}\right)\right)} - \frac{2\text{Subst}\left(\int \frac{-a^2+b^2+ab\sec(c+dx)}{a+b\sec(c+dx)}\, dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)}$$

$$= \frac{2\sqrt{x}}{a^2} + \frac{2b^2 \tan\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right) d \left(a+b \sec\left(c+d\sqrt{x}\right)\right)} - \frac{\left(2b(2a^2-b^2)\right)\text{Subst}\left(\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\, dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}$$

$$= \frac{2\sqrt{x}}{a^2} + \frac{2b^2 \tan\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right) d \left(a+b \sec\left(c+d\sqrt{x}\right)\right)} - \frac{\left(2(2a^2-b^2)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\, dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}$$

$$= \frac{2\sqrt{x}}{a^2} + \frac{2b^2 \tan\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right) d \left(a+b \sec\left(c+d\sqrt{x}\right)\right)} - \frac{\left(4(2a^2-b^2)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\, dx, x, \tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{a^2\left(a^2-b^2\right) d}$$

$$= \frac{2\sqrt{x}}{a^2} - \frac{4b(2a^2-b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{2b^2 \tan\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right) d \left(a+b \sec\left(c+d\sqrt{x}\right)\right)}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\, dx = \frac{2\left(-\frac{2b(-2a^2+b^2)\,\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{a(a^2-b^2)\left(c+d\sqrt{x}\right)\cos\left(c+d\sqrt{x}\right)+b\left((a^2-b^2)\left(c+d\sqrt{x}\right)+ab\sin\left(c+d\sqrt{x}\right)\right)}{b+a\cos\left(c+d\sqrt{x}\right)}\right)}{a^2(a-b)(a+b)d}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]])^2),x]
```

```
[Out] (2*((-2*b*(-2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*Sqrt[x])/2])/Sqrt[a^2
 - b^2]])/Sqrt[a^2 - b^2] + (a*(a^2 - b^2)*(c + d*Sqrt[x])*Cos[c + d*Sqrt[x
]] + b*((a^2 - b^2)*(c + d*Sqrt[x]) + a*b*Sin[c + d*Sqrt[x]]))/(b + a*Cos[c
 + d*Sqrt[x]])))/(a^2*(a - b)*(a + b)*d)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{\frac{4\arctan\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{a^2}+\frac{4b\left(-\frac{ba\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^2a-\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^2b-a-b\right)}-\frac{\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{a^2}}{d}$
default	$\frac{\frac{4\arctan\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{a^2}+\frac{4b\left(-\frac{ba\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^2a-\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^2b-a-b\right)}-\frac{\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{a^2}}{d}$

```
[In] int(1/(a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(2/a^2*arctan(tan(1/2*c+1/2*d*x^(1/2)))+2*b/a^2*(-b*a/(a^2-b^2)*tan(1/2
*c+1/2*d*x^(1/2))/(tan(1/2*c+1/2*d*x^(1/2))^2*a-tan(1/2*c+1/2*d*x^(1/2))^2*
b-a-b)-(2*a^2-b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*c+
1/2*d*x^(1/2))/((a-b)*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(110) = 220.

Time = 0.33 (sec) , antiderivative size = 574, normalized size of antiderivative = 4.52

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$= \left[\frac{2\left(a^5-2a^3b^2+ab^4\right)d\sqrt{x}\cos\left(d\sqrt{x}+c\right)+2\left(a^4b-2a^2b^3+b^5\right)d\sqrt{x}+\left(\left(2a^3b-ab^3\right)\sqrt{a^2-b^2}\cos\left(d\sqrt{x}\right.\right.}{\left(a^7-2a^5\right.}\right.$$

```
[In] integrate(1/(a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*cos(d*sqrt(x) + c) + 2*(a^4*b - 2*a
^2*b^3 + b^5)*d*sqrt(x) + ((2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*cos(d*sqrt(x)
+ c) + (2*a^2*b^2 - b^4)*sqrt(a^2 - b^2))*log((2*a*b*cos(d*sqrt(x) + c) - (
a^2 - 2*b^2)*cos(d*sqrt(x) + c)^2 + 2*a^2 - b^2 - 2*(sqrt(a^2 - b^2)*b*cos(
d*sqrt(x) + c) + sqrt(a^2 - b^2)*a)*sin(d*sqrt(x) + c))/(a^2*cos(d*sqrt(x)
+ c)^2 + 2*a*b*cos(d*sqrt(x) + c) + b^2)) + 2*(a^3*b^2 - a*b^4)*sin(d*sqrt(
x) + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*sqrt(x) + c) + (a^6*b - 2*a^4
```

```
*b^3 + a^2*b^5)*d), 2*((a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*cos(d*sqrt(x) +
c) + (a^4*b - 2*a^2*b^3 + b^5)*d*sqrt(x) - ((2*a^3*b - a*b^3)*sqrt(-a^2 + b
^2)*cos(d*sqrt(x) + c) + (2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2))*arctan(-(sqrt(
-a^2 + b^2)*b*cos(d*sqrt(x) + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*sin(d*s
qrt(x) + c))) + (a^3*b^2 - a*b^4)*sin(d*sqrt(x) + c))/((a^7 - 2*a^5*b^2 + a
^3*b^4)*d*cos(d*sqrt(x) + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/(a+b*sec(c+d*x**(1/2)))**2/x**(1/2),x)
[Out] Integral(1/(sqrt(x)*(a + b*sec(c + d*sqrt(x)))**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.54

$$\begin{aligned}\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx &= -\frac{4\,b^2\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)}{\left(a^3d-ab^2d\right)\left(a\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)^2-a-b\right)}\\&\quad+\frac{4\left(2\,a^2b-b^3\right)\left(\pi\left\lfloor\frac{d\sqrt{x}+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4d-a^2b^2d\right)\sqrt{-a^2+b^2}}\\&\quad+\frac{2\left(d\sqrt{x}+c\right)}{a^2d}\end{aligned}$$

```
[In] integrate(1/(a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -4*b^2*tan(1/2*d*sqrt(x) + 1/2*c)/((a^3*d - a*b^2*d)*(a*tan(1/2*d*sqrt(x) +
 1/2*c)^2 - b*tan(1/2*d*sqrt(x) + 1/2*c)^2 - a - b)) + 4*(2*a^2*b - b^3)*(p
i*floor(1/2*(d*sqrt(x) + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
sqrt(x) + 1/2*c) - b*tan(1/2*d*sqrt(x) + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*d
 - a^2*b^2*d)*sqrt(-a^2 + b^2)) + 2*(d*sqrt(x) + c)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 18.69 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.60

$$\int \frac{1}{\sqrt{x}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$
$$=\frac{\frac{b^2\,4\mathrm{i}}{a\,d\,(a^2-b^2)}+\frac{b^3\,\mathrm{e}^{c\,1\mathrm{i}+d\,\sqrt{x}\,1\mathrm{i}}\,4\mathrm{i}}{a^2\,d\,(a^2-b^2)}}{a+a\,\mathrm{e}^{c\,2\mathrm{i}+d\,\sqrt{x}\,2\mathrm{i}}+2\,b\,\mathrm{e}^{c\,1\mathrm{i}+d\,\sqrt{x}\,1\mathrm{i}}}+\frac{2\,\sqrt{x}}{a^2}$$
$$+\frac{\ln\left(\mathrm{e}^{c\,1\mathrm{i}+d\,\sqrt{x}\,1\mathrm{i}}\,(4\,a^2\,b-2\,b^3)-\frac{(4\,a^2\,b-2\,b^3)\,(a^2-b^2)\,\left(a+b\,\mathrm{e}^{c\,1\mathrm{i}+d\,\sqrt{x}\,1\mathrm{i}}\right)\,1\mathrm{i}}{(a+b)^{3/2}\,(a-b)^{3/2}}\right)\,(4\,a^2\,b-2\,b^3)}{a^2\,d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$
$$-\frac{2\,b\,\ln\left(\mathrm{e}^{c\,1\mathrm{i}+d\,\sqrt{x}\,1\mathrm{i}}\,(4\,a^2\,b-2\,b^3)+\frac{b\,(a^2-b^2)\,(2\,a^2-b^2)\,\left(a+b\,\mathrm{e}^{c\,1\mathrm{i}+d\,\sqrt{x}\,1\mathrm{i}}\right)\,2\mathrm{i}}{(a+b)^{3/2}\,(a-b)^{3/2}}\right)\,(2\,a^2-b^2)}{a^2\,d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

```
[In] int(1/(x^(1/2)*(a + b/cos(c + d*x^(1/2)))^2),x)
```

```
[Out] ((b^2*4i)/(a*d*(a^2 - b^2)) + (b^3*exp(c*1i + d*x^(1/2)*1i)*4i)/(a^2*d*(a^2
 - b^2)))/(a + a*exp(c*2i + d*x^(1/2)*2i) + 2*b*exp(c*1i + d*x^(1/2)*1i)) +
 (2*x^(1/2))/a^2 + (log(exp(c*1i + d*x^(1/2)*1i)*(4*a^2*b - 2*b^3) - ((4*a^
2*b - 2*b^3)*(a^2 - b^2)*(a + b*exp(c*1i + d*x^(1/2)*1i))*1i)/((a + b)^(3/2
)*(a - b)^(3/2)))*(4*a^2*b - 2*b^3))/(a^2*d*(a + b)^(3/2)*(a - b)^(3/2)) -
(2*b*log(exp(c*1i + d*x^(1/2)*1i)*(4*a^2*b - 2*b^3) + (b*(a^2 - b^2)*(2*a^2
 - b^2)*(a + b*exp(c*1i + d*x^(1/2)*1i))*2i)/((a + b)^(3/2)*(a - b)^(3/2)))
*(2*a^2 - b^2))/(a^2*d*(a + b)^(3/2)*(a - b)^(3/2))
```

3.69 $\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])^2),x]
[Out] Defer[Int][1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 44.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{3}{2}}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(1/x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x)
[Out] int(1/x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(sqrt(x)/(b^2*x^2*sec(d*sqrt(x) + c)^2 + 2*a*b*x^2*sec(d*sqrt(x) +
c) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{x^{\frac{3}{2}}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x**(3/2)/(a+b*sec(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x**(3/2)*(a + b*sec(c + d*sqrt(x)))**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2 x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(1/((b*sec(d*sqrt(x) + c) + a)^2*x^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 13.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{x^{3/2}\left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(1/(x^(3/2)*(a + b/cos(c + d*x^(1/2)))^2),x)
[Out] int(1/(x^(3/2)*(a + b/cos(c + d*x^(1/2)))^2), x)
```

3.70 $\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^(5/2)/(a+b*sec(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])^2),x]
[Out] Defer[Int][1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 47.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{5}{2}}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(1/x^(5/2)/(a+b*sec(c+d*x^(1/2)))^2,x)
[Out] int(1/x^(5/2)/(a+b*sec(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{5/2}\left(a+b\sec\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\sec\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(sqrt(x)/(b^2*x^3*sec(d*sqrt(x) + c)^2 + 2*a*b*x^3*sec(d*sqrt(x) +
c) + a^2*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 45.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2} \left(a+b \sec \left(c+d\sqrt{x}\right)\right)^2} \, dx = \int \frac{1}{x^{\frac{5}{2}} \left(a+b \sec \left(c+d\sqrt{x}\right)\right)^2} \, dx$$

```
[In] integrate(1/x**(5/2)/(a+b*sec(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x**(5/2)*(a + b*sec(c + d*sqrt(x)))**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2} \left(a+b \sec \left(c+d\sqrt{x}\right)\right)^2} \, dx = \text{Timed out}$$

```
[In] integrate(1/x^(5/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2} \left(a+b \sec \left(c+d\sqrt{x}\right)\right)^2} \, dx = \int \frac{1}{\left(b \sec \left(d\sqrt{x}+c\right)+a\right)^2 x^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/x^(5/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(1/((b*sec(d*sqrt(x) + c) + a)^2*x^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 13.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2} \left(a+b \sec \left(c+d\sqrt{x}\right)\right)^2} \, dx = \int \frac{1}{x^{5/2} \left(a+\frac{b}{\cos\left(c+d\sqrt{x}\right)}\right)^2} \, dx$$

```
[In] int(1/(x^(5/2)*(a + b/cos(c + d*x^(1/2)))^2),x)
[Out] int(1/(x^(5/2)*(a + b/cos(c + d*x^(1/2)))^2), x)
```

3.71 $\int (ex)^m (a + b\sec(c + dx^n))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m (a + b\sec(c + dx^n))^p \, dx = x^{-m}(ex)^m \text{Int}(x^m (a + b\sec(c + dx^n))^p, x)$$

```
[Out] (e*x)^m*Unintegrable(x^m*(a+b*sec(c+d*x^n))^p,x)/(x^m)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (a + b\sec(c + dx^n))^p \, dx = \int (ex)^m (a + b\sec(c + dx^n))^p \, dx$$

```
[In] Int[(e*x)^m*(a + b*Sec[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*(a + b*Sec[c + d*x^n])^p, x])/x^m
```

Rubi steps

$$\text{integral} = \left(x^{-m}(ex)^m\right) \int x^m (a + b\sec(c + dx^n))^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 4.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\sec(c+dx^n))^p \, dx = \int (ex)^m (a+b\sec(c+dx^n))^p \, dx$$

```
[In] Integrate[(e*x)^m*(a + b*Sec[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^m*(a + b*Sec[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m (a+b\sec(c+d\,x^n))^p dx$$

```
[In] int((e*x)^m*(a+b*sec(c+d*x^n))^p,x)
[Out] int((e*x)^m*(a+b*sec(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\sec(c+dx^n))^p \, dx = \int (ex)^m (b\sec(dx^n+c)+a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sec(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*sec(d*x^n + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 47.83 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m (a+b\sec(c+dx^n))^p \, dx = \int (ex)^m (a+b\sec(c+dx^n))^p \, dx$$

```
[In] integrate((e*x)**m*(a+b*sec(c+d*x**n))**p,x)
[Out] Integral((e*x)**m*(a + b*sec(c + d*x**n))**p, x)
```

Maxima [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\sec(c + dx^n))^p \, dx = \int (ex)^m (b\sec(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sec(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*sec(d*x^n + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\sec(c + dx^n))^p \, dx = \int (ex)^m (b\sec(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sec(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*sec(d*x^n + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 12.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (ex)^m (a + b\sec(c + dx^n))^p \, dx = \int \left(a + \frac{b}{\cos(c + d\,x^n)}\right)^p (e\,x)^m \, dx$$

```
[In] int((a + b/cos(c + d*x^n))^p*(e*x)^m,x)
[Out] int((a + b/cos(c + d*x^n))^p*(e*x)^m, x)
```

3.72 $\int (ex)^{-1+n} (a + b\sec (c + dx^n))\, dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int (ex)^{-1+n} (a + b\sec (c + dx^n))\, dx = \frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \text{arctanh}(\sin (c + dx^n))}{den}$$

```
[Out] a*(e*x)^n/e/n+b*(e*x)^n*arctanh(sin(c+d*x^n))/d/e/n/(x^n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {14, 4293, 4289, 3855}

$$\int (ex)^{-1+n} (a + b\sec (c + dx^n))\, dx = \frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \text{arctanh}(\sin (c + dx^n))}{den}$$

```
[In] Int[(e*x)^(-1 + n)*(a + b*Sec[c + d*x^n]),x]
```

```
[Out] (a*(e*x)^n)/(e*n) + (b*(e*x)^n*ArcTanh[Sin[c + d*x^n]])/(d*e*n*x^n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4293

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a(ex)^{-1+n} + b(ex)^{-1+n}\sec\left(c+dx^n\right)\right)\,dx \\
&= \frac{a(ex)^n}{en} + b\int (ex)^{-1+n}\sec\left(c+dx^n\right)\,dx \\
&= \frac{a(ex)^n}{en} + \frac{\left(bx^{-n}(ex)^n\right)\int x^{-1+n}\sec\left(c+dx^n\right)\,dx}{e} \\
&= \frac{a(ex)^n}{en} + \frac{\left(bx^{-n}(ex)^n\right)\text{Subst}\left(\int \sec(c+dx)\,dx, x, x^n\right)}{en} \\
&= \frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n\text{arctanh}(\sin\left(c+dx^n\right))}{den}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int (ex)^{-1+n}\left(a+b\sec\left(c+dx^n\right)\right)\,dx = \frac{x^{-n}(ex)^n\left(adx^n + b\text{arctanh}(\sin\left(c+dx^n\right))\right)}{den}$$

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Sec[c + d*x^n]),x]
[Out] ((e*x)^n*(a*d*x^n + b*ArcTanh[Sin[c + d*x^n]]))/(d*e*n*x^n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.98 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.61

method	result
risch	$\frac{ax\,\mathrm{e}^{\frac{(-1+n)\left(-i\pi\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)+i\pi\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2+i\pi\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2-i\pi\,\mathrm{csgn}(iex)^3+2\ln(x)+2\ln(e)\right)}{2}}}{n} - \frac{2i\arctan\left(\mathrm{e}^{i(c\cdots}\right.}{}$

[In] int((e*x)^(-1+n)*(a+b*sec(c+d*x^n)),x,method=_RETURNVERBOSE)

[Out] a/n*x*exp(1/2*(-1+n)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csgn(I*e)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x)+2*ln(e)))-2*I*arctan(exp(I*(c+d*x^n)))/d/e*e^n/n*b*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int (ex)^{-1+n}\left(a+b\sec\left(c+dx^n\right)\right)\,dx = \frac{2\,ade^{n-1}x^n + be^{n-1}\log\left(\sin\left(dx^n+c\right)+1\right) - be^{n-1}\log\left(-\sin\left(dx^n+c\right)+1\right)}{2\,dn}$$

[In] integrate((e*x)^(-1+n)*(a+b*sec(c+d*x^n)),x, algorithm="fricas")

[Out] 1/2*(2*a*d*e^(n - 1)*x^n + b*e^(n - 1)*log(sin(d*x^n + c) + 1) - b*e^(n - 1)*log(-sin(d*x^n + c) + 1))/(d*n)

Sympy [F]

$$\int (ex)^{-1+n}\left(a+b\sec\left(c+dx^n\right)\right)\,dx = \int (ex)^{n-1}\left(a+b\sec\left(c+dx^n\right)\right)\,dx$$

[In] integrate((e*x)**(-1+n)*(a+b*sec(c+d*x**n)),x)

[Out] Integral((e*x)**(n - 1)*(a + b*sec(c + d*x**n)), x)

Maxima [F]

$$\int (ex)^{-1+n}(a+b\sec(c+dx^n))\,dx = \int (b\sec(dx^n+c)+a)(ex)^{n-1}\,dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sec(c+d*x^n)),x, algorithm="maxima")
```

```
[Out] 2*b*e^n*integrate((x^n*cos(2*d*x^n + 2*c)*cos(d*x^n + c) + x^n*sin(2*d*x^n
+ 2*c)*sin(d*x^n + c) + x^n*cos(d*x^n + c))/(e*x*cos(2*d*x^n + 2*c)^2 + e*x
*sin(2*d*x^n + 2*c)^2 + 2*e*x*cos(2*d*x^n + 2*c) + e*x), x) + (e*x)^n*a/(e*
n)
```

Giac [F]

$$\int (ex)^{-1+n}(a+b\sec(c+dx^n))\,dx = \int (b\sec(dx^n+c)+a)(ex)^{n-1}\,dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sec(c+d*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x^n + c) + a)*(e*x)^(n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 14.72 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.36

$$\int (ex)^{-1+n}(a+b\sec(c+dx^n))\,dx$$
$$= \frac{(e\,x)^n\left(b\,\ln\left(-b\,(e\,x)^{n-1}\,2\mathrm{i}-2\,b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\,(e\,x)^{n-1}\right)-b\,\ln\left(b\,(e\,x)^{n-1}\,2\mathrm{i}-2\,b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\,(e\,x)^{n-1}\right)+a\,d\,x^n\right)}{d\,e\,n\,x^n}$$

```
[In] int((a + b/cos(c + d*x^n))*(e*x)^(n - 1),x)
```

```
[Out] ((e*x)^n*(b*log(- b*(e*x)^(n - 1)*2i - 2*b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n
 - 1)) - b*log(b*(e*x)^(n - 1)*2i - 2*b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n -
1)) + a*d*x^n))/(d*e*n*x^n)
```

3.73 $\int (ex)^{-1+2n} (a+b\sec(c+dx^n))\, dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int (ex)^{-1+2n}(a+b\sec(c+dx^n))\,dx = \frac{a(ex)^{2n}}{2en} - \frac{2ibx^{-n}(ex)^{2n}\arctan\left(e^{i(c+dx^n)}\right)}{den} + \frac{ibx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-ie^{i(c+dx^n)}\right)}{d^2en} - \frac{ibx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,ie^{i(c+dx^n)}\right)}{d^2en}$$

```
[Out] 1/2*a*(e*x)^(2*n)/e/n-2*I*b*(e*x)^(2*n)*arctan(exp(I*(c+d*x^n)))/d/e/n/(x^n
)+I*b*(e*x)^(2*n)*polylog(2,-I*exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-I*b*(e*x
)^(2*n)*polylog(2,I*exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {14, 4293, 4289, 4266, 2317, 2438}

$$\int (ex)^{-1+2n}(a+b\sec(c+dx^n))\,dx = \frac{a(ex)^{2n}}{2en} - \frac{2ibx^{-n}(ex)^{2n}\arctan\left(e^{i(c+dx^n)}\right)}{den} + \frac{ibx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-ie^{i(dx^n+c)}\right)}{d^2en} - \frac{ibx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,ie^{i(dx^n+c)}\right)}{d^2en}$$

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Sec[c + d*x^n]),x]
```

```
[Out] (a*(e*x)^(2*n))/(2*e*n) - ((2*I)*b*(e*x)^(2*n)*ArcTan[E^(I*(c + d*x^n))])/(
d*e*n*x^n) + (I*b*(e*x)^(2*n)*PolyLog[2, (-I)*E^(I*(c + d*x^n))])/(d^2*e*n*
```

```
x^(2*n)) - (I*b*(e*x)^(2*n)*PolyLog[2, I*E^(I*(c + d*x^n))])/(d^2*e*n*x^(2*
n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4293

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a(ex)^{-1+2n} + b(ex)^{-1+2n}\sec\left(c+dx^n\right)\right)\,dx\\ &= \frac{a(ex)^{2n}}{2en} + b\int (ex)^{-1+2n}\sec\left(c+dx^n\right)\,dx\end{aligned}$$

$$
\begin{aligned}
&= \frac{a(ex)^{2n}}{2en} + \frac{\left(bx^{-2n}(ex)^{2n}\right) \int x^{-1+2n} \sec\left(c+dx^n\right)\, dx}{e} \\
&= \frac{a(ex)^{2n}}{2en} + \frac{\left(bx^{-2n}(ex)^{2n}\right) \operatorname{Subst}\left(\int x \sec(c+dx)\, dx, x, x^n\right)}{en} \\
&= \frac{a(ex)^{2n}}{2en} - \frac{2ibx^{-n}(ex)^{2n} \arctan\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad - \frac{\left(bx^{-2n}(ex)^{2n}\right) \operatorname{Subst}\left(\int \log\left(1-ie^{i(c+dx)}\right)\, dx, x, x^n\right)}{den} \\
&\quad + \frac{\left(bx^{-2n}(ex)^{2n}\right) \operatorname{Subst}\left(\int \log\left(1+ie^{i(c+dx)}\right)\, dx, x, x^n\right)}{den} \\
&= \frac{a(ex)^{2n}}{2en} - \frac{2ibx^{-n}(ex)^{2n} \arctan\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad + \frac{\left(ibx^{-2n}(ex)^{2n}\right) \operatorname{Subst}\left(\int \frac{\log(1-ix)}{x}\, dx, x, e^{i(c+dx^n)}\right)}{d^2en} \\
&\quad - \frac{\left(ibx^{-2n}(ex)^{2n}\right) \operatorname{Subst}\left(\int \frac{\log(1+ix)}{x}\, dx, x, e^{i(c+dx^n)}\right)}{d^2en} \\
&= \frac{a(ex)^{2n}}{2en} - \frac{2ibx^{-n}(ex)^{2n} \arctan\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad + \frac{ibx^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, -ie^{i(c+dx^n)}\right)}{d^2en} - \frac{ibx^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, ie^{i(c+dx^n)}\right)}{d^2en}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.26

$$
\int (ex)^{-1+2n} \left(a + b\sec\left(c+dx^n\right)\right)\, dx
$$

$$
= \frac{(ex)^{2n} \cos\left(c+dx^n\right) \left(a + \frac{bx^{-2n}\left(\left(-2c+\pi-2dx^n\right)\left(\log\left(1-ie^{-i(c+dx^n)}\right)-\log\left(1+ie^{-i(c+dx^n)}\right)\right)-(-2c+\pi)\log\left(\cot\left(\frac{1}{4}(2c+\pi+2dx^n)\right)\right)}{d^2}\right.}{2en\left(b+a\cos\left(c+dx^n\right)\right)}
$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sec[c + d*x^n]),x]
```

```
[Out] ((e*x)^(2*n)*Cos[c + d*x^n]*(a + (b*((-2*c + Pi - 2*d*x^n)*(Log[1 - I/E^(I*
(c + d*x^n))] - Log[1 + I/E^(I*(c + d*x^n))]) - (-2*c + Pi)*Log[Cot[(2*c +
Pi + 2*d*x^n)/4]] + (2*I)*(PolyLog[2, (-I)/E^(I*(c + d*x^n))] - PolyLog[2,
I/E^(I*(c + d*x^n))])))/(d^2*x^(2*n)))*(a + b*Sec[c + d*x^n]))/(2*e*n*(b +
a*Cos[c + d*x^n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.93 (sec) , antiderivative size = 829, normalized size of antiderivative = 5.56

method	result	size
risch	Expression too large to display	829

```
[In] int((e*x)^(2*n-1)*(a+b*sec(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] I/e/n/d*b*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(e^n)^2*ln(1+exp(I*x^n
*d)*(-exp(2*I*c))^(1/2))*(-exp(2*I*c))^(1/2)*x^n*exp(-1/2*I*(2*Pi*n*csgn(I*
e*x)^3-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n
*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn(I*e*x)^3+Pi*csgn(I*e)*csgn(I*e*x)^
2+Pi*csgn(I*x)*csgn(I*e*x)^2+2*c))-I/e/n/d*b*(-1)^(1/2*csgn(I*e)*csgn(I*x)*
csgn(I*e*x))*(e^n)^2*ln(1-exp(I*x^n*d)*(-exp(2*I*c))^(1/2))*(-exp(2*I*c))^(
1/2)*x^n*exp(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*
Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn
(I*e*x)^3+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^2+2*c))+1/e/n
/d^2*b*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(e^n)^2*(-exp(2*I*c))^(1/
2)*dilog(1+exp(I*x^n*d)*(-exp(2*I*c))^(1/2))*exp(-1/2*I*(2*Pi*n*csgn(I*e*x)
^3-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csg
n(I*e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn(I*e*x)^3+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi
*csgn(I*x)*csgn(I*e*x)^2+2*c))-1/e/n/d^2*b*(-1)^(1/2*csgn(I*e)*csgn(I*x)*cs
gn(I*e*x))*(e^n)^2*(-exp(2*I*c))^(1/2)*dilog(1-exp(I*x^n*d)*(-exp(2*I*c))^(
1/2))*exp(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*
n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn(I*
e*x)^3+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^2+2*c))+1/2*a/n*
x*exp(1/2*(2*n-1)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csgn(I*e)*csg
n(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x)+2*ln(e))
)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(133) = 266.

Time = 0.30 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.15

$$\int (ex)^{-1+2n}\left(a+b\sec\left(c+dx^n\right)\right)\,dx$$
$$=\frac{ad^2e^{2n-1}x^{2n}-bce^{2n-1}\log\left(\cos\left(dx^n+c\right)+i\,\sin\left(dx^n+c\right)+i\right)+bce^{2n-1}\log\left(\cos\left(dx^n+c\right)-i\,\sin\left(dx^n+\right.\right.}{}$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*e^(2*n - 1)*x^(2*n) - b*c*e^(2*n - 1)*log(cos(d*x^n + c) + I*sin
(d*x^n + c) + I) + b*c*e^(2*n - 1)*log(cos(d*x^n + c) - I*sin(d*x^n + c) +
I) - b*c*e^(2*n - 1)*log(-cos(d*x^n + c) + I*sin(d*x^n + c) + I) + b*c*e^(2
*n - 1)*log(-cos(d*x^n + c) - I*sin(d*x^n + c) + I) - I*b*e^(2*n - 1)*dilog
(I*cos(d*x^n + c) + sin(d*x^n + c)) - I*b*e^(2*n - 1)*dilog(I*cos(d*x^n + c
) - sin(d*x^n + c)) + I*b*e^(2*n - 1)*dilog(-I*cos(d*x^n + c) + sin(d*x^n +
 c)) + I*b*e^(2*n - 1)*dilog(-I*cos(d*x^n + c) - sin(d*x^n + c)) + (b*d*e^(
2*n - 1)*x^n + b*c*e^(2*n - 1))*log(I*cos(d*x^n + c) + sin(d*x^n + c) + 1)
- (b*d*e^(2*n - 1)*x^n + b*c*e^(2*n - 1))*log(I*cos(d*x^n + c) - sin(d*x^n
+ c) + 1) + (b*d*e^(2*n - 1)*x^n + b*c*e^(2*n - 1))*log(-I*cos(d*x^n + c) +
 sin(d*x^n + c) + 1) - (b*d*e^(2*n - 1)*x^n + b*c*e^(2*n - 1))*log(-I*cos(d
*x^n + c) - sin(d*x^n + c) + 1))/(d^2*n)
```

Sympy [F]

$$\int (ex)^{-1+2n}(a+b\sec(c+dx^n))\,dx=\int (ex)^{2n-1}(a+b\sec(c+dx^n))\,dx$$

```
[In] integrate((e*x)**(-1+2*n)*(a+b*sec(c+d*x**n)),x)
[Out] Integral((e*x)**(2*n - 1)*(a + b*sec(c + d*x**n)), x)
```

Maxima [F]

$$\int (ex)^{-1+2n}(a+b\sec(c+dx^n))\,dx=\int (b\sec(dx^n+c)+a)(ex)^{2n-1}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n)),x, algorithm="maxima")
[Out] 2*b*e^(2*n)*integrate((x^(2*n)*cos(2*d*x^n + 2*c)*cos(d*x^n + c) + x^(2*n)*
sin(2*d*x^n + 2*c)*sin(d*x^n + c) + x^(2*n)*cos(d*x^n + c))/(e*x*cos(2*d*x^
n + 2*c)^2 + e*x*sin(2*d*x^n + 2*c)^2 + 2*e*x*cos(2*d*x^n + 2*c) + e*x), x)
 + 1/2*(e*x)^(2*n)*a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+2n}(a+b\sec(c+dx^n))\,dx=\int (b\sec(dx^n+c)+a)(ex)^{2n-1}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n)),x, algorithm="giac")
[Out] integrate((b*sec(d*x^n + c) + a)*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}\left(a+b\sec\left(c+dx^n\right)\right)\,dx=\int\left(a+\frac{b}{\cos\left(c+d\,x^n\right)}\right)\left(e\,x\right)^{2\,n-1}\,dx$$

```
[In] int((a + b/cos(c + d*x^n))*(e*x)^(2*n - 1),x)
[Out] int((a + b/cos(c + d*x^n))*(e*x)^(2*n - 1), x)
```

3.74 $\int (ex)^{-1+3n} (a+b\sec(c+dx^n))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 235

$$\begin{aligned}\int (ex)^{-1+3n}(a+b\sec(c+dx^n))\,dx =& \frac{a(ex)^{3n}}{3en}-\frac{2ibx^{-n}(ex)^{3n}\arctan\left(e^{i(c+dx^n)}\right)}{den}\\ &+\frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-ie^{i(c+dx^n)}\right)}{d^2en}\\ &-\frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,ie^{i(c+dx^n)}\right)}{d^2en}\\ &-\frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-ie^{i(c+dx^n)}\right)}{d^3en}\\ &+\frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,ie^{i(c+dx^n)}\right)}{d^3en}\end{aligned}$$

```
[Out] 1/3*a*(e*x)^(3*n)/e/n-2*I*b*(e*x)^(3*n)*arctan(exp(I*(c+d*x^n)))/d/e/n/(x^n
)+2*I*b*(e*x)^(3*n)*polylog(2,-I*exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-2*I*b*
(e*x)^(3*n)*polylog(2,I*exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-2*b*(e*x)^(3*n)
*polylog(3,-I*exp(I*(c+d*x^n)))/d^3/e/n/(x^(3*n))+2*b*(e*x)^(3*n)*polylog(3
,I*exp(I*(c+d*x^n)))/d^3/e/n/(x^(3*n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used

$= \{14, 4293, 4289, 4266, 2611, 2320, 6724\}$

$$\int (ex)^{-1+3n} \left(a + b\sec\left(c + dx^n\right)\right) dx = \frac{a(ex)^{3n}}{3en} - \frac{2ibx^{-n}(ex)^{3n}\arctan\left(e^{i(c+dx^n)}\right)}{den} - \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3, -ie^{i(dx^n+c)}\right)}{d^3en} + \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3, ie^{i(dx^n+c)}\right)}{d^3en} + \frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2, -ie^{i(dx^n+c)}\right)}{d^2en} - \frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2, ie^{i(dx^n+c)}\right)}{d^2en}$$

```
[In] Int[(e*x)^(-1 + 3*n)*(a + b*Sec[c + d*x^n]),x]
```

```
[Out] (a*(e*x)^(3*n))/(3*e*n) - ((2*I)*b*(e*x)^(3*n)*ArcTan[E^(I*(c + d*x^n))])/(
d*e*n*x^n) + ((2*I)*b*(e*x)^(3*n)*PolyLog[2, (-I)*E^(I*(c + d*x^n))])/(d^2*
e*n*x^(2*n)) - ((2*I)*b*(e*x)^(3*n)*PolyLog[2, I*E^(I*(c + d*x^n))])/(d^2*e
*n*x^(2*n)) - (2*b*(e*x)^(3*n)*PolyLog[3, (-I)*E^(I*(c + d*x^n))])/(d^3*e*n
*x^(3*n)) + (2*b*(e*x)^(3*n)*PolyLog[3, I*E^(I*(c + d*x^n))])/(d^3*e*n*x^(3
*n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4293

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a(ex)^{-1+3n}+b(ex)^{-1+3n}\sec\left(c+dx^n\right)\right)\,dx\\
&= \frac{a(ex)^{3n}}{3en}+b\int (ex)^{-1+3n}\sec\left(c+dx^n\right)\,dx\\
&= \frac{a(ex)^{3n}}{3en}+\frac{\left(bx^{-3n}(ex)^{3n}\right)\int x^{-1+3n}\sec\left(c+dx^n\right)\,dx}{e}\\
&= \frac{a(ex)^{3n}}{3en}+\frac{\left(bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x^2\sec(c+dx)\,dx,x,x^n\right)}{en}\\
&= \frac{a(ex)^{3n}}{3en}-\frac{2ibx^{-n}(ex)^{3n}\arctan\left(e^{i(c+dx^n)}\right)}{den}\\
&\quad -\frac{\left(2bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1-ie^{i(c+dx)}\right)\,dx,x,x^n\right)}{den}\\
&\quad +\frac{\left(2bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1+ie^{i(c+dx)}\right)\,dx,x,x^n\right)}{den}
\end{aligned}
$$

$$= \frac{a(ex)^{3n}}{3en} - \frac{2ibx^{-n}(ex)^{3n}\arctan\left(e^{i(c+dx^n)}\right)}{den} + \frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-ie^{i(c+dx^n)}\right)}{d^2en} - \frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,ie^{i(c+dx^n)}\right)}{d^2en} - \frac{(2ibx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)\,dx,x,x^n\right)}{d^2en} + \frac{(2ibx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)\,dx,x,x^n\right)}{d^2en}$$

$$= \frac{a(ex)^{3n}}{3en} - \frac{2ibx^{-n}(ex)^{3n}\arctan\left(e^{i(c+dx^n)}\right)}{den} + \frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-ie^{i(c+dx^n)}\right)}{d^2en} - \frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,ie^{i(c+dx^n)}\right)}{d^2en} - \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{d^3en} + \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,ix)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{d^3en}$$

$$= \frac{a(ex)^{3n}}{3en} - \frac{2ibx^{-n}(ex)^{3n}\arctan\left(e^{i(c+dx^n)}\right)}{den} + \frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-ie^{i(c+dx^n)}\right)}{d^2en} - \frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,ie^{i(c+dx^n)}\right)}{d^2en} - \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-ie^{i(c+dx^n)}\right)}{d^3en} + \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,ie^{i(c+dx^n)}\right)}{d^3en}$$

Mathematica [F]

$$\int (ex)^{-1+3n}\left(a+b\sec\left(c+dx^n\right)\right)\,dx = \int (ex)^{-1+3n}\left(a+b\sec\left(c+dx^n\right)\right)\,dx$$

```
[In] Integrate[(e*x)^(-1 + 3*n)*(a + b*Sec[c + d*x^n]),x]
[Out] Integrate[(e*x)^(-1 + 3*n)*(a + b*Sec[c + d*x^n]), x]
```

Maple [F]

$$\int (ex)^{3n-1} (a + b\sec(c + d\,x^n))\,dx$$

```
[In] int((e*x)^(3*n-1)*(a+b*sec(c+d*x^n)),x)
```

```
[Out] int((e*x)^(3*n-1)*(a+b*sec(c+d*x^n)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 655 vs. 2(215) = 430.

Time = 0.34 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.79

$$\int (ex)^{-1+3n}(a + b\sec(c + dx^n))\,dx$$
$$= \frac{2\,ad^3e^{3\,n-1}x^{3\,n} - 6i\,bde^{3\,n-1}x^n\mathrm{Li}_2(i\,\cos(dx^n+c)+\sin(dx^n+c)) - 6i\,bde^{3\,n-1}x^n\mathrm{Li}_2(i\,\cos(dx^n+c)-\sin}{}$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*e^(3*n - 1)*x^(3*n) - 6*I*b*d*e^(3*n - 1)*x^n*dilog(I*cos(d*x^
n + c) + sin(d*x^n + c)) - 6*I*b*d*e^(3*n - 1)*x^n*dilog(I*cos(d*x^n + c) -
 sin(d*x^n + c)) + 6*I*b*d*e^(3*n - 1)*x^n*dilog(-I*cos(d*x^n + c) + sin(d*
x^n + c)) + 6*I*b*d*e^(3*n - 1)*x^n*dilog(-I*cos(d*x^n + c) - sin(d*x^n + c
)) + 3*b*c^2*e^(3*n - 1)*log(cos(d*x^n + c) + I*sin(d*x^n + c) + I) - 3*b*c
^2*e^(3*n - 1)*log(cos(d*x^n + c) - I*sin(d*x^n + c) + I) + 3*b*c^2*e^(3*n
- 1)*log(-cos(d*x^n + c) + I*sin(d*x^n + c) + I) - 3*b*c^2*e^(3*n - 1)*log(
-cos(d*x^n + c) - I*sin(d*x^n + c) + I) - 6*b*e^(3*n - 1)*polylog(3, I*cos(
d*x^n + c) + sin(d*x^n + c)) + 6*b*e^(3*n - 1)*polylog(3, I*cos(d*x^n + c)
- sin(d*x^n + c)) - 6*b*e^(3*n - 1)*polylog(3, -I*cos(d*x^n + c) + sin(d*x^
n + c)) + 6*b*e^(3*n - 1)*polylog(3, -I*cos(d*x^n + c) - sin(d*x^n + c)) +
3*(b*d^2*e^(3*n - 1)*x^(2*n) - b*c^2*e^(3*n - 1))*log(I*cos(d*x^n + c) + si
n(d*x^n + c) + 1) - 3*(b*d^2*e^(3*n - 1)*x^(2*n) - b*c^2*e^(3*n - 1))*log(I
*cos(d*x^n + c) - sin(d*x^n + c) + 1) + 3*(b*d^2*e^(3*n - 1)*x^(2*n) - b*c^
2*e^(3*n - 1))*log(-I*cos(d*x^n + c) + sin(d*x^n + c) + 1) - 3*(b*d^2*e^(3*
n - 1)*x^(2*n) - b*c^2*e^(3*n - 1))*log(-I*cos(d*x^n + c) - sin(d*x^n + c)
+ 1))/(d^3*n)
```

Sympy [F]

$$\int (ex)^{-1+3n}(a+b\sec(c+dx^n))\,dx=\int (ex)^{3n-1}(a+b\sec(c+dx^n))\,dx$$

```
[In] integrate((e*x)**(-1+3*n)*(a+b*sec(c+d*x**n)),x)
[Out] Integral((e*x)**(3*n - 1)*(a + b*sec(c + d*x**n)), x)
```

Maxima [F]

$$\int (ex)^{-1+3n}(a+b\sec(c+dx^n))\,dx=\int (b\sec(dx^n+c)+a)(ex)^{3n-1}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n)),x, algorithm="maxima")
[Out] 2*b*e^(3*n)*integrate((x^(3*n)*cos(2*d*x^n + 2*c)*cos(d*x^n + c) + x^(3*n)*
sin(2*d*x^n + 2*c)*sin(d*x^n + c) + x^(3*n)*cos(d*x^n + c))/(e*x*cos(2*d*x^
n + 2*c)^2 + e*x*sin(2*d*x^n + 2*c)^2 + 2*e*x*cos(2*d*x^n + 2*c) + e*x), x)
 + 1/3*(e*x)^(3*n)*a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+3n}(a+b\sec(c+dx^n))\,dx=\int (b\sec(dx^n+c)+a)(ex)^{3n-1}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n)),x, algorithm="giac")
[Out] integrate((b*sec(d*x^n + c) + a)*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}(a+b\sec(c+dx^n))\,dx=\int \left(a+\frac{b}{\cos(c+d\,x^n)}\right)(e\,x)^{3\,n-1}\,dx$$

```
[In] int((a + b/cos(c + d*x^n))*(e*x)^(3*n - 1),x)
[Out] int((a + b/cos(c + d*x^n))*(e*x)^(3*n - 1), x)
```

3.75 $\int (ex)^{-1+n} (a+b\sec(c+dx^n))^2\, dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int (ex)^{-1+n} (a+b\sec(c+dx^n))^2\, dx = \frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \operatorname{arctanh}(\sin(c+dx^n))}{den} + \frac{b^2x^{-n}(ex)^n \tan(c+dx^n)}{den}$$

```
[Out] a^2*(e*x)^n/e/n+2*a*b*(e*x)^n*arctanh(sin(c+d*x^n))/d/e/n/(x^n)+b^2*(e*x)^n
*tan(c+d*x^n)/d/e/n/(x^n)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4293, 4289, 3858, 3855, 3852, 8}

$$\int (ex)^{-1+n} (a+b\sec(c+dx^n))^2\, dx = \frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \operatorname{arctanh}(\sin(c+dx^n))}{den} + \frac{b^2x^{-n}(ex)^n \tan(c+dx^n)}{den}$$

```
[In] Int[(e*x)^(-1 + n)*(a + b*Sec[c + d*x^n])^2,x]
```

```
[Out] (a^2*(e*x)^n)/(e*n) + (2*a*b*(e*x)^n*ArcTanh[Sin[c + d*x^n]])/(d*e*n*x^n) +
 (b^2*(e*x)^n*Tan[c + d*x^n])/(d*e*n*x^n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4293

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(x^{-n}(ex)^n)\int x^{-1+n}(a+b\sec(c+dx^n))^2\,dx}{e}\\
&= \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int(a+b\sec(c+dx))^2\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^n}{en}+\frac{(2abx^{-n}(ex)^n)\,\text{Subst}\left(\int\sec(c+dx)\,dx,x,x^n\right)}{en}\\
&\quad+\frac{(b^2x^{-n}(ex)^n)\,\text{Subst}\left(\int\sec^2(c+dx)\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^n}{en}+\frac{2abx^{-n}(ex)^n\text{arctanh}(\sin(c+dx^n))}{den}\\
&\quad-\frac{(b^2x^{-n}(ex)^n)\,\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx^n)\right)}{den}\\
&= \frac{a^2(ex)^n}{en}+\frac{2abx^{-n}(ex)^n\text{arctanh}(\sin(c+dx^n))}{den}+\frac{b^2x^{-n}(ex)^n\tan(c+dx^n)}{den}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int (ex)^{-1+n} (a + b \sec (c + dx^n))^2 \, dx = \frac{x^{-n}(ex)^n (a^2 dx^n + 2ab\text{arctanh}(\sin (c + dx^n)) + b^2 \tan (c + dx^n))}{den}$$

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Sec[c + d*x^n])^2,x]
[Out] ((e*x)^n*(a^2*d*x^n + 2*a*b*ArcTanh[Sin[c + d*x^n]] + b^2*Tan[c + d*x^n]))/
(d*e*n*x^n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.02 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.49

method	result
risch	$\frac{a^2 x\, \mathrm{e}^{\frac{(-1+n)\left(-i\pi\, \mathrm{csgn}(ie)\, \mathrm{csgn}(ix)\, \mathrm{csgn}(iex)+i\pi\, \mathrm{csgn}(ie)\, \mathrm{csgn}(iex)^2+i\pi\, \mathrm{csgn}(ix)\, \mathrm{csgn}(iex)^2-i\pi\, \mathrm{csgn}(iex)^3+2\ln(x)+2\ln(e)\right)}{2}}}{n} + 2ix\, \mathrm{e}^{\frac{(-1+n)(}{}}$

```
[In] int((e*x)^(-1+n)*(a+b*sec(c+d*x^n))^2,x,method=_RETURNVERBOSE)
[Out] a^2/n*x*exp(1/2*(-1+n)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csgn(I*e
)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x)+2*l
n(e)))+2*I*x*exp(1/2*(-1+n)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csg
n(I*e)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x
)+2*ln(e)))*b^2/d/n/(x^n)/(1+exp(2*I*(c+d*x^n)))-4*I*arctan(exp(I*(c+d*x^n)
))/d/e*e^n/n*a*b*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(csgn(I*e*x)-csgn(I*x))*(-
csgn(I*e*x)+csgn(I*e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.43

$$\int (ex)^{-1+n} (a + b \sec (c + dx^n))^2 \, dx = \frac{a^2 de^{n-1} x^n \cos (dx^n + c) + abe^{n-1} \cos (dx^n + c) \log (\sin (dx^n + c) + 1) - abe^{n-1} \cos (dx^n + c) \log (-\sin (d}{dn \cos (dx^n + c)}$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sec(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*e^(n - 1)*x^n*cos(d*x^n + c) + a*b*e^(n - 1)*cos(d*x^n + c)*log(sin(
d*x^n + c) + 1) - a*b*e^(n - 1)*cos(d*x^n + c)*log(-sin(d*x^n + c) + 1) + b
^2*e^(n - 1)*sin(d*x^n + c))/(d*n*cos(d*x^n + c))
```

Sympy [F]

$$\int (ex)^{-1+n}\left(a+b\sec\left(c+dx^n\right)\right)^2\,dx=\int (ex)^{n-1}\left(a+b\sec\left(c+dx^n\right)\right)^2\,dx$$

```
[In] integrate((e*x)**(-1+n)*(a+b*sec(c+d*x**n))**2,x)
[Out] Integral((e*x)**(n - 1)*(a + b*sec(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+n}\left(a+b\sec\left(c+dx^n\right)\right)^2\,dx=\int \left(b\sec\left(dx^n+c\right)+a\right)^2(ex)^{n-1}\,dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")
[Out] (e*x)^n*a^2/(e*n) + 2*(b^2*e^n*sin(2*d*x^n + 2*c) + 2*(a*b*d*e^(n + 1)*n*co
s(2*d*x^n + 2*c)^2 + a*b*d*e^(n + 1)*n*sin(2*d*x^n + 2*c)^2 + 2*a*b*d*e^(n
+ 1)*n*cos(2*d*x^n + 2*c) + a*b*d*e^(n + 1)*n)*integrate((x^n*cos(2*d*x^n +
 2*c)*cos(d*x^n + c) + x^n*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + x^n*cos(d*x^
n + c))/(e*x*cos(2*d*x^n + 2*c)^2 + e*x*sin(2*d*x^n + 2*c)^2 + 2*e*x*cos(2*
d*x^n + 2*c) + e*x), x))/(d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n +
2*c)^2 + 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n)
```

Giac [F]

$$\int (ex)^{-1+n}\left(a+b\sec\left(c+dx^n\right)\right)^2\,dx=\int \left(b\sec\left(dx^n+c\right)+a\right)^2(ex)^{n-1}\,dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sec(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((b*sec(d*x^n + c) + a)^2*(e*x)^(n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 15.51 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.28

$$\int (ex)^{-1+n} (a + b \sec (c + dx^n))^2 \, dx$$
$$= \frac{a^2 x (e x)^{n-1}}{n} + \frac{b^2 x (e x)^{n-1} 2i}{d n x^n \left(e^{c 2i + d x^n 2i} + 1\right)}$$
$$+ \frac{2 a b x \ln \left(-a b (e x)^{n-1} 4i - 4 a b e^{c 1i} e^{d x^n 1i} (e x)^{n-1}\right) (e x)^{n-1}}{d n x^n}$$
$$- \frac{2 a b x \ln \left(a b (e x)^{n-1} 4i - 4 a b e^{c 1i} e^{d x^n 1i} (e x)^{n-1}\right) (e x)^{n-1}}{d n x^n}$$

```
[In] int((a + b/cos(c + d*x^n))^2*(e*x)^(n - 1),x)
```

```
[Out] (a^2*x*(e*x)^(n - 1))/n + (b^2*x*(e*x)^(n - 1)*2i)/(d*n*x^n*(exp(c*2i + d*x
^n*2i) + 1)) + (2*a*b*x*log(- a*b*(e*x)^(n - 1)*4i - 4*a*b*exp(c*1i)*exp(d*
x^n*1i)*(e*x)^(n - 1))*(e*x)^(n - 1))/(d*n*x^n) - (2*a*b*x*log(a*b*(e*x)^(n
 - 1)*4i - 4*a*b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1))*(e*x)^(n - 1))/(d*n
*x^n)
```

3.76 $\int (ex)^{-1+2n} (a+b\sec(c+dx^n))^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 221

$$\int (ex)^{-1+2n}(a+b\sec(c+dx^n))^2\,dx = \frac{a^2(ex)^{2n}}{2en} - \frac{4iabx^{-n}(ex)^{2n}\arctan\left(e^{i(c+dx^n)}\right)}{den} + \frac{b^2x^{-2n}(ex)^{2n}\log(\cos(c+dx^n))}{d^2en} + \frac{2iabx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-ie^{i(c+dx^n)}\right)}{d^2en} - \frac{2iabx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,ie^{i(c+dx^n)}\right)}{d^2en} + \frac{b^2x^{-n}(ex)^{2n}\tan(c+dx^n)}{den}$$

```
[Out] 1/2*a^2*(e*x)^(2*n)/e/n-4*I*a*b*(e*x)^(2*n)*arctan(exp(I*(c+d*x^n)))/d/e/n/
(x^n)+b^2*(e*x)^(2*n)*ln(cos(c+d*x^n))/d^2/e/n/(x^(2*n))+2*I*a*b*(e*x)^(2*n
)*polylog(2,-I*exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-2*I*a*b*(e*x)^(2*n)*poly
log(2,I*exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))+b^2*(e*x)^(2*n)*tan(c+d*x^n)/d/
e/n/(x^n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

= {4293, 4289, 4275, 4266, 2317, 2438, 4269, 3556}

$$\int (ex)^{-1+2n} (a + b \sec (c + dx^n))^2 \, dx = \frac{a^2 (ex)^{2n}}{2en} - \frac{4iabx^{-n}(ex)^{2n} \arctan \left(e^{i(c+dx^n)}\right)}{den} + \frac{2iabx^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, -ie^{i(dx^n+c)}\right)}{d^2 en} - \frac{2iabx^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, ie^{i(dx^n+c)}\right)}{d^2 en} + \frac{b^2 x^{-2n}(ex)^{2n} \log (\cos (c + dx^n))}{d^2 en} + \frac{b^2 x^{-n}(ex)^{2n} \tan (c + dx^n)}{den}$$

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Sec[c + d*x^n])^2,x]
```

```
[Out] (a^2*(e*x)^(2*n))/(2*e*n) - ((4*I)*a*b*(e*x)^(2*n)*ArcTan[E^(I*(c + d*x^n))
])/(d*e*n*x^n) + (b^2*(e*x)^(2*n)*Log[Cos[c + d*x^n]])/(d^2*e*n*x^(2*n)) +
((2*I)*a*b*(e*x)^(2*n)*PolyLog[2, (-I)*E^(I*(c + d*x^n))])/(d^2*e*n*x^(2*n)
) - ((2*I)*a*b*(e*x)^(2*n)*PolyLog[2, I*E^(I*(c + d*x^n))])/(d^2*e*n*x^(2*n
)) + (b^2*(e*x)^(2*n)*Tan[c + d*x^n])/(d*e*n*x^n)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4293

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n}(a+b\sec(c+dx^n))^2 \, dx}{e} \\
&= \frac{(x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int x(a+b\sec(c+dx))^2 \, dx, x, x^n\right)}{en} \\
&= \frac{(x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \left(a^2x+2abx\sec(c+dx)+b^2x\sec^2(c+dx)\right) \, dx, x, x^n\right)}{en} \\
&= \frac{a^2(ex)^{2n}}{2en} + \frac{(2abx^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int x\sec(c+dx) \, dx, x, x^n\right)}{en} \\
&\quad + \frac{(b^2x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int x\sec^2(c+dx) \, dx, x, x^n\right)}{en} \\
&= \frac{a^2(ex)^{2n}}{2en} - \frac{4iabx^{-n}(ex)^{2n}\arctan\left(e^{i(c+dx^n)}\right)}{den} + \frac{b^2x^{-n}(ex)^{2n}\tan(c+dx^n)}{den} \\
&\quad - \frac{(2abx^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \log\left(1-ie^{i(c+dx)}\right) \, dx, x, x^n\right)}{den} \\
&\quad + \frac{(2abx^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \log\left(1+ie^{i(c+dx)}\right) \, dx, x, x^n\right)}{den} \\
&\quad - \frac{(b^2x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \tan(c+dx) \, dx, x, x^n\right)}{den}
\end{aligned}$$

$$= \frac{a^2(ex)^{2n}}{2en} - \frac{4iabx^{-n}(ex)^{2n}\arctan\left(e^{i(c+dx^n)}\right)}{den} + \frac{b^2x^{-2n}(ex)^{2n}\log\left(\cos\left(c+dx^n\right)\right)}{d^2en} + \frac{b^2x^{-n}(ex)^{2n}\tan\left(c+dx^n\right)}{den} + \frac{\left(2iabx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{d^2en} - \frac{\left(2iabx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{d^2en}$$

$$= \frac{a^2(ex)^{2n}}{2en} - \frac{4iabx^{-n}(ex)^{2n}\arctan\left(e^{i(c+dx^n)}\right)}{den} + \frac{b^2x^{-2n}(ex)^{2n}\log\left(\cos\left(c+dx^n\right)\right)}{d^2en} + \frac{2iabx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-ie^{i(c+dx^n)}\right)}{d^2en} - \frac{2iabx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,ie^{i(c+dx^n)}\right)}{d^2en} + \frac{b^2x^{-n}(ex)^{2n}\tan\left(c+dx^n\right)}{den}$$

Mathematica [A] (verified)

Time = 5.78 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.57

$$\int (ex)^{-1+2n}\left(a+b\sec\left(c+dx^n\right)\right)^2\,dx$$

$$= \frac{x^{-2n}(ex)^{2n}\left(8ab\arctan(\cot(c))\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx^n}{2}\right)\right) - \frac{4ab\csc(c)\left((dx^n-\arctan(\cot(c)))\left(\log\left(1-e^{i(dx^n-\cdots}\right.\right.\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sec[c + d*x^n])^2,x]
```

```
[Out] ((e*x)^(2*n)*(8*a*b*ArcTan[Cot[c]]*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x^n)/2]]
- (4*a*b*Csc[c]*((d*x^n - ArcTan[Cot[c]])*(Log[1 - E^(I*(d*x^n - ArcTan[Cot
[c]]))] - Log[1 + E^(I*(d*x^n - ArcTan[Cot[c]]))]) + I*PolyLog[2, -E^(I*(d*
x^n - ArcTan[Cot[c]]))] - I*PolyLog[2, E^(I*(d*x^n - ArcTan[Cot[c]]))]))/Sq
rt[Csc[c]^2] + (2*b^2*d*x^n*Sin[(d*x^n)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c
+ d*x^n)/2] - Sin[(c + d*x^n)/2])) + (2*b^2*d*x^n*Sin[(d*x^n)/2])/((Cos[c/2
] + Sin[c/2])*(Cos[(c + d*x^n)/2] + Sin[(c + d*x^n)/2])) - 2*b^2*d*x^n*Tan[
c] + d*x^n*(a^2*d*x^n + 2*b^2*Tan[c]) + 2*b^2*(Log[Cos[c + d*x^n]] + d*x^n*
Tan[c])))/(2*d^2*e*n*x^(2*n))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.27 (sec) , antiderivative size = 1100, normalized size of antiderivative = 4.98

method	result	size
risch	Expression too large to display	1100

```
[In] int((e*x)^(2*n-1)*(a+b*sec(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2/n*x*exp(1/2*(2*n-1)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csg
n(I*e)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x
)+2*ln(e)))+2*I*x*exp(1/2*(2*n-1)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*
Pi*csgn(I*e)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+
2*ln(x)+2*ln(e)))*b^2/d/n/(x^n)/(1+exp(2*I*(c+d*x^n)))+2*I*b/d/n*(e^n)^2/e*
a*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(-exp(2*I*c))^(1/2)*ln(1+exp(I
*x^n*d)*(-exp(2*I*c))^(1/2))*x^n*exp(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*n*cs
gn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*csgn(
I*x)*csgn(I*e*x)-Pi*csgn(I*e*x)^3+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*c
sgn(I*e*x)^2+2*c))-2*I*b/d/n*(e^n)^2/e*a*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn
(I*e*x))*(-exp(2*I*c))^(1/2)*ln(1-exp(I*x^n*d)*(-exp(2*I*c))^(1/2))*x^n*exp
(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*
x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn(I*e*x)^3+Pi
*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^2+2*c))+2*b/d^2/n*(e^n)^2
/e*a*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(-exp(2*I*c))^(1/2)*dilog(1
+exp(I*x^n*d)*(-exp(2*I*c))^(1/2))*exp(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*n*
csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*csg
n(I*x)*csgn(I*e*x)-Pi*csgn(I*e*x)^3+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)
*csgn(I*e*x)^2+2*c))-2*b/d^2/n*(e^n)^2/e*a*(-1)^(1/2*csgn(I*e)*csgn(I*x)*cs
gn(I*e*x))*(-exp(2*I*c))^(1/2)*dilog(1-exp(I*x^n*d)*(-exp(2*I*c))^(1/2))*ex
p(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I
*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn(I*e*x)^3+P
i*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^2+2*c))+b^2/d^2/n*(e^n)^
2/e*exp(1/2*I*csgn(I*e*x)*Pi*(2*n-1)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+
csgn(I*e)))*ln(1+exp(2*I*(c+d*x^n)))-2*b^2/d^2/n*(e^n)^2/e*exp(1/2*I*csgn(I
*e*x)*Pi*(2*n-1)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))*ln(exp(I
*x^n*d))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(207) = 414.

Time = 0.32 (sec) , antiderivative size = 656, normalized size of antiderivative = 2.97

$$\int (ex)^{-1+2n}(a+b\sec(c+dx^n))^2\,dx$$
$$=\frac{a^2d^2e^{2n-1}x^{2n}\cos(dx^n+c)+2b^2de^{2n-1}x^n\sin(dx^n+c)-2i\,abe^{2n-1}\cos(dx^n+c)\,\mathrm{Li}_2(i\cos(dx^n+c)+\mathrm{s}$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d^2*e^(2*n - 1)*x^(2*n)*cos(d*x^n + c) + 2*b^2*d*e^(2*n - 1)*x^n*s
in(d*x^n + c) - 2*I*a*b*e^(2*n - 1)*cos(d*x^n + c)*dilog(I*cos(d*x^n + c) +
 sin(d*x^n + c)) - 2*I*a*b*e^(2*n - 1)*cos(d*x^n + c)*dilog(I*cos(d*x^n + c
) - sin(d*x^n + c)) + 2*I*a*b*e^(2*n - 1)*cos(d*x^n + c)*dilog(-I*cos(d*x^n
 + c) + sin(d*x^n + c)) + 2*I*a*b*e^(2*n - 1)*cos(d*x^n + c)*dilog(-I*cos(d
*x^n + c) - sin(d*x^n + c)) - (2*a*b*c - b^2)*e^(2*n - 1)*cos(d*x^n + c)*lo
g(cos(d*x^n + c) + I*sin(d*x^n + c) + I) + (2*a*b*c + b^2)*e^(2*n - 1)*cos(
d*x^n + c)*log(cos(d*x^n + c) - I*sin(d*x^n + c) + I) - (2*a*b*c - b^2)*e^(
2*n - 1)*cos(d*x^n + c)*log(-cos(d*x^n + c) + I*sin(d*x^n + c) + I) + (2*a*
b*c + b^2)*e^(2*n - 1)*cos(d*x^n + c)*log(-cos(d*x^n + c) - I*sin(d*x^n + c
) + I) + 2*(a*b*d*e^(2*n - 1)*x^n + a*b*c*e^(2*n - 1))*cos(d*x^n + c)*log(I
*cos(d*x^n + c) + sin(d*x^n + c) + 1) - 2*(a*b*d*e^(2*n - 1)*x^n + a*b*c*e^
(2*n - 1))*cos(d*x^n + c)*log(I*cos(d*x^n + c) - sin(d*x^n + c) + 1) + 2*(a
*b*d*e^(2*n - 1)*x^n + a*b*c*e^(2*n - 1))*cos(d*x^n + c)*log(-I*cos(d*x^n +
 c) + sin(d*x^n + c) + 1) - 2*(a*b*d*e^(2*n - 1)*x^n + a*b*c*e^(2*n - 1))*c
os(d*x^n + c)*log(-I*cos(d*x^n + c) - sin(d*x^n + c) + 1))/(d^2*n*cos(d*x^n
 + c))
```

Sympy [F]

$$\int (ex)^{-1+2n}(a+b\sec(c+dx^n))^2\,dx=\int (ex)^{2n-1}(a+b\sec(c+dx^n))^2\,dx$$

```
[In] integrate((e*x)**(-1+2*n)*(a+b*sec(c+d*x**n))**2,x)
```

```
[Out] Integral((e*x)**(2*n - 1)*(a + b*sec(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+2n}(a+b\sec(c+dx^n))^2\,dx = \int (b\sec(dx^n+c)+a)^2(ex)^{2n-1}\,dx$$

[In] integrate((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")

[Out] 1/2*(e*x)^(2*n)*a^2/(e*n) + (2*b^2*e^(2*n)*x^n*sin(2*d*x^n + 2*c) + (d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 + 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n)*integrate(2*(2*a*b*d*e^(2*n)*x^(2*n)*cos(2*d*x^n + 2*c)*cos(d*x^n + c) + 2*a*b*d*e^(2*n)*x^(2*n)*cos(d*x^n + c) + (2*a*b*d*e^(2*n)*x^(2*n)*sin(d*x^n + c) - b^2*e^(2*n)*x^n)*sin(2*d*x^n + 2*c))/(d*e*x*cos(2*d*x^n + 2*c)^2 + d*e*x*sin(2*d*x^n + 2*c)^2 + 2*d*e*x*cos(2*d*x^n + 2*c) + d*e*x), x))/(d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 + 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n)

Giac [F]

$$\int (ex)^{-1+2n}(a+b\sec(c+dx^n))^2\,dx = \int (b\sec(dx^n+c)+a)^2(ex)^{2n-1}\,dx$$

[In] integrate((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="giac")

[Out] integrate((b*sec(d*x^n + c) + a)^2*(e*x)^(2*n - 1), x)

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}(a+b\sec(c+dx^n))^2\,dx = \int \left(a+\frac{b}{\cos(c+d\,x^n)}\right)^2 (e\,x)^{2\,n-1}\,dx$$

[In] int((a + b/cos(c + d*x^n))^2*(e*x)^(2*n - 1),x)

[Out] int((a + b/cos(c + d*x^n))^2*(e*x)^(2*n - 1), x)

3.77 $\int (ex)^{-1+3n} (a + b \sec (c + dx^n))^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 390

$$\begin{aligned}\int (ex)^{-1+3n} (a + b \sec (c + dx^n))^2 \, dx = {} & \frac{a^2 (ex)^{3n}}{3en} - \frac{ib^2 x^{-n} (ex)^{3n}}{den} \\ & - \frac{4iabx^{-n} (ex)^{3n} \arctan \left(e^{i(c+dx^n)}\right)}{den} \\ & + \frac{2b^2 x^{-2n} (ex)^{3n} \log \left(1 + e^{2i(c+dx^n)}\right)}{d^2 en} \\ & + \frac{4iabx^{-2n} (ex)^{3n} \operatorname{PolyLog} \left(2, -ie^{i(c+dx^n)}\right)}{d^2 en} \\ & - \frac{4iabx^{-2n} (ex)^{3n} \operatorname{PolyLog} \left(2, ie^{i(c+dx^n)}\right)}{d^2 en} \\ & - \frac{ib^2 x^{-3n} (ex)^{3n} \operatorname{PolyLog} \left(2, -e^{2i(c+dx^n)}\right)}{d^3 en} \\ & - \frac{4abx^{-3n} (ex)^{3n} \operatorname{PolyLog} \left(3, -ie^{i(c+dx^n)}\right)}{d^3 en} \\ & + \frac{4abx^{-3n} (ex)^{3n} \operatorname{PolyLog} \left(3, ie^{i(c+dx^n)}\right)}{d^3 en} \\ & + \frac{b^2 x^{-n} (ex)^{3n} \tan (c + dx^n)}{den}\end{aligned}$$

```
[Out] 1/3*a^2*(e*x)^(3*n)/e/n-I*b^2*(e*x)^(3*n)/d/e/n/(x^n)-4*I*a*b*(e*x)^(3*n)*a
rctan(exp(I*(c+d*x^n)))/d/e/n/(x^n)+2*b^2*(e*x)^(3*n)*ln(1+exp(2*I*(c+d*x^n
)))/d^2/e/n/(x^(2*n))+4*I*a*b*(e*x)^(3*n)*polylog(2,-I*exp(I*(c+d*x^n)))/d^
2/e/n/(x^(2*n))-4*I*a*b*(e*x)^(3*n)*polylog(2,I*exp(I*(c+d*x^n)))/d^2/e/n/(
x^(2*n))-I*b^2*(e*x)^(3*n)*polylog(2,-exp(2*I*(c+d*x^n)))/d^3/e/n/(x^(3*n))
-4*a*b*(e*x)^(3*n)*polylog(3,-I*exp(I*(c+d*x^n)))/d^3/e/n/(x^(3*n))+4*a*b*(
e*x)^(3*n)*polylog(3,I*exp(I*(c+d*x^n)))/d^3/e/n/(x^(3*n))+b^2*(e*x)^(3*n)*
tan(c+d*x^n)/d/e/n/(x^n)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4293, 4289, 4275, 4266, 2611, 2320, 6724, 4269, 3800, 2221, 2317, 2438}

$$\int (ex)^{-1+3n}(a+b\sec(c+dx^n))^2\,dx=\frac{a^2(ex)^{3n}}{3en}-\frac{4iabx^{-n}(ex)^{3n}\arctan\left(e^{i(c+dx^n)}\right)}{den}-\frac{4abx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-ie^{i(dx^n+c)}\right)}{d^3en}+\frac{4abx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,ie^{i(dx^n+c)}\right)}{d^3en}+\frac{4iabx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-ie^{i(dx^n+c)}\right)}{d^2en}-\frac{4iabx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,ie^{i(dx^n+c)}\right)}{d^2en}-\frac{ib^2x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(2,-e^{2i(dx^n+c)}\right)}{d^3en}+\frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+e^{2i(c+dx^n)}\right)}{d^2en}+\frac{b^2x^{-n}(ex)^{3n}\tan(c+dx^n)}{den}-\frac{ib^2x^{-n}(ex)^{3n}}{den}$$

```
[In] Int[(e*x)^(-1 + 3*n)*(a + b*Sec[c + d*x^n])^2,x]
```

```
[Out] (a^2*(e*x)^(3*n))/(3*e*n) - (I*b^2*(e*x)^(3*n))/(d*e*n*x^n) - ((4*I)*a*b*(e
*x)^(3*n)*ArcTan[E^(I*(c + d*x^n))])/(d*e*n*x^n) + (2*b^2*(e*x)^(3*n)*Log[1
 + E^((2*I)*(c + d*x^n))])/(d^2*e*n*x^(2*n)) + ((4*I)*a*b*(e*x)^(3*n)*PolyL
og[2, (-I)*E^(I*(c + d*x^n))])/(d^2*e*n*x^(2*n)) - ((4*I)*a*b*(e*x)^(3*n)*P
olyLog[2, I*E^(I*(c + d*x^n))])/(d^2*e*n*x^(2*n)) - (I*b^2*(e*x)^(3*n)*Poly
Log[2, -E^((2*I)*(c + d*x^n))])/(d^3*e*n*x^(3*n)) - (4*a*b*(e*x)^(3*n)*Poly
Log[3, (-I)*E^(I*(c + d*x^n))])/(d^3*e*n*x^(3*n)) + (4*a*b*(e*x)^(3*n)*Poly
Log[3, I*E^(I*(c + d*x^n))])/(d^3*e*n*x^(3*n)) + (b^2*(e*x)^(3*n)*Tan[c + d
*x^n])/(d*e*n*x^n)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4293

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x^{-3n}(ex)^{3n}\right)\int x^{-1+3n}(a+b\sec\left(c+dx^n\right))^2\,dx}{e}\\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x^2(a+b\sec(c+dx))^2\,dx,x,x^n\right)}{en}\\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int\left(a^2x^2+2abx^2\sec(c+dx)+b^2x^2\sec^2(c+dx)\right)\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^{3n}}{3en}+\frac{\left(2abx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x^2\sec(c+dx)\,dx,x,x^n\right)}{en}\\
&\quad+\frac{\left(b^2x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x^2\sec^2(c+dx)\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^{3n}}{3en}-\frac{4iabx^{-n}(ex)^{3n}\arctan\left(e^{i(c+dx^n)}\right)}{den}+\frac{b^2x^{-n}(ex)^{3n}\tan\left(c+dx^n\right)}{den}\\
&\quad-\frac{\left(4abx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1-ie^{i(c+dx)}\right)\,dx,x,x^n\right)}{den}\\
&\quad+\frac{\left(4abx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1+ie^{i(c+dx)}\right)\,dx,x,x^n\right)}{den}\\
&\quad-\frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\tan(c+dx)\,dx,x,x^n\right)}{den}
\end{aligned}
$$

$$= \frac{a^2(ex)^{3n}}{3en} - \frac{ib^2x^{-n}(ex)^{3n}}{den} - \frac{4iabx^{-n}(ex)^{3n}\arctan\left(e^{i(c+dx^n)}\right)}{den} + \frac{4iabx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-ie^{i(c+dx^n)}\right)}{d^2en} - \frac{4iabx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,ie^{i(c+dx^n)}\right)}{d^2en} + \frac{b^2x^{-n}(ex)^{3n}\tan(c+dx^n)}{den} - \frac{(4iabx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-ie^{i(c+dx)}\right)\,dx,x,x^n\right)}{d^2en} + \frac{(4iabx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,ie^{i(c+dx)}\right)\,dx,x,x^n\right)}{d^2en} + \frac{(4ib^2x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{e^{2i(c+dx)}x}{1+e^{2i(c+dx)}}\,dx,x,x^n\right)}{den}$$

$$= \frac{a^2(ex)^{3n}}{3en} - \frac{ib^2x^{-n}(ex)^{3n}}{den} - \frac{4iabx^{-n}(ex)^{3n}\arctan\left(e^{i(c+dx^n)}\right)}{den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+e^{2i(c+dx^n)}\right)}{d^2en} + \frac{4iabx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-ie^{i(c+dx^n)}\right)}{d^2en} - \frac{4iabx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,ie^{i(c+dx^n)}\right)}{d^2en} + \frac{b^2x^{-n}(ex)^{3n}\tan(c+dx^n)}{den} - \frac{(4abx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{d^3en} + \frac{(4abx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{d^3en} - \frac{(2b^2x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \log\left(1+e^{2i(c+dx)}\right)\,dx,x,x^n\right)}{d^2en}$$

$$= \frac{a^2(ex)^{3n}}{3en} - \frac{ib^2x^{-n}(ex)^{3n}}{den} - \frac{4iabx^{-n}(ex)^{3n}\arctan\left(e^{i(c+dx^n)}\right)}{den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+e^{2i(c+dx^n)}\right)}{d^2en} + \frac{4iabx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-ie^{i(c+dx^n)}\right)}{d^2en} - \frac{4iabx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,ie^{i(c+dx^n)}\right)}{d^2en} - \frac{4abx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-ie^{i(c+dx^n)}\right)}{d^3en} + \frac{4abx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,ie^{i(c+dx^n)}\right)}{d^3en} + \frac{b^2x^{-n}(ex)^{3n}\tan(c+dx^n)}{den} + \frac{(ib^2x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2i(c+dx^n)}\right)}{d^3en}$$

$$
\begin{aligned}
&= \frac{a^2(ex)^{3n}}{3en} - \frac{ib^2x^{-n}(ex)^{3n}}{den} - \frac{4iabx^{-n}(ex)^{3n}\arctan\left(e^{i(c+dx^n)}\right)}{den} \\
&+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+e^{2i(c+dx^n)}\right)}{d^2en} + \frac{4iabx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-ie^{i(c+dx^n)}\right)}{d^2en} \\
&- \frac{4iabx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,ie^{i(c+dx^n)}\right)}{d^2en} - \frac{ib^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-e^{2i(c+dx^n)}\right)}{d^3en} \\
&- \frac{4abx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-ie^{i(c+dx^n)}\right)}{d^3en} \\
&+ \frac{4abx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,ie^{i(c+dx^n)}\right)}{d^3en} + \frac{b^2x^{-n}(ex)^{3n}\tan\left(c+dx^n\right)}{den}
\end{aligned}
$$

Mathematica [F]

$$
\int (ex)^{-1+3n}\left(a+b\sec\left(c+dx^n\right)\right)^2\,dx = \int (ex)^{-1+3n}\left(a+b\sec\left(c+dx^n\right)\right)^2\,dx
$$

```
[In] Integrate[(e*x)^(-1 + 3*n)*(a + b*Sec[c + d*x^n])^2,x]
[Out] Integrate[(e*x)^(-1 + 3*n)*(a + b*Sec[c + d*x^n])^2, x]
```

Maple [F]

$$
\int (ex)^{3n-1}\left(a+b\sec\left(c+d\,x^n\right)\right)^2dx
$$

```
[In] int((e*x)^(3*n-1)*(a+b*sec(c+d*x^n))^2,x)
[Out] int((e*x)^(3*n-1)*(a+b*sec(c+d*x^n))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1032 vs. 2(365) = 730.

Time = 0.36 (sec) , antiderivative size = 1032, normalized size of antiderivative = 2.65

$$
\int (ex)^{-1+3n}\left(a+b\sec\left(c+dx^n\right)\right)^2\,dx = \text{Too large to display}
$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="fricas")
[Out] 1/3*(a^2*d^3*e^(3*n - 1)*x^(3*n)*cos(d*x^n + c) + 3*b^2*d^2*e^(3*n - 1)*x^(
2*n)*sin(d*x^n + c) - 6*a*b*e^(3*n - 1)*cos(d*x^n + c)*polylog(3, I*cos(d*x
^n + c) + sin(d*x^n + c)) + 6*a*b*e^(3*n - 1)*cos(d*x^n + c)*polylog(3, I*c
os(d*x^n + c) - sin(d*x^n + c)) - 6*a*b*e^(3*n - 1)*cos(d*x^n + c)*polylog(
```

```
3, -I*cos(d*x^n + c) + sin(d*x^n + c)) + 6*a*b*e^(3*n - 1)*cos(d*x^n + c)*p
olylog(3, -I*cos(d*x^n + c) - sin(d*x^n + c)) + 3*(a*b*c^2 - b^2*c)*e^(3*n
- 1)*cos(d*x^n + c)*log(cos(d*x^n + c) + I*sin(d*x^n + c) + I) - 3*(a*b*c^2
 + b^2*c)*e^(3*n - 1)*cos(d*x^n + c)*log(cos(d*x^n + c) - I*sin(d*x^n + c)
+ I) + 3*(a*b*c^2 - b^2*c)*e^(3*n - 1)*cos(d*x^n + c)*log(-cos(d*x^n + c) +
 I*sin(d*x^n + c) + I) - 3*(a*b*c^2 + b^2*c)*e^(3*n - 1)*cos(d*x^n + c)*log
(-cos(d*x^n + c) - I*sin(d*x^n + c) + I) - 3*(2*I*a*b*d*e^(3*n - 1)*x^n - I
*b^2*e^(3*n - 1))*cos(d*x^n + c)*dilog(I*cos(d*x^n + c) + sin(d*x^n + c)) -
 3*(2*I*a*b*d*e^(3*n - 1)*x^n + I*b^2*e^(3*n - 1))*cos(d*x^n + c)*dilog(I*c
os(d*x^n + c) - sin(d*x^n + c)) - 3*(-2*I*a*b*d*e^(3*n - 1)*x^n + I*b^2*e^(
3*n - 1))*cos(d*x^n + c)*dilog(-I*cos(d*x^n + c) + sin(d*x^n + c)) - 3*(-2*
I*a*b*d*e^(3*n - 1)*x^n - I*b^2*e^(3*n - 1))*cos(d*x^n + c)*dilog(-I*cos(d*
x^n + c) - sin(d*x^n + c)) + 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) + b^2*d*e^(3*n
- 1)*x^n - (a*b*c^2 - b^2*c)*e^(3*n - 1))*cos(d*x^n + c)*log(I*cos(d*x^n +
c) + sin(d*x^n + c) + 1) - 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) - b^2*d*e^(3*n -
1)*x^n - (a*b*c^2 + b^2*c)*e^(3*n - 1))*cos(d*x^n + c)*log(I*cos(d*x^n + c)
 - sin(d*x^n + c) + 1) + 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) + b^2*d*e^(3*n - 1)
*x^n - (a*b*c^2 - b^2*c)*e^(3*n - 1))*cos(d*x^n + c)*log(-I*cos(d*x^n + c)
+ sin(d*x^n + c) + 1) - 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) - b^2*d*e^(3*n - 1)*
x^n - (a*b*c^2 + b^2*c)*e^(3*n - 1))*cos(d*x^n + c)*log(-I*cos(d*x^n + c) -
 sin(d*x^n + c) + 1))/(d^3*n*cos(d*x^n + c))
```

Sympy [F]

$$\int (ex)^{-1+3n}(a+b\sec(c+dx^n))^2\,dx=\int (ex)^{3n-1}(a+b\sec(c+dx^n))^2\,dx$$

```
[In] integrate((e*x)**(-1+3*n)*(a+b*sec(c+d*x**n))**2,x)
[Out] Integral((e*x)**(3*n - 1)*(a + b*sec(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+3n}(a+b\sec(c+dx^n))^2\,dx=\int (b\sec(dx^n+c)+a)^2(ex)^{3n-1}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")
[Out] 1/3*(e*x)^(3*n)*a^2/(e*n) + (2*b^2*e^(3*n)*x^(2*n)*sin(2*d*x^n + 2*c) + (d*
e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 + 2*d*e*n*cos(2*d*x^n
 + 2*c) + d*e*n)*integrate(4*(a*b*d*e^(3*n)*x^(3*n)*cos(2*d*x^n + 2*c)*cos(
d*x^n + c) + a*b*d*e^(3*n)*x^(3*n)*cos(d*x^n + c) + (a*b*d*e^(3*n)*x^(3*n)*
sin(d*x^n + c) - b^2*e^(3*n)*x^(2*n))*sin(2*d*x^n + 2*c))/(d*e*x*cos(2*d*x^
```

```
n + 2*c)^2 + d*e*x*sin(2*d*x^n + 2*c)^2 + 2*d*e*x*cos(2*d*x^n + 2*c) + d*e*
x), x))/(d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 + 2*d*e*n*
cos(2*d*x^n + 2*c) + d*e*n)
```

Giac [F]

$$\int (ex)^{-1+3n} \left(a + b\sec \left(c + dx^n\right)\right)^2 \, dx = \int (b\sec \left(dx^n + c\right) + a)^2 (ex)^{3n-1} \, dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((b*sec(d*x^n + c) + a)^2*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} \left(a + b\sec \left(c + dx^n\right)\right)^2 \, dx = \int \left(a + \frac{b}{\cos\left(c + d\,x^n\right)}\right)^2 (e\,x)^{3\,n-1} \, dx$$

```
[In] int((a + b/cos(c + d*x^n))^2*(e*x)^(3*n - 1),x)
[Out] int((a + b/cos(c + d*x^n))^2*(e*x)^(3*n - 1), x)
```

3.78 $\int \frac{(ex)^{-1+n}}{a+b\sec(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{(ex)^{-1+n}}{a+b\sec(c+dx^n)}\,dx = \frac{(ex)^n}{aen} - \frac{2bx^{-n}(ex)^n\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}den}$$

```
[Out] (e*x)^n/a/e/n-2*b*(e*x)^n*arctanh((a-b)^(1/2)*tan(1/2*c+1/2*d*x^n)/(a+b)^(1
/2))/a/d/e/n/(x^n)/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4293, 4289, 3868, 2738, 214}

$$\int \frac{(ex)^{-1+n}}{a+b\sec(c+dx^n)}\,dx = \frac{(ex)^n}{aen} - \frac{2bx^{-n}(ex)^n\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{aden\sqrt{a-b}\sqrt{a+b}}$$

```
[In] Int[(e*x)^(-1 + n)/(a + b*Sec[c + d*x^n]),x]
```

```
[Out] (e*x)^n/(a*e*n) - (2*b*(e*x)^n*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x^n)/2])/Sqr
t[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d*e*n*x^n)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4293

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x^{-n}(ex)^n\right)\int \frac{x^{-1+n}}{a+b\sec(c+dx^n)}\,dx}{e}\\
&= \frac{\left(x^{-n}(ex)^n\right)\operatorname{Subst}\left(\int \frac{1}{a+b\sec(c+dx)}\,dx, x, x^n\right)}{en}\\
&= \frac{(ex)^n}{aen} - \frac{\left(x^{-n}(ex)^n\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx, x, x^n\right)}{aen}\\
&= \frac{(ex)^n}{aen} - \frac{\left(2x^{-n}(ex)^n\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx^n)\right)\right)}{aden}\\
&= \frac{(ex)^n}{aen} - \frac{2bx^{-n}(ex)^n\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}den}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{(ex)^{-1+n}}{a+b\sec(c+dx^n)}\,dx=\frac{(ex)^n\left(d+cx^{-n}+\frac{2bx^{-n}\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{aden}$$

```
[In] Integrate[(e*x)^(-1 + n)/(a + b*Sec[c + d*x^n]),x]
[Out] ((e*x)^n*(d + c/x^n + (2*b*ArcTanh[((-a + b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 -
 b^2]])/(Sqrt[a^2 - b^2]*x^n)))/(a*d*e*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.61

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(-1+n)\left(-i\pi\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)+i\pi\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2+i\pi\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2-i\pi\,\mathrm{csgn}(iex)^3+2\ln(x)+2\ln(e)\right)}{2}}}{an}+\frac{2i\arctan\left(\frac{2a\,\mathrm{e}^{i(}}{2\sqrt{}}\right.}{}$

```
[In] int((e*x)^(-1+n)/(a+b*sec(c+d*x^n)),x,method=_RETURNVERBOSE)
[Out] 1/a/n*x*exp(1/2*(-1+n)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csgn(I*e
)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x)+2*l
n(e)))+2*I*arctan(1/2*(2*a*exp(I*(d*x^n+2*c))+2*exp(I*c)*b)/(a^2*exp(2*I*c)
-exp(2*I*c)*b^2)^(1/2))/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)/d/e*e^n/n/a*b
*exp(1/2*I*(-Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*n*csgn(I*e)*csgn(I*e*x
)^2+Pi*n*csgn(I*x)*csgn(I*e*x)^2-Pi*n*csgn(I*e*x)^3+Pi*csgn(I*e)*csgn(I*x)*
csgn(I*e*x)-Pi*csgn(I*e)*csgn(I*e*x)^2-Pi*csgn(I*x)*csgn(I*e*x)^2+Pi*csgn(I
*e*x)^3+2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.45

$$\int \frac{(ex)^{-1+n}}{a+b\sec(c+dx^n)}\,dx$$
$$= \left[\frac{2(a^2-b^2)de^{n-1}x^n + \sqrt{a^2-b^2}be^{n-1}\log\left(\frac{2ab\cos(dx^n+c)-(a^2-2b^2)\cos(dx^n+c)^2+2a^2-b^2-2\left(\sqrt{a^2-b^2}b\cos(dx^n+c)+\sqrt{a^2-b^2}\right.}{a^2\cos(dx^n+c)^2+2ab\cos(dx^n+c)+b^2}\right.}{2(a^3-ab^2)dn}\right.$$

```
[In] integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2 - b^2)*d*e^(n - 1)*x^n + sqrt(a^2 - b^2)*b*e^(n - 1)*log((2*a*
b*cos(d*x^n + c) - (a^2 - 2*b^2)*cos(d*x^n + c)^2 + 2*a^2 - b^2 - 2*(sqrt(a
^2 - b^2)*b*cos(d*x^n + c) + sqrt(a^2 - b^2)*a)*sin(d*x^n + c))/(a^2*cos(d*
x^n + c)^2 + 2*a*b*cos(d*x^n + c) + b^2)))/((a^3 - a*b^2)*d*n), ((a^2 - b^2
)*d*e^(n - 1)*x^n - sqrt(-a^2 + b^2)*b*e^(n - 1)*arctan(-(sqrt(-a^2 + b^2)*
b*cos(d*x^n + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*sin(d*x^n + c))))/((a^3
 - a*b^2)*d*n)]
```

Sympy [F]

$$\int \frac{(ex)^{-1+n}}{a+b\sec(c+dx^n)}\,dx = \int \frac{(ex)^{n-1}}{a+b\sec(c+dx^n)}\,dx$$

```
[In] integrate((e*x)**(-1+n)/(a+b*sec(c+d*x**n)),x)
```

```
[Out] Integral((e*x)**(n - 1)/(a + b*sec(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+n}}{a+b\sec(c+dx^n)}\,dx = \int \frac{(ex)^{n-1}}{b\sec(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n)),x, algorithm="maxima")
```

```
[Out] -(2*a*b*e^(n + 1)*n*integrate((a*x^n*cos(2*d*x^n + 2*c)*cos(d*x^n + c) + 2*
b*x^n*cos(d*x^n + c)^2 + a*x^n*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 2*b*x^n*
sin(d*x^n + c)^2 + a*x^n*cos(d*x^n + c))/(a^3*e*x*cos(2*d*x^n + 2*c)^2 + 4*
a*b^2*e*x*cos(d*x^n + c)^2 + a^3*e*x*sin(2*d*x^n + 2*c)^2 + 4*a^2*b*e*x*sin
(2*d*x^n + 2*c)*sin(d*x^n + c) + 4*a*b^2*e*x*sin(d*x^n + c)^2 + 4*a^2*b*e*x
*cos(d*x^n + c) + a^3*e*x + 2*(2*a^2*b*e*x*cos(d*x^n + c) + a^3*e*x)*cos(2*
d*x^n + 2*c)), x) - e^n*x^n)/(a*e*n)
```

Giac [F]

$$\int \frac{(ex)^{-1+n}}{a+b\sec(c+dx^n)}\,dx = \int \frac{(ex)^{n-1}}{b\sec(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n)),x, algorithm="giac")
[Out] integrate((e*x)^(n - 1)/(b*sec(d*x^n + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 15.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.56

$$\int \frac{(ex)^{-1+n}}{a+b\sec(c+dx^n)}\,dx = \frac{x\,(e\,x)^{n-1}}{a\,n} + \frac{b\,x\,\ln\left(2\,b\,x\,e^{c\,1i}\,e^{d\,x^n\,1i}\,(e\,x)^{n-1} - \frac{b\,x\,\left(a+b\,e^{c\,1i}\,e^{d\,x^n\,1i}\right)\,(e\,x)^{n-1}\,2i}{\sqrt{a+b}\,\sqrt{a-b}}\right)\,(e\,x)^{n-1}}{a\,d\,n\,x^n\,\sqrt{a+b}\,\sqrt{a-b}} - \frac{b\,x\,\ln\left(2\,b\,x\,e^{c\,1i}\,e^{d\,x^n\,1i}\,(e\,x)^{n-1} + \frac{b\,x\,\left(a+b\,e^{c\,1i}\,e^{d\,x^n\,1i}\right)\,(e\,x)^{n-1}\,2i}{\sqrt{a+b}\,\sqrt{a-b}}\right)\,(e\,x)^{n-1}}{a\,d\,n\,x^n\,\sqrt{a+b}\,\sqrt{a-b}}$$

```
[In] int((e*x)^(n - 1)/(a + b/cos(c + d*x^n)),x)
[Out] (x*(e*x)^(n - 1))/(a*n) + (b*x*log(2*b*x*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n -
 1) - (b*x*(a + b*exp(c*1i)*exp(d*x^n*1i))*(e*x)^(n - 1)*2i)/((a + b)^(1/2)
*(a - b)^(1/2)))*(e*x)^(n - 1))/(a*d*n*x^n*(a + b)^(1/2)*(a - b)^(1/2)) - (
b*x*log(2*b*x*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1) + (b*x*(a + b*exp(c*1i)
*exp(d*x^n*1i))*(e*x)^(n - 1)*2i)/((a + b)^(1/2)*(a - b)^(1/2)))*(e*x)^(n -
 1))/(a*d*n*x^n*(a + b)^(1/2)*(a - b)^(1/2))
```

3.79 $\int \frac{(ex)^{-1+2n}}{a+b\sec(c+dx^n)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 328

$$
\begin{aligned}
\int \frac{(ex)^{-1+2n}}{a+b\sec(c+dx^n)}\,dx &= \frac{(ex)^{2n}}{2aen}+\frac{ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} \\
&\quad -\frac{ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} \\
&\quad +\frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} \\
&\quad -\frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}
\end{aligned}
$$

```
[Out] 1/2*(e*x)^(2*n)/a/e/n+I*b*(e*x)^(2*n)*ln(1+a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)
^(1/2)))/a/d/e/n/(x^n)/(-a^2+b^2)^(1/2)-I*b*(e*x)^(2*n)*ln(1+a*exp(I*(c+d*x
^n))/(b+(-a^2+b^2)^(1/2)))/a/d/e/n/(x^n)/(-a^2+b^2)^(1/2)+b*(e*x)^(2*n)*pol
ylog(2,-a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a/d^2/e/n/(x^(2*n))/(-a^2+
b^2)^(1/2)-b*(e*x)^(2*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2))
)/a/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4293, 4289, 4276, 3402, 2296, 2221, 2317, 2438}

$$\int \frac{(ex)^{-1+2n}}{a+b\sec(c+dx^n)}\,dx=\frac{bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}}-\frac{bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}}+\frac{ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{aden\sqrt{b^2-a^2}}-\frac{ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{\sqrt{b^2-a^2}+b}\right)}{aden\sqrt{b^2-a^2}}+\frac{(ex)^{2n}}{2aen}$$

```
[In] Int[(e*x)^(-1 + 2*n)/(a + b*Sec[c + d*x^n]),x]
```

```
[Out] (e*x)^(2*n)/(2*a*e*n) + (I*b*(e*x)^(2*n)*Log[1 + (a*E^(I*(c + d*x^n)))/(b -
 Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d*e*n*x^n) - (I*b*(e*x)^(2*n)*Log[
1 + (a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d*e*
n*x^n) + (b*(e*x)^(2*n)*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 +
 b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) - (b*(e*x)^(2*n)*PolyLog[2,
-((a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2*e
*n*x^(2*n))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4293

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(x^{-2n}(ex)^{2n}\right)\int \frac{x^{-1+2n}}{a+b\sec(c+dx^n)}\,dx}{e}$$

$$= \frac{\left(x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{x}{a+b\sec(c+dx)}\,dx, x, x^n\right)}{en}$$

$$= \frac{\left(x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \left(\frac{x}{a}-\frac{bx}{a(b+a\cos(c+dx))}\right)\,dx, x, x^n\right)}{en}$$

$$=\frac{(ex)^{2n}}{2aen}-\frac{\left(bx^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int\frac{x}{b+a\cos(c+dx)}\,dx,x,x^n\right)}{aen}$$

$$=\frac{(ex)^{2n}}{2aen}-\frac{\left(2bx^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int\frac{e^{i(c+dx)}x}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx,x,x^n\right)}{aen}$$

$$=\frac{(ex)^{2n}}{2aen}-\frac{\left(2bx^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int\frac{e^{i(c+dx)}x}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,x^n\right)}{\sqrt{-a^2+b^2}en}+\frac{\left(2bx^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int\frac{e^{i(c+dx)}x}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,x^n\right)}{\sqrt{-a^2+b^2}en}$$

$$=\frac{(ex)^{2n}}{2aen}+\frac{ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}-\frac{ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}-\frac{\left(ibx^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a\sqrt{-a^2+b^2}den}+\frac{\left(ibx^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a\sqrt{-a^2+b^2}den}$$

$$=\frac{(ex)^{2n}}{2aen}+\frac{ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}-\frac{ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}-\frac{\left(bx^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{a\sqrt{-a^2+b^2}d^2en}+\frac{\left(bx^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{a\sqrt{-a^2+b^2}d^2en}$$

$$=\frac{(ex)^{2n}}{2aen}+\frac{ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}-\frac{ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}+\frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}-\frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 861 vs. 2(328) = 656.

Time = 2.36 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.62

$$\int \frac{(ex)^{-1+2n}}{a+b\sec(c+dx^n)}\,dx$$
$$= \frac{(ex)^{2n}\left(b+a\cos(c+dx^n)\right)\left(1-\frac{2bx^{-2n}\left(2(c+dx^n)\operatorname{arctanh}\left(\frac{(a+b)\cot\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)-2\left(c+\arccos\left(-\frac{b}{a}\right)\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}\right.}{\sqrt{a^{2}}}\right.\right.}{}\right.}{}$$

```
[In] Integrate[(e*x)^(-1 + 2*n)/(a + b*Sec[c + d*x^n]),x]
```

```
[Out] ((e*x)^(2*n)*(b + a*Cos[c + d*x^n])*(1 - (2*b*(2*(c + d*x^n)*ArcTanh[((a +
b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] - 2*(c + ArcCos[-(b/a)])*ArcTanh[((
a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + (ArcCos[-(b/a)] - (2*I)*ArcTa
nh[((a + b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + (2*I)*ArcTanh[((a - b)*T
an[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[Sqrt[a^2 - b^2]/(Sqrt[2]*Sqrt[a]*E
^((I/2)*(c + d*x^n))*Sqrt[b + a*Cos[c + d*x^n]])] + (ArcCos[-(b/a)] + (2*I)
*(ArcTanh[((a + b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] - ArcTanh[((a - b)*
Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[(Sqrt[a^2 - b^2]*E^((I/2)*(c + d
*x^n)))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Cos[c + d*x^n]])] - (ArcCos[-(b/a)] - (
2*I)*ArcTanh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[((a + b)*(a
 - b - I*Sqrt[a^2 - b^2])*(1 + I*Tan[(c + d*x^n)/2]))/(a*(a + b + Sqrt[a^2
- b^2]*Tan[(c + d*x^n)/2]))] - (ArcCos[-(b/a)] + (2*I)*ArcTanh[((a - b)*Tan
[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[((a + b)*((-I)*a + I*b + Sqrt[a^2 -
b^2])*(I + Tan[(c + d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^n)
/2]))] + I*(PolyLog[2, ((b - I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Ta
n[(c + d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))] - Poly
Log[2, ((b + I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]
))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))])))/(Sqrt[a^2 - b^2]*d^
2*x^(2*n)))*Sec[c + d*x^n])/(2*a*e*n*(a + b*Sec[c + d*x^n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 752, normalized size of antiderivative = 2.29

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(2n-1)\left(-i\pi\ \mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)+i\pi\ \mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2+i\pi\ \mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2-i\pi\ \mathrm{csgn}(iex)^3+2\ln(x)+2\ln(e)\right)}{2}}}{2an} + \frac{\left(ix^n d\ln\left(\frac{a\,\mathrm{e}^{i(d}}{}\right.\right.}{}$

```
[In] int((e*x)^(2*n-1)/(a+b*sec(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/n*x*exp(1/2*(2*n-1)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csgn(
I*e)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x)+
2*ln(e)))+1/(a^2-b^2)*(I*x^n*d*ln((a*exp(I*(d*x^n+2*c))+exp(I*c)*b+(exp(2*I
*c)*b^2-a^2*exp(2*I*c))^(1/2))/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^
(1/2)))-I*x^n*d*ln((a*exp(I*(d*x^n+2*c))+exp(I*c)*b-(exp(2*I*c)*b^2-a^2*exp
(2*I*c))^(1/2))/(exp(I*c)*b-(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2)))-dilog(a
/(exp(I*c)*b-(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*exp(I*(d*x^n+2*c))+1/(e
xp(I*c)*b-(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*exp(I*c)*b-1/(exp(I*c)*b-(
exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2)
)+dilog(a/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*exp(I*(d*x^n+2
*c))+1/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*exp(I*c)*b+1/(exp
(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*(exp(2*I*c)*b^2-a^2*exp(2*I*
c))^(1/2)))*(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2)/d^2/n/e*(e^n)^2*b/a*exp(-
1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi*n*csgn(I*e)*csgn(I*e*x)^
2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x)^3-Pi*csgn(I*e)*csgn(I*x
)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^2-Pi*csgn
(I*e*x)^3+2*c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(300) = 600$.

Time = 0.53 (sec) , antiderivative size = 1268, normalized size of antiderivative = 3.87

$$\int \frac{(ex)^{-1+2n}}{a+b\sec(c+dx^n)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*a*b*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) + 2
*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) + I*a*b*c*e^(2*n -
1)*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n + c) + 2
*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - I*a*b*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^
2)*log(-2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a
^2) - 2*b) + I*a*b*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2)*log(-2*a*cos(d*x^n
+ c) - 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) + (a^2 - b^
2)*d^2*e^(2*n - 1)*x^(2*n) - a*b*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2)*dilog(-
((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (I*a*sqrt(-(a^2 - b^2)/a^2
```

```
) + I*b)*sin(d*x^n + c) + a)/a + 1) - a*b*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2
)*dilog(-((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (-I*a*sqrt(-(a^2
- b^2)/a^2) - I*b)*sin(d*x^n + c) + a)/a + 1) + a*b*e^(2*n - 1)*sqrt(-(a^2
- b^2)/a^2)*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) + (I*a*sqr
t(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c) - a)/a + 1) + a*b*e^(2*n - 1)*sqr
t(-(a^2 - b^2)/a^2)*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) +
(-I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) - a)/a + 1) + (I*a*b*d*e
^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) + I*a*b*c*e^(2*n - 1)*sqrt(-(a^2 - b^
2)/a^2))*log(((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (I*a*sqrt(-(a
^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) + a)/a) + (-I*a*b*d*e^(2*n - 1)*x^n*sq
rt(-(a^2 - b^2)/a^2) - I*a*b*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2))*log(((a*
sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (-I*a*sqrt(-(a^2 - b^2)/a^2) -
 I*b)*sin(d*x^n + c) + a)/a) + (I*a*b*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a
^2) + I*a*b*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2))*log(-((a*sqrt(-(a^2 - b^2
)/a^2) - b)*cos(d*x^n + c) + (I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n +
 c) - a)/a) + (-I*a*b*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) - I*a*b*c*e^
(2*n - 1)*sqrt(-(a^2 - b^2)/a^2))*log(-((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(
d*x^n + c) + (-I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) - a)/a))/((
a^3 - a*b^2)*d^2*n)
```

Sympy [F]

$$\int \frac{(ex)^{-1+2n}}{a+b\sec(c+dx^n)}\,dx = \int \frac{(ex)^{2n-1}}{a+b\sec(c+dx^n)}\,dx$$

```
[In] integrate((e*x)**(-1+2*n)/(a+b*sec(c+d*x**n)),x)
[Out] Integral((e*x)**(2*n - 1)/(a + b*sec(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+2n}}{a+b\sec(c+dx^n)}\,dx = \int \frac{(ex)^{2n-1}}{b\sec(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n)),x, algorithm="maxima")
[Out] -1/2*(4*a*b*e^(2*n + 1)*n*integrate((a*x^(2*n)*cos(2*d*x^n + 2*c)*cos(d*x^n
 + c) + 2*b*x^(2*n)*cos(d*x^n + c)^2 + a*x^(2*n)*sin(2*d*x^n + 2*c)*sin(d*x
^n + c) + 2*b*x^(2*n)*sin(d*x^n + c)^2 + a*x^(2*n)*cos(d*x^n + c))/(a^3*e*x
*cos(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x*cos(d*x^n + c)^2 + a^3*e*x*sin(2*d*x^n
+ 2*c)^2 + 4*a^2*b*e*x*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 4*a*b^2*e*x*sin(
d*x^n + c)^2 + 4*a^2*b*e*x*cos(d*x^n + c) + a^3*e*x + 2*(2*a^2*b*e*x*cos(d*
x^n + c) + a^3*e*x)*cos(2*d*x^n + 2*c)), x) - e^(2*n)*x^(2*n))/(a*e*n)
```

Giac [F]

$$\int \frac{(ex)^{-1+2n}}{a+b\sec(c+dx^n)}\,dx = \int \frac{(ex)^{2n-1}}{b\sec(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n)),x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)/(b*sec(d*x^n + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+2n}}{a+b\sec(c+dx^n)}\,dx = \int \frac{(e\,x)^{2n-1}}{a+\frac{b}{\cos(c+d\,x^n)}}\,dx$$

```
[In] int((e*x)^(2*n - 1)/(a + b/cos(c + d*x^n)),x)
[Out] int((e*x)^(2*n - 1)/(a + b/cos(c + d*x^n)), x)
```

3.80 $\int \frac{(ex)^{-1+3n}}{a+b\sec(c+dx^n)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 485

$$\begin{aligned}\int \frac{(ex)^{-1+3n}}{a+b\sec(c+dx^n)}\,dx &= \frac{(ex)^{3n}}{3aen}+\frac{ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}\\
&\quad-\frac{ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}\\
&\quad+\frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}\\
&\quad-\frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}\\
&\quad+\frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en}\\
&\quad-\frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en}\end{aligned}$$

```
[Out] 1/3*(e*x)^(3*n)/a/e/n+I*b*(e*x)^(3*n)*ln(1+a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)
^(1/2)))/a/d/e/n/(x^n)/(-a^2+b^2)^(1/2)-I*b*(e*x)^(3*n)*ln(1+a*exp(I*(c+d*x
^n))/(b+(-a^2+b^2)^(1/2)))/a/d/e/n/(x^n)/(-a^2+b^2)^(1/2)+2*b*(e*x)^(3*n)*p
olylog(2,-a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a/d^2/e/n/(x^(2*n))/(-a^
2+b^2)^(1/2)-2*b*(e*x)^(3*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1
/2)))/a/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)+2*I*b*(e*x)^(3*n)*polylog(3,-a*e
xp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a/d^3/e/n/(x^(3*n))/(-a^2+b^2)^(1/2)-
2*I*b*(e*x)^(3*n)*polylog(3,-a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a/d^3
/e/n/(x^(3*n))/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4293, 4289, 4276, 3402, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{(ex)^{-1+3n}}{a+b\sec(c+dx^n)}\,dx = \frac{2ibx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^3en\sqrt{b^2-a^2}} - \frac{2ibx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^3en\sqrt{b^2-a^2}} + \frac{2bx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} - \frac{2bx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} + \frac{ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{aden\sqrt{b^2-a^2}} - \frac{ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{\sqrt{b^2-a^2}+b}\right)}{aden\sqrt{b^2-a^2}} + \frac{(ex)^{3n}}{3aen}$$

```
[In] Int[(e*x)^(-1 + 3*n)/(a + b*Sec[c + d*x^n]),x]
```

```
[Out] (e*x)^(3*n)/(3*a*e*n) + (I*b*(e*x)^(3*n)*Log[1 + (a*E^(I*(c + d*x^n)))/(b -
 Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d*e*n*x^n) - (I*b*(e*x)^(3*n)*Log[
1 + (a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d*e*
n*x^n) + (2*b*(e*x)^(3*n)*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2
 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) - (2*b*(e*x)^(3*n)*PolyLog
[2, -((a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d
^2*e*n*x^(2*n)) + ((2*I)*b*(e*x)^(3*n)*PolyLog[3, -((a*E^(I*(c + d*x^n)))/(
b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3*e*n*x^(3*n)) - ((2*I)*b*(e
*x)^(3*n)*PolyLog[3, -((a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2]))])/(a*S
qrt[-a^2 + b^2]*d^3*e*n*x^(3*n))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4293

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\left(x^{-3n}(ex)^{3n}\right)\int \frac{x^{-1+3n}}{a+b\sec(c+dx^n)}\,dx}{e}$$

$$= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x^2}{a+b\sec(c+dx)}\,dx, x, x^n\right)}{en}$$

$$= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \left(\frac{x^2}{a}-\frac{bx^2}{a(b+a\cos(c+dx))}\right)\,dx, x, x^n\right)}{en}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{\left(bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x^2}{b+a\cos(c+dx)}\,dx, x, x^n\right)}{aen}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{\left(2bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, x^n\right)}{aen}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{\left(2bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^n\right)}{\sqrt{-a^2+b^2}en} + \frac{\left(2bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^n\right)}{\sqrt{-a^2+b^2}en}$$

$$= \frac{(ex)^{3n}}{3aen} + \frac{ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{\left(2ibx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}den} + \frac{\left(2ibx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}den}$$

$$= \frac{(ex)^{3n}}{3aen} + \frac{ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} - \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a\sqrt{-a^2+b^2}d^2en} + \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a\sqrt{-a^2+b^2}d^2en}$$

$$= \frac{(ex)^{3n}}{3aen} + \frac{ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} + \frac{(2ibx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a\sqrt{-a^2+b^2}d^3en} - \frac{(2ibx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a\sqrt{-a^2+b^2}d^3en}$$

$$= \frac{(ex)^{3n}}{3aen} + \frac{ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} + \frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en} - \frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en}$$

Mathematica [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\sec(c+dx^n)}\,dx = \int \frac{(ex)^{-1+3n}}{a+b\sec(c+dx^n)}\,dx$$

```
[In] Integrate[(e*x)^(-1 + 3*n)/(a + b*Sec[c + d*x^n]),x]
[Out] Integrate[(e*x)^(-1 + 3*n)/(a + b*Sec[c + d*x^n]), x]
```

Maple [F]

$$\int \frac{(ex)^{3n-1}}{a+b\sec(c+d\,x^n)}dx$$

```
[In] int((e*x)^(3*n-1)/(a+b*sec(c+d*x^n)),x)
[Out] int((e*x)^(3*n-1)/(a+b*sec(c+d*x^n)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1711 vs. $2(445) = 890$.

Time = 0.51 (sec) , antiderivative size = 1711, normalized size of antiderivative = 3.53

$$\int \frac{(ex)^{-1+3n}}{a+b\sec(c+dx^n)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n)),x, algorithm="fricas")
[Out] -1/6*(6*a*b*d*e^(3*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2)*dilog(-((a*sqrt(-(a^2
- b^2)/a^2) + b)*cos(d*x^n + c) - (I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*
x^n + c) + a)/a + 1) + 6*a*b*d*e^(3*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2)*dilog
(-((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (-I*a*sqrt(-(a^2 - b^2)/
a^2) - I*b)*sin(d*x^n + c) + a)/a + 1) - 6*a*b*d*e^(3*n - 1)*x^n*sqrt(-(a^2
 - b^2)/a^2)*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) + (I*a*sq
rt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c) - a)/a + 1) - 6*a*b*d*e^(3*n - 1
)*x^n*sqrt(-(a^2 - b^2)/a^2)*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^
n + c) + (-I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) - a)/a + 1) - 3
*I*a*b*c^2*e^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) + 2*I*
a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) + 3*I*a*b*c^2*e^(3*n -
 1)*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n + c) +
2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - 3*I*a*b*c^2*e^(3*n - 1)*sqrt(-(a^2 - b^
2)/a^2)*log(-2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b
^2)/a^2) - 2*b) + 3*I*a*b*c^2*e^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2)*log(-2*a*c
```

```
os(d*x^n + c) - 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) -
2*(a^2 - b^2)*d^3*e^(3*n - 1)*x^(3*n) + 6*I*a*b*e^(3*n - 1)*sqrt(-(a^2 - b^
2)/a^2)*polylog(3, -((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) + (I*a*s
qrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c))/a) - 6*I*a*b*e^(3*n - 1)*sqrt(
-(a^2 - b^2)/a^2)*polylog(3, -((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c
) + (-I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c))/a) + 6*I*a*b*e^(3*n
 - 1)*sqrt(-(a^2 - b^2)/a^2)*polylog(3, ((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos
(d*x^n + c) - (I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c))/a) - 6*I*a
*b*e^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2)*polylog(3, ((a*sqrt(-(a^2 - b^2)/a^2)
 - b)*cos(d*x^n + c) - (-I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c))/
a) - 3*(I*a*b*d^2*e^(3*n - 1)*x^(2*n)*sqrt(-(a^2 - b^2)/a^2) - I*a*b*c^2*e^
(3*n - 1)*sqrt(-(a^2 - b^2)/a^2))*log(((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d
*x^n + c) - (I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) + a)/a) - 3*(
-I*a*b*d^2*e^(3*n - 1)*x^(2*n)*sqrt(-(a^2 - b^2)/a^2) + I*a*b*c^2*e^(3*n -
1)*sqrt(-(a^2 - b^2)/a^2))*log(((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n +
c) - (-I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c) + a)/a) - 3*(I*a*b*
d^2*e^(3*n - 1)*x^(2*n)*sqrt(-(a^2 - b^2)/a^2) - I*a*b*c^2*e^(3*n - 1)*sqrt
(-(a^2 - b^2)/a^2))*log(-((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) + (
I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c) - a)/a) - 3*(-I*a*b*d^2*e^
(3*n - 1)*x^(2*n)*sqrt(-(a^2 - b^2)/a^2) + I*a*b*c^2*e^(3*n - 1)*sqrt(-(a^2
 - b^2)/a^2))*log(-((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) + (-I*a*s
qrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) - a)/a))/((a^3 - a*b^2)*d^3*n)
```

Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\sec(c+dx^n)}\,dx=\int \frac{(ex)^{3n-1}}{a+b\sec(c+dx^n)}\,dx$$

```
[In] integrate((e*x)**(-1+3*n)/(a+b*sec(c+d*x**n)),x)
```

```
[Out] Integral((e*x)**(3*n - 1)/(a + b*sec(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\sec(c+dx^n)}\,dx=\int \frac{(ex)^{3n-1}}{b\sec(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n)),x, algorithm="maxima")
```

```
[Out] -1/3*(6*a*b*e^(3*n + 1)*n*integrate((a*x^(3*n)*cos(2*d*x^n + 2*c)*cos(d*x^n
 + c) + 2*b*x^(3*n)*cos(d*x^n + c)^2 + a*x^(3*n)*sin(2*d*x^n + 2*c)*sin(d*x
^n + c) + 2*b*x^(3*n)*sin(d*x^n + c)^2 + a*x^(3*n)*cos(d*x^n + c))/(a^3*e*x
```

```
*cos(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x*cos(d*x^n + c)^2 + a^3*e*x*sin(2*d*x^n
+ 2*c)^2 + 4*a^2*b*e*x*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 4*a*b^2*e*x*sin(
d*x^n + c)^2 + 4*a^2*b*e*x*cos(d*x^n + c) + a^3*e*x + 2*(2*a^2*b*e*x*cos(d*
x^n + c) + a^3*e*x)*cos(2*d*x^n + 2*c)), x) - e^(3*n)*x^(3*n))/(a*e*n)
```

Giac [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\sec(c+dx^n)}\,dx=\int \frac{(ex)^{3n-1}}{b\sec(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n)),x, algorithm="giac")
[Out] integrate((e*x)^(3*n - 1)/(b*sec(d*x^n + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{a+b\sec(c+dx^n)}\,dx=\int \frac{(e\,x)^{3\,n-1}}{a+\frac{b}{\cos(c+d\,x^n)}}\,dx$$

```
[In] int((e*x)^(3*n - 1)/(a + b/cos(c + d*x^n)),x)
[Out] int((e*x)^(3*n - 1)/(a + b/cos(c + d*x^n)), x)
```

3.81 $\int \frac{(ex)^{-1+n}}{(a+b\sec(c+dx^n))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{(ex)^{-1+n}}{(a+b\sec(c+dx^n))^2}\,dx = \frac{(ex)^n}{a^2en} - \frac{2b(2a^2-b^2)\,x^{-n}(ex)^n\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}den} + \frac{b^2x^{-n}(ex)^n\tan(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\sec(c+dx^n))}$$

```
[Out] (e*x)^n/a^2/e/n-2*b*(2*a^2-b^2)*(e*x)^n*arctanh((a-b)^(1/2)*tan(1/2*c+1/2*d
*x^n)/(a+b)^(1/2))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d/e/n/(x^n)+b^2*(e*x)^n*tan(
c+d*x^n)/a/(a^2-b^2)/d/e/n/(x^n)/(a+b*sec(c+d*x^n))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4293, 4289, 3870, 4004, 3916, 2738, 214}

$$\int \frac{(ex)^{-1+n}}{(a+b\sec(c+dx^n))^2}\,dx = -\frac{2b(2a^2-b^2)\,x^{-n}(ex)^n\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a^2den(a-b)^{3/2}(a+b)^{3/2}} + \frac{b^2x^{-n}(ex)^n\tan(c+dx^n)}{aden\,(a^2-b^2)\,(a+b\sec(c+dx^n))} + \frac{(ex)^n}{a^2en}$$

```
[In] Int[(e*x)^(-1 + n)/(a + b*Sec[c + d*x^n])^2,x]
```

```
[Out] (e*x)^n/(a^2*e*n) - (2*b*(2*a^2 - b^2)*(e*x)^n*ArcTanh[(Sqrt[a - b]*Tan[(c
+ d*x^n)/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d*e*n*x^n) + (b
^2*(e*x)^n*Tan[c + d*x^n])/(a*(a^2 - b^2)*d*e*n*x^n*(a + b*Sec[c + d*x^n]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4293

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{(x^{-n}(ex)^n)\int \frac{x^{-1+n}}{(a+b\sec(c+dx^n))^2}\,dx}{e}$$

$$= \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{(a+b\sec(c+dx))^2}\,dx, x, x^n\right)}{en}$$

$$= \frac{b^2x^{-n}(ex)^n\tan(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\sec(c+dx^n))} - \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{-a^2+b^2+ab\sec(c+dx)}{a+b\sec(c+dx)}\,dx, x, x^n\right)}{a(a^2-b^2)\,en}$$

$$= \frac{(ex)^n}{a^2en} + \frac{b^2x^{-n}(ex)^n\tan(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\sec(c+dx^n))} + \frac{((-a^2b+b(-a^2+b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx, x, x^n\right)}{a^2(a^2-b^2)\,en}$$

$$= \frac{(ex)^n}{a^2en} + \frac{b^2x^{-n}(ex)^n\tan(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\sec(c+dx^n))} + \frac{((-a^2b+b(-a^2+b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx, x, x^n\right)}{a^2b(a^2-b^2)\,en}$$

$$= \frac{(ex)^n}{a^2en} + \frac{b^2x^{-n}(ex)^n\tan(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\sec(c+dx^n))} + \frac{(2(-a^2b+b(-a^2+b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx^n)\right)\right)}{a^2b(a^2-b^2)\,den}$$

$$= \frac{(ex)^n}{a^2en} - \frac{2b(2a^2-b^2)\,x^{-n}(ex)^n\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}den} + \frac{b^2x^{-n}(ex)^n\tan(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\sec(c+dx^n))}$$

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.22

$$\int \frac{(ex)^{-1+n}}{(a+b\sec(c+dx^n))^2}\,dx$$

$$= \frac{x^{-n}(ex)^n\left(-2b(-2a^2+b^2)\,\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx^n)) + \sqrt{a^2-b^2}(a(a^2-b^2)(c+d\right.}{a^2(a-b)(a+b)\sqrt{a^2-b^2}den\,(b+a\cos(c+dx}$$

```
[In] Integrate[(e*x)^(-1 + n)/(a + b*Sec[c + d*x^n])^2,x]
```

```
[Out] ((e*x)^n*(-2*b*(-2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x^n)/2])/Sqrt[a^
2 - b^2]]*(b + a*Cos[c + d*x^n]) + Sqrt[a^2 - b^2]*(a*(a^2 - b^2)*(c + d*x^
n)*Cos[c + d*x^n] + b*((a^2 - b^2)*(c + d*x^n) + a*b*Sin[c + d*x^n]))))/(a^
2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d*e*n*x^n*(b + a*Cos[c + d*x^n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.72 (sec) , antiderivative size = 638, normalized size of antiderivative = 4.06

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(-1+n)\left(-i\pi\ \mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)+i\pi\ \mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2+i\pi\ \mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2-i\pi\,\mathrm{csgn}(iex)^3+2\ln(x)+2\ln(e)\right)}{2}}}{a^2 n} + \frac{2ib^2e^n(-1)^{\frac{\mathrm{csgn}}{}}}{}$

```
[In] int((e*x)^(-1+n)/(a+b*sec(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/n*x*exp(1/2*(-1+n)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csgn(I
*e)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x)+2
*ln(e)))+2*I*b^2/a^2/(a^2-b^2)/d/n/(a*exp(2*I*(c+d*x^n))+2*b*exp(I*(c+d*x^n
))+a)*e^n*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(b*exp(1/2*I*(-Pi*n*cs
gn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*n*csgn(I*e)*csgn(I*e*x)^2+Pi*n*csgn(I*x)*c
sgn(I*e*x)^2-Pi*n*csgn(I*e*x)^3-Pi*csgn(I*e)*csgn(I*e*x)^2-Pi*csgn(I*x)*csg
n(I*e*x)^2+Pi*csgn(I*e*x)^3+2*d*x^n+2*c))+exp(1/2*I*Pi*csgn(I*e*x)*(-csgn(I
*e)*csgn(I*x)*n+csgn(I*e)*csgn(I*e*x)*n+csgn(I*x)*csgn(I*e*x)*n-csgn(I*e*x)
^2*n-csgn(I*e)*csgn(I*e*x)-csgn(I*x)*csgn(I*e*x)+csgn(I*e*x)^2))*a)/e+2*I*a
rctan(1/2*(2*a*exp(I*(d*x^n+2*c))+2*exp(I*c)*b)/(a^2*exp(2*I*c)-exp(2*I*c)*
b^2)^(1/2))/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)/d/e*e^n/n/(-a^2+b^2)*(-2*
a^2+b^2)/a^2*b*exp(1/2*I*(-Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*n*csgn(I
*e)*csgn(I*e*x)^2+Pi*n*csgn(I*x)*csgn(I*e*x)^2-Pi*n*csgn(I*e*x)^3+Pi*csgn(I
*e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn(I*e)*csgn(I*e*x)^2-Pi*csgn(I*x)*csgn(I*e*
x)^2+Pi*csgn(I*e*x)^3+2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 628, normalized size of antiderivative = 4.00

$$\int \frac{(ex)^{-1+n}}{(a+b\sec(c+dx^n))^2}\,dx$$

$$= \left[\frac{2\,(a^5-2\,a^3b^2+ab^4)de^{n-1}x^n\cos(dx^n+c)+2\,(a^4b-2\,a^2b^3+b^5)de^{n-1}x^n+2\,(a^3b^2-ab^4)e^{n-1}\sin(dx^n}{2\,((a}\right.$$

```
[In] integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*e^(n - 1)*x^n*cos(d*x^n + c) + 2*(a^4*b
 - 2*a^2*b^3 + b^5)*d*e^(n - 1)*x^n + 2*(a^3*b^2 - a*b^4)*e^(n - 1)*sin(d*x
^n + c) + ((2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*e^(n - 1)*cos(d*x^n + c) + (2*
a^2*b^2 - b^4)*sqrt(a^2 - b^2)*e^(n - 1))*log((2*a*b*cos(d*x^n + c) - (a^2
- 2*b^2)*cos(d*x^n + c)^2 + 2*a^2 - b^2 - 2*(sqrt(a^2 - b^2)*b*cos(d*x^n +
c) + sqrt(a^2 - b^2)*a)*sin(d*x^n + c))/(a^2*cos(d*x^n + c)^2 + 2*a*b*cos(d
*x^n + c) + b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*cos(d*x^n + c) + (a^6*b
 - 2*a^4*b^3 + a^2*b^5)*d*n), ((a^5 - 2*a^3*b^2 + a*b^4)*d*e^(n - 1)*x^n*co
s(d*x^n + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*e^(n - 1)*x^n + (a^3*b^2 - a*b^4
)*e^(n - 1)*sin(d*x^n + c) - ((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*e^(n - 1)*
cos(d*x^n + c) + (2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*e^(n - 1))*arctan(-(sqr
t(-a^2 + b^2)*b*cos(d*x^n + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*sin(d*x^n
 + c))))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*cos(d*x^n + c) + (a^6*b - 2*a^4*b
^3 + a^2*b^5)*d*n)]
```

Sympy [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\sec(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(a+b\sec(c+dx^n))^2}\,dx$$

```
[In] integrate((e*x)**(-1+n)/(a+b*sec(c+d*x**n))**2,x)
```

```
[Out] Integral((e*x)**(n - 1)/(a + b*sec(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\sec(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(b\sec(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")
```

```
[Out] ((a^4 - a^2*b^2)*d*e^n*x^n*cos(2*d*x^n + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*e^n*x
^n*cos(d*x^n + c)^2 + (a^4 - a^2*b^2)*d*e^n*x^n*sin(2*d*x^n + 2*c)^2 + 2*a*
b^3*e^n*sin(d*x^n + c) + 4*(a^2*b^2 - b^4)*d*e^n*x^n*sin(d*x^n + c)^2 + 4*(
a^3*b - a*b^3)*d*e^n*x^n*cos(d*x^n + c) + (a^4 - a^2*b^2)*d*e^n*x^n - 2*(a*
b^3*e^n*sin(d*x^n + c) - 2*(a^3*b - a*b^3)*d*e^n*x^n*cos(d*x^n + c) - (a^4
- a^2*b^2)*d*e^n*x^n)*cos(2*d*x^n + 2*c) - 2*((2*a^8*b - 3*a^6*b^3 + a^4*b^
5)*d*e^(n + 1)*n*cos(2*d*x^n + 2*c)^2*cos(c) + 4*(2*a^6*b^3 - 3*a^4*b^5 + a
^2*b^7)*d*e^(n + 1)*n*cos(d*x^n + c)^2*cos(c) + (2*a^8*b - 3*a^6*b^3 + a^4*
b^5)*d*e^(n + 1)*n*cos(c)*sin(2*d*x^n + 2*c)^2 + 4*(2*a^7*b^2 - 3*a^5*b^4 +
 a^3*b^6)*d*e^(n + 1)*n*cos(c)*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 4*(2*a^6
```

```
*b^3 - 3*a^4*b^5 + a^2*b^7)*d*e^(n + 1)*n*cos(c)*sin(d*x^n + c)^2 + 4*(2*a^
7*b^2 - 3*a^5*b^4 + a^3*b^6)*d*e^(n + 1)*n*cos(d*x^n + c)*cos(c) + (2*a^8*b
 - 3*a^6*b^3 + a^4*b^5)*d*e^(n + 1)*n*cos(c) + 2*(2*(2*a^7*b^2 - 3*a^5*b^4
+ a^3*b^6)*d*e^(n + 1)*n*cos(d*x^n + c)*cos(c) + (2*a^8*b - 3*a^6*b^3 + a^4
*b^5)*d*e^(n + 1)*n*cos(c))*cos(2*d*x^n + 2*c))*integrate((a^3*x^n*cos(2*d*
x^n + 2*c)*cos(d*x^n) + a^3*x^n*sin(2*d*x^n + 2*c)*sin(d*x^n) + 2*(a^2*b -
b^3)*x^n*cos(d*x^n)^2*cos(c) + 2*(a^2*b - b^3)*x^n*cos(c)*sin(d*x^n)^2 + (a
^3 - a*b^2)*x^n*cos(d*x^n) - (a*b^2*x^n*cos(d*x^n)*cos(2*c) + a*b^2*x^n*sin
(d*x^n)*sin(2*c))*cos(2*d*x^n) - (a*b^2*x^n*cos(2*c)*sin(d*x^n) - a*b^2*x^n
*cos(d*x^n)*sin(2*c))*sin(2*d*x^n))/(a^8*e*x*cos(2*d*x^n + 2*c)^2 + a^8*e*x
*sin(2*d*x^n + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*e*x*cos(2
*d*x^n)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*
b^4 + a^2*b^6)*sin(c)^2)*e*x*cos(d*x^n)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)
*e*x*cos(d*x^n)*cos(c) + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*e*x*sin(
2*d*x^n)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4
*b^4 + a^2*b^6)*sin(c)^2)*e*x*sin(d*x^n)^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5
)*e*x*sin(d*x^n)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*e*x - 2*(2*((a^5*b^3
- a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*e*x*cos(d
*x^n) + (a^6*b^2 - a^4*b^4)*e*x*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*si
n(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*e*x*sin(d*x^n))*cos(2*d*x^n)
- 2*(a^6*b^2*e*x*cos(2*d*x^n)*cos(2*c) - a^6*b^2*e*x*sin(2*d*x^n)*sin(2*c)
- 2*(a^7*b - a^5*b^3)*e*x*cos(d*x^n)*cos(c) + 2*(a^7*b - a^5*b^3)*e*x*sin(d
*x^n)*sin(c) - (a^8 - a^6*b^2)*e*x)*cos(2*d*x^n + 2*c) + 2*(2*((a^5*b^3 - a
^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*e*x*cos(d*x^
n) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*
sin(c))*e*x*sin(d*x^n) + (a^6*b^2 - a^4*b^4)*e*x*sin(2*c))*sin(2*d*x^n) - 2
*(a^6*b^2*e*x*cos(2*c)*sin(2*d*x^n) + a^6*b^2*e*x*cos(2*d*x^n)*sin(2*c) - 2
*(a^7*b - a^5*b^3)*e*x*cos(c)*sin(d*x^n) - 2*(a^7*b - a^5*b^3)*e*x*cos(d*x^
n)*sin(c))*sin(2*d*x^n + 2*c)), x) - 2*((2*a^8*b - 3*a^6*b^3 + a^4*b^5)*d*e
^(n + 1)*n*cos(2*d*x^n + 2*c)^2*sin(c) + 4*(2*a^6*b^3 - 3*a^4*b^5 + a^2*b^7
)*d*e^(n + 1)*n*cos(d*x^n + c)^2*sin(c) + (2*a^8*b - 3*a^6*b^3 + a^4*b^5)*d
*e^(n + 1)*n*sin(2*d*x^n + 2*c)^2*sin(c) + 4*(2*a^7*b^2 - 3*a^5*b^4 + a^3*b
^6)*d*e^(n + 1)*n*sin(2*d*x^n + 2*c)*sin(d*x^n + c)*sin(c) + 4*(2*a^6*b^3 -
 3*a^4*b^5 + a^2*b^7)*d*e^(n + 1)*n*sin(d*x^n + c)^2*sin(c) + 4*(2*a^7*b^2
- 3*a^5*b^4 + a^3*b^6)*d*e^(n + 1)*n*cos(d*x^n + c)*sin(c) + (2*a^8*b - 3*a
^6*b^3 + a^4*b^5)*d*e^(n + 1)*n*sin(c) + 2*(2*(2*a^7*b^2 - 3*a^5*b^4 + a^3*
b^6)*d*e^(n + 1)*n*cos(d*x^n + c)*sin(c) + (2*a^8*b - 3*a^6*b^3 + a^4*b^5)*
d*e^(n + 1)*n*sin(c))*cos(2*d*x^n + 2*c))*integrate((a^3*x^n*cos(d*x^n)*sin
(2*d*x^n + 2*c) - a^3*x^n*cos(2*d*x^n + 2*c)*sin(d*x^n) + 2*(a^2*b - b^3)*x
^n*cos(d*x^n)^2*sin(c) + 2*(a^2*b - b^3)*x^n*sin(d*x^n)^2*sin(c) - (a^3 - a
*b^2)*x^n*sin(d*x^n) + (a*b^2*x^n*cos(2*c)*sin(d*x^n) - a*b^2*x^n*cos(d*x^n
)*sin(2*c))*cos(2*d*x^n) - (a*b^2*x^n*cos(d*x^n)*cos(2*c) + a*b^2*x^n*sin(d
*x^n)*sin(2*c))*sin(2*d*x^n))/(a^8*e*x*cos(2*d*x^n + 2*c)^2 + a^8*e*x*sin(2
*d*x^n + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*e*x*cos(2*d*x^n
)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 +
```

```
a^2*b^6)*sin(c)^2)*e*x*cos(d*x^n)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*e*x*c
os(d*x^n)*cos(c) + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*e*x*sin(2*d*x^
n)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 +
 a^2*b^6)*sin(c)^2)*e*x*sin(d*x^n)^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*e*x*
sin(d*x^n)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*e*x - 2*(2*((a^5*b^3 - a^3*
b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*e*x*cos(d*x^n)
+ (a^6*b^2 - a^4*b^4)*e*x*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c)
 - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*e*x*sin(d*x^n))*cos(2*d*x^n) - 2*(a
^6*b^2*e*x*cos(2*d*x^n)*cos(2*c) - a^6*b^2*e*x*sin(2*d*x^n)*sin(2*c) - 2*(a
^7*b - a^5*b^3)*e*x*cos(d*x^n)*cos(c) + 2*(a^7*b - a^5*b^3)*e*x*sin(d*x^n)*
sin(c) - (a^8 - a^6*b^2)*e*x)*cos(2*d*x^n + 2*c) + 2*(2*((a^5*b^3 - a^3*b^5
)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*e*x*cos(d*x^n) - 2
*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c)
)*e*x*sin(d*x^n) + (a^6*b^2 - a^4*b^4)*e*x*sin(2*c))*sin(2*d*x^n) - 2*(a^6*
b^2*e*x*cos(2*c)*sin(2*d*x^n) + a^6*b^2*e*x*cos(2*d*x^n)*sin(2*c) - 2*(a^7*
b - a^5*b^3)*e*x*cos(c)*sin(d*x^n) - 2*(a^7*b - a^5*b^3)*e*x*cos(d*x^n)*sin
(c))*sin(2*d*x^n + 2*c)), x) + 2*(a*b^3*e^n*cos(d*x^n + c) + a^2*b^2*e^n +
2*(a^3*b - a*b^3)*d*e^n*x^n*sin(d*x^n + c))*sin(2*d*x^n + 2*c))/((a^6 - a^4
*b^2)*d*e*n*cos(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*cos(d*x^n +
c)^2 + (a^6 - a^4*b^2)*d*e*n*sin(2*d*x^n + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*e
*n*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 4*(a^4*b^2 - a^2*b^4)*d*e*n*sin(d*x^
n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n
 + 2*(2*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n)*cos
(2*d*x^n + 2*c))
```

Giac [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\sec(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(b\sec(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((e*x)^(n - 1)/(b*sec(d*x^n + c) + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 18.23 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.94

$$\int \frac{(ex)^{-1+n}}{(a+b\sec(c+dx^n))^2}\,dx = \frac{\frac{b^2\,x\,(e\,x)^{n-1}\,2\mathrm{i}}{a\,d\,n\,x^n\,(a^2-b^2)}+\frac{b^3\,x\,\mathrm{e}^{c\,1\mathrm{i}+d\,x^n\,1\mathrm{i}}\,(e\,x)^{n-1}\,2\mathrm{i}}{a^2\,d\,n\,x^n\,(a^2-b^2)}}{a+a\,\mathrm{e}^{c\,2\mathrm{i}+d\,x^n\,2\mathrm{i}}+2\,b\,\mathrm{e}^{c\,1\mathrm{i}+d\,x^n\,1\mathrm{i}}}+\frac{x\,(e\,x)^{n-1}}{a^2\,n}$$
$$+\frac{b\,x\,\ln\left(-2\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\left(b^3\,x\,(e\,x)^{n-1}-2\,a^2\,b\,x\,(e\,x)^{n-1}\right)-\frac{b\,x\,(a^4-a^2\,b^2)\left(a+b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\right)(e\,x)^{n-1}\,(2\,a^2-b^2)\,2\mathrm{i}}{a^2\,(a+b)^{3/2}\,(a-b)^{3/2}}\right)(e}{a^2\,d\,n\,x^n\,(a+b)^{3/2}\,(a-b)^{3/2}}$$
$$-\frac{b\,x\,\ln\left(-2\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\left(b^3\,x\,(e\,x)^{n-1}-2\,a^2\,b\,x\,(e\,x)^{n-1}\right)+\frac{b\,x\,(a^4-a^2\,b^2)\left(a+b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\right)(e\,x)^{n-1}\,(2\,a^2-b^2)\,2\mathrm{i}}{a^2\,(a+b)^{3/2}\,(a-b)^{3/2}}\right)(e}{a^2\,d\,n\,x^n\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

```
[In] int((e*x)^(n - 1)/(a + b/cos(c + d*x^n))^2,x)
```

```
[Out] ((b^2*x*(e*x)^(n - 1)*2i)/(a*d*n*x^n*(a^2 - b^2)) + (b^3*x*exp(c*1i + d*x^n
*1i)*(e*x)^(n - 1)*2i)/(a^2*d*n*x^n*(a^2 - b^2)))/(a + a*exp(c*2i + d*x^n*2
i) + 2*b*exp(c*1i + d*x^n*1i)) + (x*(e*x)^(n - 1))/(a^2*n) + (b*x*log(- 2*e
xp(c*1i)*exp(d*x^n*1i)*(b^3*x*(e*x)^(n - 1) - 2*a^2*b*x*(e*x)^(n - 1)) - (b
*x*(a^4 - a^2*b^2)*(a + b*exp(c*1i)*exp(d*x^n*1i))*(e*x)^(n - 1)*(2*a^2 - b
^2)*2i)/(a^2*(a + b)^(3/2)*(a - b)^(3/2)))*(e*x)^(n - 1)*(2*a^2 - b^2))/(a^
2*d*n*x^n*(a + b)^(3/2)*(a - b)^(3/2)) - (b*x*log((b*x*(a^4 - a^2*b^2)*(a +
 b*exp(c*1i)*exp(d*x^n*1i))*(e*x)^(n - 1)*(2*a^2 - b^2)*2i)/(a^2*(a + b)^(3
/2)*(a - b)^(3/2)) - 2*exp(c*1i)*exp(d*x^n*1i)*(b^3*x*(e*x)^(n - 1) - 2*a^2
*b*x*(e*x)^(n - 1)))*(e*x)^(n - 1)*(2*a^2 - b^2))/(a^2*d*n*x^n*(a + b)^(3/2
)*(a - b)^(3/2))
```

3.82 $\int \frac{(ex)^{-1+2n}}{(a+b\sec(c+dx^n))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 757

$$\begin{aligned}
\int \frac{(ex)^{-1+2n}}{(a+b\sec(c+dx^n))^2}\,dx &= \frac{(ex)^{2n}}{2a^2en} - \frac{ib^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den} \\
&+ \frac{2ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} \\
&+ \frac{ib^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den} \\
&- \frac{2ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} \\
&+ \frac{b^2x^{-2n}(ex)^{2n}\log(b+a\cos(c+dx^n))}{a^2(a^2-b^2)d^2en} \\
&- \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2en} \\
&+ \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} \\
&+ \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2en} \\
&- \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} \\
&+ \frac{b^2x^{-n}(ex)^{2n}\sin(c+dx^n)}{a(a^2-b^2)den(b+a\cos(c+dx^n))}
\end{aligned}$$

```
[Out] 1/2*(e*x)^(2*n)/a^2/e/n+b^2*(e*x)^(2*n)*ln(b+a*cos(c+d*x^n))/a^2/(a^2-b^2)/
d^2/e/n/(x^(2*n))-I*b^3*(e*x)^(2*n)*ln(1+a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(
1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)+I*b^3*(e*x)^(2*n)*ln(1+a*exp(I*(c+d
*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)-b^3*(e*x)^(2*
n)*polylog(2,-a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)
/d^2/e/n/(x^(2*n))+b^3*(e*x)^(2*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b+(-a^2+b
^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))+b^2*(e*x)^(2*n)*sin(c+d*
x^n)/a/(a^2-b^2)/d/e/n/(x^n)/(b+a*cos(c+d*x^n))+2*I*b*(e*x)^(2*n)*ln(1+a*ex
p(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/d/e/n/(x^n)/(-a^2+b^2)^(1/2)-2*I*b
*(e*x)^(2*n)*ln(1+a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/d/e/n/(x^n)/
(-a^2+b^2)^(1/2)+2*b*(e*x)^(2*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b-(-a^2+b^2
)^(1/2)))/a^2/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)-2*b*(e*x)^(2*n)*polylog(2,
-a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(2*n))/(-a^2+b^2)^
(1/2)
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules

used = {4293, 4289, 4276, 3405, 3402, 2296, 2221, 2317, 2438, 2747, 31}

$$\int \frac{(ex)^{-1+2n}}{(a+b\sec(c+dx^n))^2}\,dx = \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}} - \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}} + \frac{b^2x^{-2n}(ex)^{2n}\log(a\cos(c+dx^n)+b)}{a^2d^2en(a^2-b^2)} + \frac{2ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{a^2den\sqrt{b^2-a^2}} - \frac{2ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{\sqrt{b^2-a^2}+b}\right)}{a^2den\sqrt{b^2-a^2}} + \frac{b^2x^{-n}(ex)^{2n}\sin(c+dx^n)}{aden(a^2-b^2)(a\cos(c+dx^n)+b)} - \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2en(b^2-a^2)^{3/2}} + \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2en(b^2-a^2)^{3/2}} - \frac{ib^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{a^2den(b^2-a^2)^{3/2}} + \frac{ib^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{\sqrt{b^2-a^2}+b}\right)}{a^2den(b^2-a^2)^{3/2}} + \frac{(ex)^{2n}}{2a^2en}$$

```
[In] Int[(e*x)^(-1 + 2*n)/(a + b*Sec[c + d*x^n])^2,x]
```

```
[Out] (e*x)^(2*n)/(2*a^2*e*n) - (I*b^3*(e*x)^(2*n)*Log[1 + (a*E^(I*(c + d*x^n)))/
(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d*e*n*x^n) + ((2*I)*b*(e*x
)^(2*n)*Log[1 + (a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a
^2 + b^2]*d*e*n*x^n) + (I*b^3*(e*x)^(2*n)*Log[1 + (a*E^(I*(c + d*x^n)))/(b
+ Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d*e*n*x^n) - ((2*I)*b*(e*x)^(
2*n)*Log[1 + (a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2
+ b^2]*d*e*n*x^n) + (b^2*(e*x)^(2*n)*Log[b + a*Cos[c + d*x^n]])/(a^2*(a^2 -
 b^2)*d^2*e*n*x^(2*n)) - (b^3*(e*x)^(2*n)*PolyLog[2, -((a*E^(I*(c + d*x^n))
)/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2*e*n*x^(2*n)) + (2*b
*(e*x)^(2*n)*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2]))])/(
a^2*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) + (b^3*(e*x)^(2*n)*PolyLog[2, -((a*E^
(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2*e*n*
x^(2*n)) - (2*b*(e*x)^(2*n)*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(b + Sqrt[-a
```

```
^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) + (b^2*(e*x)^(2*n)*Sin
[c + d*x^n])/(a*(a^2 - b^2)*d*e*n*x^n*(b + a*Cos[c + d*x^n]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
```

```
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4293

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(x^{-2n}(ex)^{2n}\right)\int \frac{x^{-1+2n}}{(a+b\sec(c+dx^n))^2}\,dx}{e}$$

$$= \frac{\left(x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{x}{(a+b\sec(c+dx))^2}\,dx, x, x^n\right)}{en}$$

$$= \frac{\left(x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \left(\frac{x}{a^2}+\frac{b^2x}{a^2(b+a\cos(c+dx))^2}-\frac{2bx}{a^2(b+a\cos(c+dx))}\right)dx, x, x^n\right)}{en}$$

$$= \frac{(ex)^{2n}}{2a^2en} - \frac{(2bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{x}{b+a\cos(c+dx)}\,dx, x, x^n\right)}{a^2en} + \frac{(b^2x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{x}{(b+a\cos(c+dx))^2}\,dx, x, x^n\right)}{a^2en}$$

$$= \frac{(ex)^{2n}}{2a^2en} + \frac{b^2x^{-n}(ex)^{2n}\sin(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cos(c+dx^n))} - \frac{(4bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, x^n\right)}{a^2en} - \frac{(b^3x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{x}{b+a\cos(c+dx)}\,dx, x, x^n\right)}{a^2(a^2-b^2)\,en} - \frac{(b^2x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{\sin(c+dx)}{b+a\cos(c+dx)}\,dx, x, x^n\right)}{a(a^2-b^2)\,den}$$

$$= \frac{(ex)^{2n}}{2a^2en} + \frac{b^2x^{-n}(ex)^{2n}\sin(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cos(c+dx^n))} - \frac{(2b^3x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, x^n\right)}{a^2(a^2-b^2)\,en} - \frac{(4bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}\,en} + \frac{(4bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}\,en} + \frac{(b^2x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{1}{b+x}\,dx, x, a\cos(c+dx^n)\right)}{a^2(a^2-b^2)\,d^2en}$$

$$= \frac{(ex)^{2n}}{2a^2en} + \frac{2ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{2ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{b^2x^{-2n}(ex)^{2n}\log\left(b+a\cos\left(c+dx^n\right)\right)}{a^2\left(a^2-b^2\right)d^2en} + \frac{b^2x^{-n}(ex)^{2n}\sin\left(c+dx^n\right)}{a\left(a^2-b^2\right)den\left(b+a\cos\left(c+dx^n\right)\right)} - \frac{\left(2b^3x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\frac{e^{i(c+dx)}x}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,x^n\right)}{a\left(a^2-b^2\right)\sqrt{-a^2+b^2}en} + \frac{\left(2b^3x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\frac{e^{i(c+dx)}x}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,x^n\right)}{a\left(a^2-b^2\right)\sqrt{-a^2+b^2}en} - \frac{\left(2ibx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{\left(2ibx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}den}$$

$$= \frac{(ex)^{2n}}{2a^2en} - \frac{ib^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} + \frac{2ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{ib^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} - \frac{2ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{b^2x^{-2n}(ex)^{2n}\log\left(b+a\cos\left(c+dx^n\right)\right)}{a^2\left(a^2-b^2\right)d^2en} + \frac{b^2x^{-n}(ex)^{2n}\sin\left(c+dx^n\right)}{a\left(a^2-b^2\right)den\left(b+a\cos\left(c+dx^n\right)\right)} - \frac{\left(2bx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{\left(2bx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{a^2\sqrt{-a^2+b^2}d^2en} - \frac{\left(ib^3x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\left(a^2-b^2\right)\sqrt{-a^2+b^2}den} + \frac{\left(ib^3x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\left(a^2-b^2\right)\sqrt{-a^2+b^2}den}$$

$$= \frac{(ex)^{2n}}{2a^2en} - \frac{ib^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} + \frac{2ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$
$$+ \frac{ib^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} - \frac{2ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$
$$+ \frac{b^2x^{-2n}(ex)^{2n}\log\left(b+a\cos\left(c+dx^n\right)\right)}{a^2\left(a^2-b^2\right)d^2en} + \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$
$$- \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{b^2x^{-n}(ex)^{2n}\sin\left(c+dx^n\right)}{a\left(a^2-b^2\right)den\left(b+a\cos\left(c+dx^n\right)\right)}$$
$$- \frac{\left(b^3x^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a^2\left(a^2-b^2\right)\sqrt{-a^2+b^2}d^2en}$$
$$+ \frac{\left(b^3x^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a^2\left(a^2-b^2\right)\sqrt{-a^2+b^2}d^2en}$$

$$= \frac{(ex)^{2n}}{2a^2en} - \frac{ib^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} + \frac{2ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$
$$+ \frac{ib^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} - \frac{2ibx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$
$$+ \frac{b^2x^{-2n}(ex)^{2n}\log\left(b+a\cos\left(c+dx^n\right)\right)}{a^2\left(a^2-b^2\right)d^2en} - \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2en}$$
$$+ \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2en}$$
$$- \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{b^2x^{-n}(ex)^{2n}\sin\left(c+dx^n\right)}{a\left(a^2-b^2\right)den\left(b+a\cos\left(c+dx^n\right)\right)}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2450 vs. 2(757) = 1514.

Time = 10.69 (sec) , antiderivative size = 2450, normalized size of antiderivative = 3.24

$$\int \frac{(ex)^{-1+2n}}{(a+b\sec(c+dx^n))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(e*x)^(-1 + 2*n)/(a + b*Sec[c + d*x^n])^2,x]
```

```
[Out] (-2*b*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*(b + a*Cos[c + d*x^n])^2*(2*(c + d*x^n)*
ArcTanh[((a + b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] - 2*(c + ArcCos[-(b/a
)])*ArcTanh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + (ArcCos[-(b/a)]
 - (2*I)*(ArcTanh[((a + b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] - ArcTanh[(
(a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[Sqrt[a^2 - b^2]/(Sqrt[2]
*Sqrt[a]*E^((I/2)*(c + d*x^n))*Sqrt[b + a*Cos[c + d*x^n]])] + (ArcCos[-(b/a
)] + (2*I)*(ArcTanh[((a + b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] - ArcTanh
[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[(Sqrt[a^2 - b^2]*E^((I
/2)*(c + d*x^n)))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Cos[c + d*x^n]])] - (ArcCos[-
(b/a)] + (2*I)*ArcTanh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[1
 - ((b - I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))/(
a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))] + (-ArcCos[-(b/a)] + (2*I)
*ArcTanh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[1 - ((b + I*Sqr
t[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))/(a*(a + b + Sqr
t[a^2 - b^2]*Tan[(c + d*x^n)/2]))] + I*(PolyLog[2, ((b - I*Sqrt[a^2 - b^2])
*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*
Tan[(c + d*x^n)/2]))] - PolyLog[2, ((b + I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a
^2 - b^2]*Tan[(c + d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/
2]))]))*Sec[c + d*x^n]^2)/((a^2 - b^2)^(3/2)*d^2*n*(a + b*Sec[c + d*x^n])^2
) + (b^3*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*(b + a*Cos[c + d*x^n])^2*(2*(c + d*x^
n)*ArcTanh[((a + b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] - 2*(c + ArcCos[-(
b/a)])*ArcTanh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + (ArcCos[-(b/
a)] - (2*I)*(ArcTanh[((a + b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] - ArcTan
h[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[Sqrt[a^2 - b^2]/(Sqrt
[2]*Sqrt[a]*E^((I/2)*(c + d*x^n))*Sqrt[b + a*Cos[c + d*x^n]])] + (ArcCos[-(
b/a)] + (2*I)*(ArcTanh[((a + b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] - ArcT
anh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[(Sqrt[a^2 - b^2]*E^
((I/2)*(c + d*x^n)))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Cos[c + d*x^n]])] - (ArcCo
s[-(b/a)] + (2*I)*ArcTanh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Lo
g[1 - ((b - I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2])
)/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))] + (-ArcCos[-(b/a)] + (2
*I)*ArcTanh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[1 - ((b + I*
Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))/(a*(a + b +
Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))] + I*(PolyLog[2, ((b - I*Sqrt[a^2 - b^
```

```
2])*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^
2]*Tan[(c + d*x^n)/2]))] - PolyLog[2, ((b + I*Sqrt[a^2 - b^2])*(a + b - Sqr
t[a^2 - b^2]*Tan[(c + d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^
n)/2]))]))*Sec[c + d*x^n]^2)/(a^2*(a^2 - b^2)^(3/2)*d^2*n*(a + b*Sec[c + d*
x^n])^2) + (x^(1 - n)*(e*x)^(-1 + 2*n)*(b + a*Cos[c + d*x^n])^2*Sec[c + d*x
^n]^2*(a^2*d*x^n*Cos[c] - b^2*d*x^n*Cos[c] + 2*b^2*Sin[c]))/(2*a^2*(a - b)*
(a + b)*d*n*(a + b*Sec[c + d*x^n])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[
c/2])) + (b^2*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*(b + a*Cos[c + d*x^n])^2*Sec[c]*
Sec[c + d*x^n]^2*(a*Cos[c]*Log[b + a*Cos[c]*Cos[d*x^n] - a*Sin[c]*Sin[d*x^n
]] + a*d*x^n*Sin[c] - ((2*I)*a*b*ArcTan[((-I)*a*Sin[c] - I*(-b + a*Cos[c])*
Tan[(d*x^n)/2])/Sqrt[-b^2 + a^2*Cos[c]^2 + a^2*Sin[c]^2]]*Sin[c])/Sqrt[-b^2
 + a^2*Cos[c]^2 + a^2*Sin[c]^2]))/(a*(a^2 - b^2)*d^2*n*(a + b*Sec[c + d*x^n
])^2*(a^2*Cos[c]^2 + a^2*Sin[c]^2)) + (b^2*x^(1 - n)*(e*x)^(-1 + 2*n)*(b +
a*Cos[c + d*x^n])*Sec[c + d*x^n]^2*(b*Sin[c] - a*Sin[d*x^n]))/(a^2*(-a + b)
*(a + b)*d*n*(a + b*Sec[c + d*x^n])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin
[c/2])) + (b^2*x^(1 - n)*(e*x)^(-1 + 2*n)*(b + a*Cos[c + d*x^n])^2*Sec[c +
d*x^n]^2*Tan[c])/(a^2*(-a^2 + b^2)*d*n*(a + b*Sec[c + d*x^n])^2) - ((2*I)*b
^3*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*ArcTan[(b + a*Cos[c + d*x^n] + I*a*Sin[c +
d*x^n])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x^n])^2*Sec[c + d*x^n]^2*Tan[c])/
(a^2*(a^2 - b^2)^(3/2)*d^2*n*(a + b*Sec[c + d*x^n])^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.74 (sec) , antiderivative size = 3010, normalized size of antiderivative = 3.98

method	result	size
risch	Expression too large to display	3010

```
[In] int((e*x)^(2*n-1)/(a+b*sec(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2/n*x*exp(1/2*(2*n-1)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csg
n(I*e)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x
)+2*ln(e)))+2*I*b^2/a^2/(a^2-b^2)/d/n*x^n/(a*exp(2*I*(c+d*x^n))+2*b*exp(I*(
c+d*x^n))+a)*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(e^n)^2*(b*exp(1/2*
I*(-2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+2*Pi*n*csgn(I*e)*csgn(I*e*x)^2+2
*Pi*n*csgn(I*x)*csgn(I*e*x)^2-2*Pi*n*csgn(I*e*x)^3-Pi*csgn(I*e)*csgn(I*e*x)
^2-Pi*csgn(I*x)*csgn(I*e*x)^2+Pi*csgn(I*e*x)^3+2*d*x^n+2*c))+exp(1/2*I*Pi*c
sgn(I*e*x)*(-2*csgn(I*e)*csgn(I*x)*n+2*csgn(I*e)*csgn(I*e*x)*n+2*csgn(I*x)*
csgn(I*e*x)*n-2*csgn(I*e*x)^2*n-csgn(I*e)*csgn(I*e*x)-csgn(I*x)*csgn(I*e*x)
+csgn(I*e*x)^2))*a)/e-2*I*b/(a^2-b^2)^2/d*(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(
1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi*n*
csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x)^3
-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)
*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*x^n*ln((-a*exp(I*(d*x^n+2*c))-exp(I*c
```

)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))/(-exp(I*c)*b+(exp(2*I*c)*b^2-a^2
*exp(2*I*c))^(1/2)))+I*b^3/a^2/(a^2-b^2)^2/d*(exp(2*I*c)*b^2-a^2*exp(2*I*c)
)^(1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi
*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x
)^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I
*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*x^n*ln((-a*exp(I*(d*x^n+2*c))-exp(
I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))/(-exp(I*c)*b+(exp(2*I*c)*b^2-
a^2*exp(2*I*c))^(1/2)))+2*I*b/(a^2-b^2)^2/d*(exp(2*I*c)*b^2-a^2*exp(2*I*c))
^(1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi*
n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x)
^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*
x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*x^n*ln((a*exp(I*(d*x^n+2*c))+exp(I*
c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2
*exp(2*I*c))^(1/2)))-I*b^3/a^2/(a^2-b^2)^2/d*(exp(2*I*c)*b^2-a^2*exp(2*I*c)
)^(1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi
*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x
)^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I
*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*x^n*ln((a*exp(I*(d*x^n+2*c))+exp(I
*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))/(exp(I*c)*b+(exp(2*I*c)*b^2-a^
2*exp(2*I*c))^(1/2)))-2*b/(a^2-b^2)^2/d^2*(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(
1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi*n*
csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x)^3
-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)
*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*dilog(-a/(-exp(I*c)*b+(exp(2*I*c)*b^2
-a^2*exp(2*I*c))^(1/2))*exp(I*(d*x^n+2*c))-1/(-exp(I*c)*b+(exp(2*I*c)*b^2-a
^2*exp(2*I*c))^(1/2))*exp(I*c)*b+1/(-exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I
c))^(1/2))(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))+b^3/a^2/(a^2-b^2)^2/d^2*
(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I
*e)*csgn(I*x)*csgn(I*e*x)-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*c
sgn(I*e*x)^2+2*Pi*n*csgn(I*e*x)^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csg
n(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*dilo
g(-a/(-exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*exp(I*(d*x^n+2*c))
-1/(-exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*exp(I*c)*b+1/(-exp(I
*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*(exp(2*I*c)*b^2-a^2*exp(2*I*c)
)^(1/2))+2*b/(a^2-b^2)^2/d^2*(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2)/n/e*(e^n
)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi*n*csgn(I*e)*csg
n(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x)^3-Pi*csgn(I*e)
*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^
2-Pi*csgn(I*e*x)^3+2*c))*dilog(a/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c)
)^(1/2))*exp(I*(d*x^n+2*c))+1/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(
1/2))*exp(I*c)*b+1/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*(exp(
2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))-b^3/a^2/(a^2-b^2)^2/d^2*(exp(2*I*c)*b^2-a
^2*exp(2*I*c))^(1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csg
n(I*e*x)-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi
*n*csgn(I*e*x)^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x

```
)^2+Pi*csgn(I*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*dilog(a/(exp(I*c)*b+(
exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*exp(I*(d*x^n+2*c))+1/(exp(I*c)*b+(exp
(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*exp(I*c)*b+1/(exp(I*c)*b+(exp(2*I*c)*b^2
-a^2*exp(2*I*c))^(1/2))*(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))-2*b^2/a^2/(a
^2-b^2)/d^2*ln(exp(I*x^n*d))/e*(e^n)^2/n*exp(1/2*I*csgn(I*e*x)*Pi*(2*n-1)*(
csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))+b^2/a^2/(a^2-b^2)/d^2*ln(a
*exp(2*I*(c+d*x^n))+2*b*exp(I*(c+d*x^n))+a)/e*(e^n)^2/n*exp(1/2*I*csgn(I*e*
x)*Pi*(2*n-1)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2503 vs. 2(705) = 1410.

Time = 0.57 (sec) , antiderivative size = 2503, normalized size of antiderivative = 3.31

$$\int \frac{(ex)^{-1+2n}}{(a+b\sec(c+dx^n))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n))^2,x, algorithm="fricas")
[Out] 1/2*((a^5 - 2*a^3*b^2 + a*b^4)*d^2*e^(2*n - 1)*x^(2*n)*cos(d*x^n + c) + (a^
4*b - 2*a^2*b^3 + b^5)*d^2*e^(2*n - 1)*x^(2*n) + 2*(a^3*b^2 - a*b^4)*d*e^(2
*n - 1)*x^n*sin(d*x^n + c) - ((2*a^4*b - a^2*b^3)*e^(2*n - 1)*sqrt(-(a^2 -
b^2)/a^2)*cos(d*x^n + c) + (2*a^3*b^2 - a*b^4)*e^(2*n - 1)*sqrt(-(a^2 - b^2
)/a^2))*dilog(-((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (I*a*sqrt(-
(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) + a)/a + 1) - ((2*a^4*b - a^2*b^3)*e
^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2)*cos(d*x^n + c) + (2*a^3*b^2 - a*b^4)*e^(2
*n - 1)*sqrt(-(a^2 - b^2)/a^2))*dilog(-((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(
d*x^n + c) - (-I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c) + a)/a + 1)
 + ((2*a^4*b - a^2*b^3)*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2)*cos(d*x^n + c) +
 (2*a^3*b^2 - a*b^4)*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2))*dilog(((a*sqrt(-(a
^2 - b^2)/a^2) - b)*cos(d*x^n + c) + (I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin
(d*x^n + c) - a)/a + 1) + ((2*a^4*b - a^2*b^3)*e^(2*n - 1)*sqrt(-(a^2 - b^2
)/a^2)*cos(d*x^n + c) + (2*a^3*b^2 - a*b^4)*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a
^2))*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) + (-I*a*sqrt(-(a^
2 - b^2)/a^2) + I*b)*sin(d*x^n + c) - a)/a + 1) + ((a^3*b^2 - a*b^4 - I*(2*
a^4*b - a^2*b^3)*c*sqrt(-(a^2 - b^2)/a^2))*e^(2*n - 1)*cos(d*x^n + c) + (a^
2*b^3 - b^5 - I*(2*a^3*b^2 - a*b^4)*c*sqrt(-(a^2 - b^2)/a^2))*e^(2*n - 1))*
log(2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2)
+ 2*b) + ((a^3*b^2 - a*b^4 + I*(2*a^4*b - a^2*b^3)*c*sqrt(-(a^2 - b^2)/a^2)
)*e^(2*n - 1)*cos(d*x^n + c) + (a^2*b^3 - b^5 + I*(2*a^3*b^2 - a*b^4)*c*sqr
t(-(a^2 - b^2)/a^2))*e^(2*n - 1))*log(2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n
+ c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) + ((a^3*b^2 - a*b^4 - I*(2*a^4*b -
 a^2*b^3)*c*sqrt(-(a^2 - b^2)/a^2))*e^(2*n - 1)*cos(d*x^n + c) + (a^2*b^3 -
```

```
 b^5 - I*(2*a^3*b^2 - a*b^4)*c*sqrt(-(a^2 - b^2)/a^2))*e^(2*n - 1))*log(-2*
a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b)
 + ((a^3*b^2 - a*b^4 + I*(2*a^4*b - a^2*b^3)*c*sqrt(-(a^2 - b^2)/a^2))*e^(2
*n - 1)*cos(d*x^n + c) + (a^2*b^3 - b^5 + I*(2*a^3*b^2 - a*b^4)*c*sqrt(-(a^
2 - b^2)/a^2))*e^(2*n - 1))*log(-2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n + c)
+ 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) - (-I*(2*a^3*b^2 - a*b^4)*d*e^(2*n - 1)
*x^n*sqrt(-(a^2 - b^2)/a^2) - I*(2*a^3*b^2 - a*b^4)*c*e^(2*n - 1)*sqrt(-(a^
2 - b^2)/a^2) + (-I*(2*a^4*b - a^2*b^3)*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)
/a^2) - I*(2*a^4*b - a^2*b^3)*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2))*cos(d*x
^n + c))*log(((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (I*a*sqrt(-(a
^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) + a)/a) - (I*(2*a^3*b^2 - a*b^4)*d*e^(
2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) + I*(2*a^3*b^2 - a*b^4)*c*e^(2*n - 1)*s
qrt(-(a^2 - b^2)/a^2) + (I*(2*a^4*b - a^2*b^3)*d*e^(2*n - 1)*x^n*sqrt(-(a^2
 - b^2)/a^2) + I*(2*a^4*b - a^2*b^3)*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2))*
cos(d*x^n + c))*log(((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (-I*a*
sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c) + a)/a) - (-I*(2*a^3*b^2 - a*b
^4)*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) - I*(2*a^3*b^2 - a*b^4)*c*e^(2
*n - 1)*sqrt(-(a^2 - b^2)/a^2) + (-I*(2*a^4*b - a^2*b^3)*d*e^(2*n - 1)*x^n*
sqrt(-(a^2 - b^2)/a^2) - I*(2*a^4*b - a^2*b^3)*c*e^(2*n - 1)*sqrt(-(a^2 - b
^2)/a^2))*cos(d*x^n + c))*log(-((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n +
c) + (I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c) - a)/a) - (I*(2*a^3*
b^2 - a*b^4)*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) + I*(2*a^3*b^2 - a*b^
4)*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2) + (I*(2*a^4*b - a^2*b^3)*d*e^(2*n -
 1)*x^n*sqrt(-(a^2 - b^2)/a^2) + I*(2*a^4*b - a^2*b^3)*c*e^(2*n - 1)*sqrt(-
(a^2 - b^2)/a^2))*cos(d*x^n + c))*log(-((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(
d*x^n + c) + (-I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) - a)/a))/((
a^7 - 2*a^5*b^2 + a^3*b^4)*d^2*n*cos(d*x^n + c) + (a^6*b - 2*a^4*b^3 + a^2*
b^5)*d^2*n)
```

Sympy [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\sec(c+dx^n))^2}\,dx=\int \frac{(ex)^{2n-1}}{(a+b\sec(c+dx^n))^2}\,dx$$

```
[In] integrate((e*x)**(-1+2*n)/(a+b*sec(c+d*x**n))**2,x)
```

```
[Out] Integral((e*x)**(2*n - 1)/(a + b*sec(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\sec(c+dx^n))^2}\,dx = \int \frac{(ex)^{2n-1}}{(b\sec(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b^3*e^(2*n)*x^n*sin(d*x^n + c) + (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n)
*cos(2*d*x^n + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*e^(2*n)*x^(2*n)*cos(d*x^n + c)^
2 + (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n)*sin(2*d*x^n + 2*c)^2 + 4*(a^2*b^2 - b
^4)*d*e^(2*n)*x^(2*n)*sin(d*x^n + c)^2 + 4*(a^3*b - a*b^3)*d*e^(2*n)*x^(2*n
)*cos(d*x^n + c) + (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n) - 2*(2*a*b^3*e^(2*n)*x
^n*sin(d*x^n + c) - 2*(a^3*b - a*b^3)*d*e^(2*n)*x^(2*n)*cos(d*x^n + c) - (a
^4 - a^2*b^2)*d*e^(2*n)*x^(2*n))*cos(2*d*x^n + 2*c) + 2*((a^6 - a^4*b^2)*d*
e*n*cos(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*cos(d*x^n + c)^2 + (
a^6 - a^4*b^2)*d*e*n*sin(2*d*x^n + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*sin(2
*d*x^n + 2*c)*sin(d*x^n + c) + 4*(a^4*b^2 - a^2*b^4)*d*e*n*sin(d*x^n + c)^2
 + 4*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n + 2*(2*
(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n)*cos(2*d*x^n
 + 2*c))*integrate(2*(a^2*b^4*e^(2*n)*x^n*cos(2*c)*sin(2*d*x^n) + a^2*b^4*e
^(2*n)*x^n*cos(2*d*x^n)*sin(2*c) - 2*(a^3*b^3 - a*b^5)*e^(2*n)*x^n*cos(c)*s
in(d*x^n) - 2*(a^3*b^3 - a*b^5)*e^(2*n)*x^n*cos(d*x^n)*sin(c) + (a^3*b^3*e^
(2*n)*x^n*sin(d*x^n + c) - (2*a^5*b - a^3*b^3)*d*e^(2*n)*x^(2*n)*cos(d*x^n
+ c))*cos(2*d*x^n + 2*c) - ((2*a^5*b - 3*a^3*b^3 + a*b^5)*d*e^(2*n)*x^(2*n)
 - (a*b^5*e^(2*n)*x^n*sin(2*c) + (2*a^3*b^3 - a*b^5)*d*e^(2*n)*x^(2*n)*cos(
2*c))*cos(2*d*x^n) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*e^(2*n)*x^(2*n)*cos
(c) + (a^2*b^4 - b^6)*e^(2*n)*x^n*sin(c))*cos(d*x^n) - (a*b^5*e^(2*n)*x^n*c
os(2*c) - (2*a^3*b^3 - a*b^5)*d*e^(2*n)*x^(2*n)*sin(2*c))*sin(2*d*x^n) - 2*
((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*e^(2*n)*x^(2*n)*sin(c) - (a^2*b^4 - b^6)*e
^(2*n)*x^n*cos(c))*sin(d*x^n))*cos(d*x^n + c) - (a^3*b^3*e^(2*n)*x^n*cos(d*
x^n + c) + a^4*b^2*e^(2*n)*x^n + (2*a^5*b - a^3*b^3)*d*e^(2*n)*x^(2*n)*sin(
d*x^n + c))*sin(2*d*x^n + 2*c) + ((a^3*b^3 - a*b^5)*e^(2*n)*x^n - (a*b^5*e^
(2*n)*x^n*cos(2*c) - (2*a^3*b^3 - a*b^5)*d*e^(2*n)*x^(2*n)*sin(2*c))*cos(2*
d*x^n) - 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*e^(2*n)*x^(2*n)*sin(c) - (a^2*b
^4 - b^6)*e^(2*n)*x^n*cos(c))*cos(d*x^n) + (a*b^5*e^(2*n)*x^n*sin(2*c) + (2
*a^3*b^3 - a*b^5)*d*e^(2*n)*x^(2*n)*cos(2*c))*sin(2*d*x^n) - 2*((2*a^4*b^2
- 3*a^2*b^4 + b^6)*d*e^(2*n)*x^(2*n)*cos(c) + (a^2*b^4 - b^6)*e^(2*n)*x^n*s
in(c))*sin(d*x^n))*sin(d*x^n + c))/(a^8*d*e*x*cos(2*d*x^n + 2*c)^2 + a^8*d*
e*x*sin(2*d*x^n + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*e*x*
cos(2*d*x^n)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2
*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*e*x*cos(d*x^n)^2 + 4*(a^7*b - 2*a^5*b^3 + a
^3*b^5)*d*e*x*cos(d*x^n)*cos(c) + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)
*d*e*x*sin(2*d*x^n)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*
b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*e*x*sin(d*x^n)^2 - 4*(a^7*b - 2*a^5*
```

```
b^3 + a^3*b^5)*d*e*x*sin(d*x^n)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*e*x
- 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*
sin(c))*d*e*x*cos(d*x^n) + (a^6*b^2 - a^4*b^4)*d*e*x*cos(2*c) + 2*((a^5*b^3
 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*e*x*si
n(d*x^n))*cos(2*d*x^n) - 2*(a^6*b^2*d*e*x*cos(2*d*x^n)*cos(2*c) - a^6*b^2*d
*e*x*sin(2*d*x^n)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*e*x*cos(d*x^n)*cos(c) +
2*(a^7*b - a^5*b^3)*d*e*x*sin(d*x^n)*sin(c) - (a^8 - a^6*b^2)*d*e*x)*cos(2*
d*x^n + 2*c) + 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b
^5)*cos(2*c)*sin(c))*d*e*x*cos(d*x^n) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos
(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*e*x*sin(d*x^n) + (a^6*b^2 - a^
4*b^4)*d*e*x*sin(2*c))*sin(2*d*x^n) - 2*(a^6*b^2*d*e*x*cos(2*c)*sin(2*d*x^n
) + a^6*b^2*d*e*x*cos(2*d*x^n)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*e*x*cos(c)*
sin(d*x^n) - 2*(a^7*b - a^5*b^3)*d*e*x*cos(d*x^n)*sin(c))*sin(2*d*x^n + 2*c
)), x) + 4*(a*b^3*e^(2*n)*x^n*cos(d*x^n + c) + a^2*b^2*e^(2*n)*x^n + (a^3*b
 - a*b^3)*d*e^(2*n)*x^(2*n)*sin(d*x^n + c))*sin(2*d*x^n + 2*c))/((a^6 - a^4
*b^2)*d*e*n*cos(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*cos(d*x^n +
c)^2 + (a^6 - a^4*b^2)*d*e*n*sin(2*d*x^n + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*e
*n*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 4*(a^4*b^2 - a^2*b^4)*d*e*n*sin(d*x^
n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n
 + 2*(2*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n)*cos
(2*d*x^n + 2*c))
```

Giac [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\sec(c+dx^n))^2}\,dx = \int \frac{(ex)^{2n-1}}{(b\sec(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)/(b*sec(d*x^n + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+2n}}{(a+b\sec(c+dx^n))^2}\,dx = \int \frac{(e\,x)^{2n-1}}{\left(a+\frac{b}{\cos(c+d\,x^n)}\right)^2}\,dx$$

```
[In] int((e*x)^(2*n - 1)/(a + b/cos(c + d*x^n))^2,x)
[Out] int((e*x)^(2*n - 1)/(a + b/cos(c + d*x^n))^2, x)
```

3.83 $\int \frac{(ex)^{-1+3n}}{(a+b\sec(c+dx^n))^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 1384

$$\int \frac{(ex)^{-1+3n}}{(a+b\sec(c+dx^n))^2}\,dx = \frac{(ex)^{3n}}{3a^2en} - \frac{ib^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en}$$

$$+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en}$$

$$- \frac{ib^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den}$$

$$+ \frac{2ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,den}$$

$$+ \frac{ib^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den}$$

$$- \frac{2ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,den}$$

$$- \frac{2ib^2x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en}$$

$$- \frac{2ib^2x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en}$$

$$- \frac{2b^3x^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en}$$

$$+ \frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^2en}$$

$$+ \frac{2b^3x^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en}$$

$$- \frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^2en}$$

$$- \frac{2ib^3x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3en}$$

$$+ \frac{4ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^3en}$$

$$+ \frac{2ib^3x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3en}$$

$$- \frac{4ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^3en}$$

$$+ b^2x^{-n}(ex)^{3n}\sin(c+dx^n)$$

```
[Out] 1/3*(e*x)^(3*n)/a^2/e/n-2*I*b^2*(e*x)^(3*n)*polylog(2,-a*exp(I*(c+d*x^n))/(
b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3/e/n/(x^(3*n))+2*b^2*(e*x)^(3*n)*ln(
1+a*exp(I*(c+d*x^n))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2/e/n/(x^(2*n))
+2*b^2*(e*x)^(3*n)*ln(1+a*exp(I*(c+d*x^n))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-
b^2)/d^2/e/n/(x^(2*n))+2*I*b^3*(e*x)^(3*n)*polylog(3,-a*exp(I*(c+d*x^n))/(b
+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3/e/n/(x^(3*n))+I*b^3*(e*x)^(3*n
)*ln(1+a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/
(x^n)-2*I*b^2*(e*x)^(3*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b-I*(a^2-b^2)^(1/2
)))/a^2/(a^2-b^2)/d^3/e/n/(x^(3*n))+2*I*b*(e*x)^(3*n)*ln(1+a*exp(I*(c+d*x^n
))/(b-(-a^2+b^2)^(1/2)))/a^2/d/e/n/(x^n)/(-a^2+b^2)^(1/2)-2*b^3*(e*x)^(3*n)
*polylog(2,-a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d
^2/e/n/(x^(2*n))+2*b^3*(e*x)^(3*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b+(-a^2+b
^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))-I*b^2*(e*x)^(3*n)/a^2/(a
^2-b^2)/d/e/n/(x^n)-4*I*b*(e*x)^(3*n)*polylog(3,-a*exp(I*(c+d*x^n))/(b+(-a^
2+b^2)^(1/2)))/a^2/d^3/e/n/(x^(3*n))/(-a^2+b^2)^(1/2)+b^2*(e*x)^(3*n)*sin(c
+d*x^n)/a/(a^2-b^2)/d/e/n/(x^n)/(b+a*cos(c+d*x^n))-2*I*b^3*(e*x)^(3*n)*poly
log(3,-a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3/e/
n/(x^(3*n))-I*b^3*(e*x)^(3*n)*ln(1+a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))
/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)+4*b*(e*x)^(3*n)*polylog(2,-a*exp(I*(c+d*x
^n))/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)-4*b*(e*x)
^(3*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(
2*n))/(-a^2+b^2)^(1/2)+4*I*b*(e*x)^(3*n)*polylog(3,-a*exp(I*(c+d*x^n))/(b-(
-a^2+b^2)^(1/2)))/a^2/d^3/e/n/(x^(3*n))/(-a^2+b^2)^(1/2)-2*I*b*(e*x)^(3*n)*
ln(1+a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/d/e/n/(x^n)/(-a^2+b^2)^(1
/2)
```

Rubi [A] (verified)

Time = 2.87 (sec) , antiderivative size = 1384, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules

used = {4293, 4289, 4276, 3405, 3402, 2296, 2221, 2611, 2320, 6724, 4618, 2317, 2438}

$$\int \frac{(ex)^{-1+3n}}{(a+b\sec(c+dx^n))^2}\,dx = -\frac{2ib^2(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b-i\sqrt{a^2-b^2}}\right)x^{-3n}}{a^2(a^2-b^2)\,d^3en} - \frac{2ib^2(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b+i\sqrt{a^2-b^2}}\right)x^{-3n}}{a^2(a^2-b^2)\,d^3en} + \frac{4ib(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2\sqrt{b^2-a^2}d^3en} - \frac{2ib^3(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2(b^2-a^2)^{3/2}\,d^3en} - \frac{4ib(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2\sqrt{b^2-a^2}d^3en} + \frac{2ib^3(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2(b^2-a^2)^{3/2}\,d^3en} + \frac{2b^2(ex)^{3n}\log\left(\frac{e^{i(dx^n+c)}a}{b-i\sqrt{a^2-b^2}}+1\right)x^{-2n}}{a^2(a^2-b^2)\,d^2en} + \frac{2b^2(ex)^{3n}\log\left(\frac{e^{i(dx^n+c)}a}{b+i\sqrt{a^2-b^2}}+1\right)x^{-2n}}{a^2(a^2-b^2)\,d^2en} + \frac{4b(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)x^{-2n}}{a^2\sqrt{b^2-a^2}d^2en} - \frac{2b^3(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)x^{-2n}}{a^2(b^2-a^2)^{3/2}\,d^2en} - \frac{4b(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)x^{-2n}}{a^2\sqrt{b^2-a^2}d^2en} + \frac{2b^3(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)x^{-2n}}{a^2(b^2-a^2)^{3/2}\,d^2en} - \frac{ib^2(ex)^{3n}x^{-n}}{a^2(a^2-b^2)\,den} + \frac{2ib(ex)^{3n}\log\left(\frac{e^{i(dx^n+c)}a}{b-\sqrt{b^2-a^2}}+1\right)x^{-n}}{a^2\sqrt{b^2-a^2}den} - \frac{ib^3(ex)^{3n}\log\left(\frac{e^{i(dx^n+c)}a}{b-\sqrt{b^2-a^2}}+1\right)x^{-n}}{a^2(b^2-a^2)^{3/2}\,den} - \frac{2ib(ex)^{3n}\log\left(\frac{e^{i(dx^n+c)}a}{b+\sqrt{b^2-a^2}}+1\right)x^{-n}}{a^2\sqrt{b^2-a^2}den} + \frac{ib^3(ex)^{3n}\log\left(\frac{e^{i(dx^n+c)}a}{b+\sqrt{b^2-a^2}}+1\right)x^{-n}}{a^2(b^2-a^2)^{3/2}\,den} + \frac{b^2(ex)^{3n}\sin(dx^n+c)\,x^{-n}}{a(a^2-b^2)\,den\,(b+a\cos(dx^n+c))} + \frac{(ex)^{3n}}{3a^2en}$$

```
[In] Int[(e*x)^(-1 + 3*n)/(a + b*Sec[c + d*x^n])^2,x]
```

```
[Out] (e*x)^(3*n)/(3*a^2*e*n) - (I*b^2*(e*x)^(3*n))/(a^2*(a^2 - b^2)*d*e*n*x^n) +
 (2*b^2*(e*x)^(3*n)*Log[1 + (a*E^(I*(c + d*x^n)))/(b - I*Sqrt[a^2 - b^2])])
/(a^2*(a^2 - b^2)*d^2*e*n*x^(2*n)) + (2*b^2*(e*x)^(3*n)*Log[1 + (a*E^(I*(c
+ d*x^n)))/(b + I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2*e*n*x^(2*n)) - (I
*b^3*(e*x)^(3*n)*Log[1 + (a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^
2*(-a^2 + b^2)^(3/2)*d*e*n*x^n) + ((2*I)*b*(e*x)^(3*n)*Log[1 + (a*E^(I*(c +
 d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d*e*n*x^n) + (I*b^
3*(e*x)^(3*n)*Log[1 + (a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(
-a^2 + b^2)^(3/2)*d*e*n*x^n) - ((2*I)*b*(e*x)^(3*n)*Log[1 + (a*E^(I*(c + d*
x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d*e*n*x^n) - ((2*I)*b
^2*(e*x)^(3*n)*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(b - I*Sqrt[a^2 - b^2]))]
)/(a^2*(a^2 - b^2)*d^3*e*n*x^(3*n)) - ((2*I)*b^2*(e*x)^(3*n)*PolyLog[2, -((
a*E^(I*(c + d*x^n)))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3*e*n*x^
(3*n)) - (2*b^3*(e*x)^(3*n)*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(b - Sqrt[-a
^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2*e*n*x^(2*n)) + (4*b*(e*x)^(3*n)*P
olyLog[2, -((a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2
+ b^2]*d^2*e*n*x^(2*n)) + (2*b^3*(e*x)^(3*n)*PolyLog[2, -((a*E^(I*(c + d*x^
n)))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2*e*n*x^(2*n)) - (
4*b*(e*x)^(3*n)*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2]))]
)/(a^2*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) - ((2*I)*b^3*(e*x)^(3*n)*PolyLog[3
, -((a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)
*d^3*e*n*x^(3*n)) + ((4*I)*b*(e*x)^(3*n)*PolyLog[3, -((a*E^(I*(c + d*x^n)))
/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3*e*n*x^(3*n)) + ((2*I)*
b^3*(e*x)^(3*n)*PolyLog[3, -((a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2]))]
)/(a^2*(-a^2 + b^2)^(3/2)*d^3*e*n*x^(3*n)) - ((4*I)*b*(e*x)^(3*n)*PolyLog[3
, -((a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d
^3*e*n*x^(3*n)) + (b^2*(e*x)^(3*n)*Sin[c + d*x^n])/(a*(a^2 - b^2)*d*e*n*x^n
*(b + a*Cos[c + d*x^n]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4293

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 4618

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2] + I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a + Rt[-a^2 + b^2,
 2] + I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\left(x^{-3n}(ex)^{3n}\right)\int \frac{x^{-1+3n}}{(a+b\sec(c+dx^n))^2}\,dx}{e}$$

$$= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x^2}{(a+b\sec(c+dx))^2}\,dx,x,x^n\right)}{en}$$

$$= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \left(\frac{x^2}{a^2}+\frac{b^2x^2}{a^2(b+a\cos(c+dx))^2}-\frac{2bx^2}{a^2(b+a\cos(c+dx))}\right)dx,x,x^n\right)}{en}$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{\left(2bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x^2}{b+a\cos(c+dx)}\,dx, x, x^n\right)}{a^2en} + \frac{\left(b^2x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x^2}{(b+a\cos(c+dx))^2}\,dx, x, x^n\right)}{a^2en}$$

$$= \frac{(ex)^{3n}}{3a^2en} + \frac{b^2x^{-n}(ex)^{3n}\sin(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cos(c+dx^n))} - \frac{\left(4bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, x^n\right)}{a^2en} - \frac{\left(b^3x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x^2}{b+a\cos(c+dx)}\,dx, x, x^n\right)}{a^2(a^2-b^2)\,en} - \frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x\sin(c+dx)}{b+a\cos(c+dx)}\,dx, x, x^n\right)}{a(a^2-b^2)\,den}$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{ib^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} + \frac{b^2x^{-n}(ex)^{3n}\sin(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cos(c+dx^n))} - \frac{\left(2b^3x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{a+2be^{i(c+dx)}+ae^{2i(c+dx)}}\,dx, x, x^n\right)}{a^2(a^2-b^2)\,en} - \frac{\left(4bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}\,en} + \frac{\left(4bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}\,en} - \frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib-\sqrt{a^2-b^2}+iae^{i(c+dx)}}\,dx, x, x^n\right)}{a(a^2-b^2)\,den} - \frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+\sqrt{a^2-b^2}+iae^{i(c+dx)}}\,dx, x, x^n\right)}{a(a^2-b^2)\,den}$$

$$
\begin{aligned}
&= \frac{(ex)^{3n}}{3a^2en} - \frac{ib^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en} \\
&\quad + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en} + \frac{2ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,den} \\
&\quad - \frac{2ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,den} + \frac{b^2x^{-n}(ex)^{3n}\sin(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cos(c+dx^n))} \\
&\quad - \frac{\left(2b^3x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int\frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,x^n\right)}{a(a^2-b^2)\sqrt{-a^2+b^2}\,en} \\
&\quad + \frac{\left(2b^3x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int\frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{i(c+dx)}}\,dx,x,x^n\right)}{a(a^2-b^2)\sqrt{-a^2+b^2}\,en} \\
&\quad - \frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{iae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)\,dx,x,x^n\right)}{a^2(a^2-b^2)\,d^2en} \\
&\quad - \frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{iae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)\,dx,x,x^n\right)}{a^2(a^2-b^2)\,d^2en} \\
&\quad - \frac{\left(4ibx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}\,den} \\
&\quad + \frac{\left(4ibx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}\,den}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(ex)^{3n}}{3a^2en} - \frac{ib^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en} \\
&+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en} - \frac{ib^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} \\
&+ \frac{2ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,den} + \frac{ib^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} \\
&- \frac{2ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,den} + \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^2en} \\
&- \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^2en} + \frac{b^2x^{-n}(ex)^{3n}\sin(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cos(c+dx^n))} \\
&+ \frac{(2ib^2x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{iax}{ib-\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+dx^n)}\right)}{a^2(a^2-b^2)\,d^3en} \\
&+ \frac{(2ib^2x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{iax}{ib+\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i(c+dx^n)}\right)}{a^2(a^2-b^2)\,d^3en} \\
&- \frac{(4bx^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, x^n\right)}{a^2\sqrt{-a^2+b^2}\,d^2en} \\
&+ \frac{(4bx^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, x^n\right)}{a^2\sqrt{-a^2+b^2}\,d^2en} \\
&- \frac{(2ib^3x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}\,den} \\
&+ \frac{(2ib^3x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int x\log\left(1+\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}\,den}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(ex)^{3n}}{3a^2en} - \frac{ib^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en} \\
&+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en} - \frac{ib^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} \\
&+ \frac{2ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,den} + \frac{ib^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} \\
&- \frac{2ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,den} - \frac{2ib^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en} \\
&- \frac{2ib^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en} \\
&- \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} \\
&+ \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^2en} \\
&+ \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} \\
&- \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^2en} + \frac{b^2x^{-n}(ex)^{3n}\sin(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cos(c+dx^n))} \\
&+ \frac{(4ibx^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{a^2\sqrt{-a^2+b^2}\,d^3en} \\
&- \frac{(4ibx^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{a^2\sqrt{-a^2+b^2}\,d^3en} \\
&- \frac{(2b^3x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}\,d^2en} \\
&+ \frac{(2b^3x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}\,d^2en}
\end{aligned}
$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{ib^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en}$$

$$+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en} - \frac{ib^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den}$$

$$+ \frac{2ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{ib^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den}$$

$$- \frac{2ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{2ib^2x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en}$$

$$- \frac{2ib^2x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en}$$

$$- \frac{2b^3x^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en}$$

$$+ \frac{4bx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$

$$+ \frac{2b^3x^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en}$$

$$- \frac{4bx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$

$$+ \frac{4ibx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en}$$

$$- \frac{4ibx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en} + \frac{b^2x^{-n}(ex)^{3n}\sin(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cos(c+dx^n))}$$

$$+ \frac{(2ib^3x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}d^3en}$$

$$- \frac{(2ib^3x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}d^3en}$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{ib^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en}$$

$$+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en} - \frac{ib^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den}$$

$$+ \frac{2ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{ib^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den}$$

$$- \frac{2ibx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{2ib^2x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en}$$

$$- \frac{2ib^2x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+i\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en}$$

$$- \frac{2b^3x^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en}$$

$$+ \frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$

$$+ \frac{2b^3x^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en}$$

$$- \frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$

$$- \frac{2ib^3x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3en}$$

$$+ \frac{4ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en}$$

$$+ \frac{2ib^3x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3en}$$

$$- \frac{4ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en} + \frac{b^2x^{-n}(ex)^{3n}\sin(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cos(c+dx^n))}$$

Mathematica [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\sec(c+dx^n))^2}\,dx=\int \frac{(ex)^{-1+3n}}{(a+b\sec(c+dx^n))^2}\,dx$$

```
[In] Integrate[(e*x)^(-1 + 3*n)/(a + b*Sec[c + d*x^n])^2,x]
[Out] Integrate[(e*x)^(-1 + 3*n)/(a + b*Sec[c + d*x^n])^2, x]
```

Maple [F]

$$\int \frac{(ex)^{3n-1}}{(a+b\sec(c+d\,x^n))^2}dx$$

```
[In] int((e*x)^(3*n-1)/(a+b*sec(c+d*x^n))^2,x)
[Out] int((e*x)^(3*n-1)/(a+b*sec(c+d*x^n))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3831 vs. 2(1282) = 2564.

Time = 0.68 (sec) , antiderivative size = 3831, normalized size of antiderivative = 2.77

$$\int \frac{(ex)^{-1+3n}}{(a+b\sec(c+dx^n))^2}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n))^2,x, algorithm="fricas")
[Out] 1/6*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d^3*e^(3*n - 1)*x^(3*n)*cos(d*x^n + c) + 2
*(a^4*b - 2*a^2*b^3 + b^5)*d^3*e^(3*n - 1)*x^(3*n) + 6*(a^3*b^2 - a*b^4)*d^
2*e^(3*n - 1)*x^(2*n)*sin(d*x^n + c) - 6*((2*a^3*b^2 - a*b^4)*d*e^(3*n - 1)
*x^n*sqrt(-(a^2 - b^2)/a^2) + (-I*a^2*b^3 + I*b^5)*e^(3*n - 1) + ((2*a^4*b
- a^2*b^3)*d*e^(3*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) + (-I*a^3*b^2 + I*a*b^4
)*e^(3*n - 1))*cos(d*x^n + c))*dilog(-((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d
*x^n + c) - (I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) + a)/a + 1) -
 6*((2*a^3*b^2 - a*b^4)*d*e^(3*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) + (I*a^2*b
^3 - I*b^5)*e^(3*n - 1) + ((2*a^4*b - a^2*b^3)*d*e^(3*n - 1)*x^n*sqrt(-(a^2
 - b^2)/a^2) + (I*a^3*b^2 - I*a*b^4)*e^(3*n - 1))*cos(d*x^n + c))*dilog(-((
a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (-I*a*sqrt(-(a^2 - b^2)/a^2)
 - I*b)*sin(d*x^n + c) + a)/a + 1) + 6*((2*a^3*b^2 - a*b^4)*d*e^(3*n - 1)*x
^n*sqrt(-(a^2 - b^2)/a^2) - (I*a^2*b^3 - I*b^5)*e^(3*n - 1) + ((2*a^4*b - a
^2*b^3)*d*e^(3*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) - (I*a^3*b^2 - I*a*b^4)*e^
(3*n - 1))*cos(d*x^n + c))*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n
```

```
+ c) + (I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c) - a)/a + 1) + 6*((
2*a^3*b^2 - a*b^4)*d*e^(3*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) - (-I*a^2*b^3 +
 I*b^5)*e^(3*n - 1) + ((2*a^4*b - a^2*b^3)*d*e^(3*n - 1)*x^n*sqrt(-(a^2 - b
^2)/a^2) - (-I*a^3*b^2 + I*a*b^4)*e^(3*n - 1))*cos(d*x^n + c))*dilog(((a*sq
rt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) + (-I*a*sqrt(-(a^2 - b^2)/a^2) + I
*b)*sin(d*x^n + c) - a)/a + 1) - 3*((-I*(2*a^4*b - a^2*b^3)*c^2*sqrt(-(a^2
- b^2)/a^2) + 2*(a^3*b^2 - a*b^4)*c)*e^(3*n - 1)*cos(d*x^n + c) + (-I*(2*a^
3*b^2 - a*b^4)*c^2*sqrt(-(a^2 - b^2)/a^2) + 2*(a^2*b^3 - b^5)*c)*e^(3*n - 1
))*log(2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^
2) + 2*b) - 3*((I*(2*a^4*b - a^2*b^3)*c^2*sqrt(-(a^2 - b^2)/a^2) + 2*(a^3*b
^2 - a*b^4)*c)*e^(3*n - 1)*cos(d*x^n + c) + (I*(2*a^3*b^2 - a*b^4)*c^2*sqrt
(-(a^2 - b^2)/a^2) + 2*(a^2*b^3 - b^5)*c)*e^(3*n - 1))*log(2*a*cos(d*x^n +
c) - 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - 3*((-I*(2*a
^4*b - a^2*b^3)*c^2*sqrt(-(a^2 - b^2)/a^2) + 2*(a^3*b^2 - a*b^4)*c)*e^(3*n
- 1)*cos(d*x^n + c) + (-I*(2*a^3*b^2 - a*b^4)*c^2*sqrt(-(a^2 - b^2)/a^2) +
2*(a^2*b^3 - b^5)*c)*e^(3*n - 1))*log(-2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n
 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) - 3*((I*(2*a^4*b - a^2*b^3)*c^2*s
qrt(-(a^2 - b^2)/a^2) + 2*(a^3*b^2 - a*b^4)*c)*e^(3*n - 1)*cos(d*x^n + c) +
 (I*(2*a^3*b^2 - a*b^4)*c^2*sqrt(-(a^2 - b^2)/a^2) + 2*(a^2*b^3 - b^5)*c)*e
^(3*n - 1))*log(-2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2
 - b^2)/a^2) - 2*b) - 3*(-I*(2*a^3*b^2 - a*b^4)*d^2*e^(3*n - 1)*x^(2*n)*sqr
t(-(a^2 - b^2)/a^2) - 2*(a^2*b^3 - b^5)*d*e^(3*n - 1)*x^n + (I*(2*a^3*b^2 -
 a*b^4)*c^2*sqrt(-(a^2 - b^2)/a^2) - 2*(a^2*b^3 - b^5)*c)*e^(3*n - 1) + (-I
*(2*a^4*b - a^2*b^3)*d^2*e^(3*n - 1)*x^(2*n)*sqrt(-(a^2 - b^2)/a^2) - 2*(a^
3*b^2 - a*b^4)*d*e^(3*n - 1)*x^n + (I*(2*a^4*b - a^2*b^3)*c^2*sqrt(-(a^2 -
b^2)/a^2) - 2*(a^3*b^2 - a*b^4)*c)*e^(3*n - 1))*cos(d*x^n + c))*log(((a*sqr
t(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (I*a*sqrt(-(a^2 - b^2)/a^2) + I*b
)*sin(d*x^n + c) + a)/a) - 3*(I*(2*a^3*b^2 - a*b^4)*d^2*e^(3*n - 1)*x^(2*n)
*sqrt(-(a^2 - b^2)/a^2) - 2*(a^2*b^3 - b^5)*d*e^(3*n - 1)*x^n + (-I*(2*a^3*
b^2 - a*b^4)*c^2*sqrt(-(a^2 - b^2)/a^2) - 2*(a^2*b^3 - b^5)*c)*e^(3*n - 1)
+ (I*(2*a^4*b - a^2*b^3)*d^2*e^(3*n - 1)*x^(2*n)*sqrt(-(a^2 - b^2)/a^2) - 2
*(a^3*b^2 - a*b^4)*d*e^(3*n - 1)*x^n + (-I*(2*a^4*b - a^2*b^3)*c^2*sqrt(-(a
^2 - b^2)/a^2) - 2*(a^3*b^2 - a*b^4)*c)*e^(3*n - 1))*cos(d*x^n + c))*log(((
a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (-I*a*sqrt(-(a^2 - b^2)/a^2)
 - I*b)*sin(d*x^n + c) + a)/a) - 3*(-I*(2*a^3*b^2 - a*b^4)*d^2*e^(3*n - 1)*
x^(2*n)*sqrt(-(a^2 - b^2)/a^2) - 2*(a^2*b^3 - b^5)*d*e^(3*n - 1)*x^n + (I*(
2*a^3*b^2 - a*b^4)*c^2*sqrt(-(a^2 - b^2)/a^2) - 2*(a^2*b^3 - b^5)*c)*e^(3*n
 - 1) + (-I*(2*a^4*b - a^2*b^3)*d^2*e^(3*n - 1)*x^(2*n)*sqrt(-(a^2 - b^2)/a
^2) - 2*(a^3*b^2 - a*b^4)*d*e^(3*n - 1)*x^n + (I*(2*a^4*b - a^2*b^3)*c^2*sq
rt(-(a^2 - b^2)/a^2) - 2*(a^3*b^2 - a*b^4)*c)*e^(3*n - 1))*cos(d*x^n + c))*
log(-((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) + (I*a*sqrt(-(a^2 - b^2
)/a^2) - I*b)*sin(d*x^n + c) - a)/a) - 3*(I*(2*a^3*b^2 - a*b^4)*d^2*e^(3*n
- 1)*x^(2*n)*sqrt(-(a^2 - b^2)/a^2) - 2*(a^2*b^3 - b^5)*d*e^(3*n - 1)*x^n +
 (-I*(2*a^3*b^2 - a*b^4)*c^2*sqrt(-(a^2 - b^2)/a^2) - 2*(a^2*b^3 - b^5)*c)*
e^(3*n - 1) + (I*(2*a^4*b - a^2*b^3)*d^2*e^(3*n - 1)*x^(2*n)*sqrt(-(a^2 - b
```

```
^2)/a^2) - 2*(a^3*b^2 - a*b^4)*d*e^(3*n - 1)*x^n + (-I*(2*a^4*b - a^2*b^3)*
c^2*sqrt(-(a^2 - b^2)/a^2) - 2*(a^3*b^2 - a*b^4)*c)*e^(3*n - 1))*cos(d*x^n
+ c))*log(-((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) + (-I*a*sqrt(-(a^
2 - b^2)/a^2) + I*b)*sin(d*x^n + c) - a)/a) - 6*((2*I*a^4*b - I*a^2*b^3)*e^
(3*n - 1)*sqrt(-(a^2 - b^2)/a^2)*cos(d*x^n + c) + (2*I*a^3*b^2 - I*a*b^4)*e
^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2))*polylog(3, -((a*sqrt(-(a^2 - b^2)/a^2) +
 b)*cos(d*x^n + c) + (I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c))/a)
- 6*((-2*I*a^4*b + I*a^2*b^3)*e^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2)*cos(d*x^n
+ c) + (-2*I*a^3*b^2 + I*a*b^4)*e^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2))*polylog
(3, -((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) + (-I*a*sqrt(-(a^2 - b^
2)/a^2) - I*b)*sin(d*x^n + c))/a) - 6*((2*I*a^4*b - I*a^2*b^3)*e^(3*n - 1)*
sqrt(-(a^2 - b^2)/a^2)*cos(d*x^n + c) + (2*I*a^3*b^2 - I*a*b^4)*e^(3*n - 1)
*sqrt(-(a^2 - b^2)/a^2))*polylog(3, ((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x
^n + c) - (I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c))/a) - 6*((-2*I*
a^4*b + I*a^2*b^3)*e^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2)*cos(d*x^n + c) + (-2*
I*a^3*b^2 + I*a*b^4)*e^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2))*polylog(3, ((a*sqr
t(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) - (-I*a*sqrt(-(a^2 - b^2)/a^2) + I*
b)*sin(d*x^n + c))/a))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d^3*n*cos(d*x^n + c) +
(a^6*b - 2*a^4*b^3 + a^2*b^5)*d^3*n)
```

Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\sec(c+dx^n))^2}\,dx = \int \frac{(ex)^{3n-1}}{(a+b\sec(c+dx^n))^2}\,dx$$

```
[In] integrate((e*x)**(-1+3*n)/(a+b*sec(c+d*x**n))**2,x)
```

```
[Out] Integral((e*x)**(3*n - 1)/(a + b*sec(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\sec(c+dx^n))^2}\,dx = \int \frac{(ex)^{3n-1}}{(b\sec(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(6*a*b^3*e^(3*n)*x^(2*n)*sin(d*x^n + c) + (a^4 - a^2*b^2)*d*e^(3*n)*x^(
3*n)*cos(2*d*x^n + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*e^(3*n)*x^(3*n)*cos(d*x^n +
 c)^2 + (a^4 - a^2*b^2)*d*e^(3*n)*x^(3*n)*sin(2*d*x^n + 2*c)^2 + 4*(a^2*b^2
 - b^4)*d*e^(3*n)*x^(3*n)*sin(d*x^n + c)^2 + 4*(a^3*b - a*b^3)*d*e^(3*n)*x^
(3*n)*cos(d*x^n + c) + (a^4 - a^2*b^2)*d*e^(3*n)*x^(3*n) - 2*(3*a*b^3*e^(3*
n)*x^(2*n)*sin(d*x^n + c) - 2*(a^3*b - a*b^3)*d*e^(3*n)*x^(3*n)*cos(d*x^n +
```

```
c) - (a^4 - a^2*b^2)*d*e^(3*n)*x^(3*n))*cos(2*d*x^n + 2*c) + 3*((a^6 - a^4
*b^2)*d*e*n*cos(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*cos(d*x^n +
c)^2 + (a^6 - a^4*b^2)*d*e*n*sin(2*d*x^n + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*e
*n*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 4*(a^4*b^2 - a^2*b^4)*d*e*n*sin(d*x^
n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n
 + 2*(2*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n)*cos
(2*d*x^n + 2*c))*integrate(2*(2*a^2*b^4*e^(3*n)*x^(2*n)*cos(2*c)*sin(2*d*x^
n) + 2*a^2*b^4*e^(3*n)*x^(2*n)*cos(2*d*x^n)*sin(2*c) - 4*(a^3*b^3 - a*b^5)*
e^(3*n)*x^(2*n)*cos(c)*sin(d*x^n) - 4*(a^3*b^3 - a*b^5)*e^(3*n)*x^(2*n)*cos
(d*x^n)*sin(c) + (2*a^3*b^3*e^(3*n)*x^(2*n)*sin(d*x^n + c) - (2*a^5*b - a^3
*b^3)*d*e^(3*n)*x^(3*n)*cos(d*x^n + c))*cos(2*d*x^n + 2*c) - ((2*a^5*b - 3*
a^3*b^3 + a*b^5)*d*e^(3*n)*x^(3*n) - (2*a*b^5*e^(3*n)*x^(2*n)*sin(2*c) + (2
*a^3*b^3 - a*b^5)*d*e^(3*n)*x^(3*n)*cos(2*c))*cos(2*d*x^n) + 2*((2*a^4*b^2
- 3*a^2*b^4 + b^6)*d*e^(3*n)*x^(3*n)*cos(c) + 2*(a^2*b^4 - b^6)*e^(3*n)*x^(
2*n)*sin(c))*cos(d*x^n) - (2*a*b^5*e^(3*n)*x^(2*n)*cos(2*c) - (2*a^3*b^3 -
a*b^5)*d*e^(3*n)*x^(3*n)*sin(2*c))*sin(2*d*x^n) - 2*((2*a^4*b^2 - 3*a^2*b^4
 + b^6)*d*e^(3*n)*x^(3*n)*sin(c) - 2*(a^2*b^4 - b^6)*e^(3*n)*x^(2*n)*cos(c)
)*sin(d*x^n))*cos(d*x^n + c) - (2*a^3*b^3*e^(3*n)*x^(2*n)*cos(d*x^n + c) +
2*a^4*b^2*e^(3*n)*x^(2*n) + (2*a^5*b - a^3*b^3)*d*e^(3*n)*x^(3*n)*sin(d*x^n
 + c))*sin(2*d*x^n + 2*c) + (2*(a^3*b^3 - a*b^5)*e^(3*n)*x^(2*n) - (2*a*b^5
*e^(3*n)*x^(2*n)*cos(2*c) - (2*a^3*b^3 - a*b^5)*d*e^(3*n)*x^(3*n)*sin(2*c))
*cos(2*d*x^n) - 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*e^(3*n)*x^(3*n)*sin(c) -
 2*(a^2*b^4 - b^6)*e^(3*n)*x^(2*n)*cos(c))*cos(d*x^n) + (2*a*b^5*e^(3*n)*x^
(2*n)*sin(2*c) + (2*a^3*b^3 - a*b^5)*d*e^(3*n)*x^(3*n)*cos(2*c))*sin(2*d*x^
n) - 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*e^(3*n)*x^(3*n)*cos(c) + 2*(a^2*b^4
 - b^6)*e^(3*n)*x^(2*n)*sin(c))*sin(d*x^n))*sin(d*x^n + c))/(a^8*d*e*x*cos(
2*d*x^n + 2*c)^2 + a^8*d*e*x*sin(2*d*x^n + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a
^4*b^4*sin(2*c)^2)*d*e*x*cos(2*d*x^n)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6
)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*e*x*cos(d*x^n)^2 +
 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*e*x*cos(d*x^n)*cos(c) + (a^4*b^4*cos(2*c
)^2 + a^4*b^4*sin(2*c)^2)*d*e*x*sin(2*d*x^n)^2 + 4*((a^6*b^2 - 2*a^4*b^4 +
a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*e*x*sin(d*x
^n)^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*e*x*sin(d*x^n)*sin(c) + (a^8 - 2*
a^6*b^2 + a^4*b^4)*d*e*x - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5
*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*e*x*cos(d*x^n) + (a^6*b^2 - a^4*b^4)*d*e
*x*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*
cos(2*c)*sin(c))*d*e*x*sin(d*x^n))*cos(2*d*x^n) - 2*(a^6*b^2*d*e*x*cos(2*d*
x^n)*cos(2*c) - a^6*b^2*d*e*x*sin(2*d*x^n)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d
*e*x*cos(d*x^n)*cos(c) + 2*(a^7*b - a^5*b^3)*d*e*x*sin(d*x^n)*sin(c) - (a^8
 - a^6*b^2)*d*e*x)*cos(2*d*x^n + 2*c) + 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*si
n(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*e*x*cos(d*x^n) - 2*((a^5*b^
3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*e*x*s
in(d*x^n) + (a^6*b^2 - a^4*b^4)*d*e*x*sin(2*c))*sin(2*d*x^n) - 2*(a^6*b^2*d
*e*x*cos(2*c)*sin(2*d*x^n) + a^6*b^2*d*e*x*cos(2*d*x^n)*sin(2*c) - 2*(a^7*b
 - a^5*b^3)*d*e*x*cos(c)*sin(d*x^n) - 2*(a^7*b - a^5*b^3)*d*e*x*cos(d*x^n)*
```

```
sin(c))*sin(2*d*x^n + 2*c)), x) + 2*(3*a*b^3*e^(3*n)*x^(2*n)*cos(d*x^n + c)
 + 3*a^2*b^2*e^(3*n)*x^(2*n) + 2*(a^3*b - a*b^3)*d*e^(3*n)*x^(3*n)*sin(d*x^
n + c))*sin(2*d*x^n + 2*c))/((a^6 - a^4*b^2)*d*e*n*cos(2*d*x^n + 2*c)^2 + 4
*(a^4*b^2 - a^2*b^4)*d*e*n*cos(d*x^n + c)^2 + (a^6 - a^4*b^2)*d*e*n*sin(2*d
*x^n + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*sin(2*d*x^n + 2*c)*sin(d*x^n + c)
 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*sin(d*x^n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n
*cos(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n + 2*(2*(a^5*b - a^3*b^3)*d*e*n*cos(
d*x^n + c) + (a^6 - a^4*b^2)*d*e*n)*cos(2*d*x^n + 2*c))
```

Giac [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\sec(c+dx^n))^2}\,dx = \int \frac{(ex)^{3n-1}}{(b\sec(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((e*x)^(3*n - 1)/(b*sec(d*x^n + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{(a+b\sec(c+dx^n))^2}\,dx = \int \frac{(e\,x)^{3\,n-1}}{\left(a+\frac{b}{\cos(c+d\,x^n)}\right)^2}\,dx$$

```
[In] int((e*x)^(3*n - 1)/(a + b/cos(c + d*x^n))^2,x)
[Out] int((e*x)^(3*n - 1)/(a + b/cos(c + d*x^n))^2, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
      print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```